LOW-COMPLEXITY MULTIUSER DETECTION AND REDUCED-RANK WIENER FILTERS FOR ULTRA-WIDEBAND MULTIPLE ACCESS *

Zhi Tian [1], *Hongya Ge* [2], *Louis L. Scharf* [3]

[1] Department of ECE, Michigan Technological University, Houghton, MI 49931 USA
[2] Department of ECE, New Jersey Institute of Technology, Newark, NJ 07102 USA
[3] Department of ECE, Colorado State University, Fort Collins, CO 80523 USA

ABSTRACT

Realizing the large user capacity planned for ultra-wideband (UWB) systems motivates multiuser detection (MUD). However, it is impractical to implement conventional chip-rate MUD methods, because UWB signaling gives rise to high detection complexity and difficulty in capturing energy scattered by dense multipath. In this paper, we develop a reception model for UWB multiple access based on *frame-rate* sampled signals in lieu of chip-rate samples. This model enables low-complexity MUD, of which we examine a reduced-rank Wiener filter for blind symbol detection. We show that frame-rate UWB samples have a small number of *distinct* eigenvalues in the data covariance matrix, resulting in warp convergence of reduced-rank filtering. The proposed MUD method exhibits good performance at low complexity, even in the presence of strong frequency-selective multipath fading.

1. INTRODUCTION

Ultra wideband (UWB) technology is attracting growing interest as a means of wresting additional capacity from the already heavily utilized store of wireless bandwidth. Capacity studies purport that a binary impulse radio system can support an order of several thousands of active users per cell [1]. However, multiple access (MA) presents major challenges for UWB in terms of both technological capability and implementation complexity. Conventional detectors including RAKE receivers experience severe performance degradation in UWB-MA environments [2]. To enhance performance and throughput, multiuser detection (MUD) techniques are well motivated [3], which have been devised for time hopping UWB-MA [2, 4, 5], direct-sequence UWB-MA [6] and frequency-hopping UWB-MA [7].

The implementation of current MUD receivers, however, is limited by their complexity. A UWB impulse radio transmits every symbol over N_f frames with one ultra-short pulse per frame, where the frame duration is N_c ($\gg 1$) times the pulse (a.k.a. chip) duration. The complexity of a detector employing chip-rate sampled signal is determined by $N_c N_f$, which would be on the order of thousands due to the ultra-wide bandwidth, let alone the implementation difficulty in sampling and operating at multi-GHz chip rates. UWB signaling gives rise to highly frequency selective channels, which not only aggravate multiple access interference (MAI), but also impose great challenges in capturing sufficient energy scattered by dense multipath, in the presence of MAI. Because of these issues unique to UWB MA, chip-rate MUD methods developed for CDMA systems become either ineffective or impractical.

To effect MUD at practical complexity, this paper develops a reception model for UWB multiple access based on frame-rate sampled signals in lieu of chip-rate samples. The number of samples per symbol is reduced from $N_f N_c$ to N_f, which represents a significant reduction in the detection complexity order. Even at such a low sampling rate, there is no loss in multipath energy combining, as we adopt effective sampling strategy to capture all received multipath components without estimating chip-rate channel state information. Our frame-rate MA model is critical in enabling low-complexity MUD.

In particular, we investigate the application of reduced rank conjugate gradient (RRCG) Wiener filters for blind MUD in UWB systems. RRCG Wiener filters are useful for low-complexity adaptive implementation of systems for communication and array processing [8, 9]. Recent work shows warp convergence of RRCG for flat-faded CDMA systems properly designed with good spreading codes and (groupwise) power control [10]. However, it is not obvious how RRCG would work for chip-rate UWB signals, which have to deal with a large signal-space dimensionality and strong frequency-selective fading. Utilizing our frame-rate UWB-MA model, we show that a UWB signal experiencing frequency selective fading can be effectively treated as a flat-faded signal at the frame level, enabling blind MUD via RRCG. We show that frame-rate UWB samples have a small number of *distinct* eigenvalues in the data covariance matrix, resulting in warp convergence of reduced-rank MUD. Corroborating simulations will be provided.

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2. UWB SIGNALING

Consider a K-user UWB system. Each user conveys information through a stream of ultra-short pulses $p(t)$ of width T_p to occupy ultra-wide bandwidth B. Every symbol $s_k[n]$, $k \in [1, K]$, is transmitted by repeating over N_f frames one pulse per frame (of frame duration $T_f \gg T_p$) [1]. Each frame has N_c chips, with chip duration $T_c = T_f/N_c \approx T_p$. User separation is accomplished by user-specific spreading codes $\mathbf{c}_k := [c_k[0], \ldots, c_k[N_f-1]]^T$, which repeat from symbol to symbol and come with several flavors: (1) time-hopping (TH) UWB; (2) direct-sequence (DS) UWB; and (3) combined TH-DS UWB. For clarity, we focus on DS UWB with zero-mean pulse amplitude modulation (PAM) [4], while generalizations to other spreading and modulation schemes are possible. For the k-th user, the transmit symbol-waveform comprising N_f frames is given by $\bar{p}_k(t) := \sum_{i=0}^{N_f-1} c_k[i]p(t - iT_f)$ of duration $T_s := N_f T_f$.

Each user's signal propagates through a frequency selective multipath channel with impulse response $\sum_{l=0}^{L} \alpha_{k,l}\delta(t - \tau_{k,l})$ [11], where taps $\alpha_{k,l}$ and delays $\tau_{k,l}$ are assumed invariant over a block of symbols. Let $\tilde{\tau}_{k,l} := \tau_{k,l} - \tau_k$ to isolate the first-path delay $\tau_k := \tau_{k,0}$ which creates the timing offset between transmitter and receiver. The receive-pulse of the k-th user is thus given by $h_k(t) = \sum_{l=0}^{L} \alpha_{k,l} p(t - \tilde{\tau}_{k,l})$, while the corresponding receive symbol-waveform is $\bar{h}_k(t) = \sum_{i=0}^{N_f-1} c_k[i]h_k(t - iT_f)$. We select T_f to be larger than the channel delay spread $\tau_{k,L}$ to avoid inter-frame interference. With MA, the received signal is

$$r(t) = \sum_{k=1}^{K}\sum_{n=0}^{\infty} \sqrt{\mathcal{E}_k} s_k[n]\bar{h}_k(t - nT_s - \tau_k) + \zeta(t), \quad (1)$$

where $\mathcal{E}_k$ is the kth user's transmission energy, and $\zeta(t)$ is AWGN with power spectral density σ_ζ^2.

3. FRAME-RATE RECEPTION MODEL

A conventional UWB receiver employs $p(t)$ as the matched filter to generate chip-rate samples. A chip-rate symbol signature vector is of length $N_f N_c$, and has a large number of zero elements due to the low-duty-cycle transmission structure. To bypass the implementation and computational difficulty in chip-rate processing, we present a new reception approach that yields frame-rate samples, thus reducing the length of a symbol signature vector to N_f.

Suppose that user 1 is the desired user, for whom synchronization has been accomplished. The receiver employs a frame-rate correlator with correlation template $g_1(t)$ to generate sampled data. The frame-rate correlator output is

$$y[i] = \int_{iT_f}^{(i+1)T_f} r(t) g_1(t - iT_f)dt. \quad (2)$$

For each symbol, the data is collected into the $N_f \times 1$ vector $\mathbf{y}[n] := [y[nN_f], \ldots, y[nN_f + N_f - 1]]^T$, which has the form

$$\mathbf{y}[n] = \sqrt{\mathcal{E}_1}\mathbf{h}_1 s_1[n] + \sum_{k=2}^{K} \sqrt{\mathcal{E}_k} s_k[n]\mathbf{h}_k + \boldsymbol{\zeta}[n], \quad (3)$$

where $\mathbf{h}_k$ is the k-th user's temporal signature vector and $\boldsymbol{\zeta}[n]$ is a zero-mean Gaussian random noise vector with covariance $\sigma_\zeta^2\mathbf{I}$. With perfect timing, the temporal signature vector of the desired user is given by $\mathbf{h}_1 = \alpha_1\mathbf{c}_1$, where $\alpha_1 := \int_0^{T_f} g_1(t)h_1(t)dt$ reflects the energy capture capability of the template $g_1(t)$. Interfering users do not need to synchronized, but we focus on the synchronous case for clarity, which yields $\mathbf{h}_k = \alpha_k\mathbf{c}_k$, where $\alpha_k := \int_0^{T_f} g_1(t)h_k(t)dt$. In all, (3) can be equivalently written as

$$\mathbf{y}[n] = \mathbf{CAs}[n] + \boldsymbol{\zeta}[n] \quad (4)$$

where $\mathbf{A} := \text{diag}\{\sqrt{\mathcal{E}_1}\alpha_1, \ldots, \sqrt{\mathcal{E}_K}\alpha_K\}$ depends on both user amplitudes and energy capture gains, $\mathbf{C} := [\mathbf{c}_1 \ldots \mathbf{c}_K]$ is the $N_f \times K$ code matrix, and $\mathbf{s}[n] := [s_1[n], \ldots, s_K[n]]^T$ contains independent (binary) PAM symbols from all users.

Different from DS CDMA receiver designs where chip-rate sampling is required, (4) enables frame-rate blind MUD for DS UWB multiple access at practical complexity. Equally important, $\mathbf{h}_1$ is proportional to its spreading code vector even under frequency-selective multipath. Thus, the frame-rate model (4) resembles a flat-faded signal model.

Some remarks are in order to address how $g_1(t)$ can be constructed. Inspection of (1) reveals that the optimal frame-rate correlation template for user 1 is $h_1(t)$, which would attain the maximum energy capture gain α_1. To select $g_1(t)$ to be close to $h_1(t)$, we resort to channel estimates that may be available as by-products during training-based timing synchronization, which is inevitable for coherent reception of the desired user. Several choices are:

- RAKE template: both ML and subspace algorithms can be used to estimate the channel impulse response parameters $\{\alpha_{1,l}, \tilde{\tau}_{1,l}\}$ [12]. Subject to implementation constraints, $L_c (\leq L)$ RAKE fingers are combined, yielding

$$g_1(t) = \sum_{l=0}^{L_c-1} \hat{\alpha}_{1,l} p(t - \hat{\tilde{\tau}}_{1,l}). \quad (5)$$

  Albeit its popularity, the RAKE template may not be effective for UWB when $L_c \ll L$ [12, 13, 14]. It nevertheless can be used when $\{\hat{\alpha}_{1,l}, \hat{\tilde{\tau}}_{1,l}\}$ are available to the receiver for other purposes, e.g., synchronization [12].

- Noisy template: using the low-complexity timing algorithm in [13] with N training symbols, a by-product is a noisy analog waveform $\hat{h}_1(t)$ in the form

$$\begin{aligned}\hat{h}_1(t) &= \frac{1}{NN_f}\sum_{i=0}^{NN_f-1} c_1[i \bmod N_f]\, r(t + iT_f) \\ &\propto h_1(t) + \xi(t), \end{aligned} \quad (6)$$

where $\xi(t)$ is zero-mean Gaussian with PSD $\sigma_\xi^2 = \sigma_\zeta^2/(NN_f\mathcal{E}_1)$. Selecting $g_1(t) = \hat{h}_1(t)$ attains asymptotically (in N) optimal energy capture. Even for a small N, σ_ξ^2 can be low when N_f is large. Estimation of $\hat{h}_1(t)$ requires only analog delay and sum units operating at *frame or symbol rates* to perform waveform averaging over segments of $r(t)$, and can be effectively implemented *blindly* in multiple access environments [14].

4. REDUCED RANK WIENER FILTER

The data structure in (4) permits application of a variety of MUD algorithms [3]. It is well known that the linear MMSE MUD, also known as the Wiener filter, is optimum among all linear detectors, in terms of maximizing the output signal to interference and noise ratio (SINR). The MMSE filter for the desired user is given by

$$\mathbf{w} = \mathbf{R}_{yy}^{-1}\mathbf{c}_1, \tag{7}$$

where $\mathbf{R}_{yy} := \mathrm{E}\{\mathbf{y}\mathbf{y}^T\} = \mathbf{C}\mathbf{A}^2\mathbf{C}^T + \sigma_w^2\mathbf{I}$ is the data covariance matrix. The decision rule for detecting $s_1[n]$ is then made by $\hat{s}_1[n] = \mathrm{sgn}\{\mathbf{w}^T\mathbf{y}\}$. Motivated by low-complexity implementation, we focus on the vector conjugate gradient (V-CG) Wiener filter, which is a reduced-rank approximation of (7).

Using the V-CG method, we start with an initial value of the filter vector $\mathbf{w}_0 = \mathbf{c}_1$, along with an initial search direction $\mathbf{d}_0 = \mathbf{r}_0$ where $\mathbf{r}_0 = \mathbf{c}_1 - \mathbf{R}_{yy}\mathbf{w}_0$ is the initial residue vector. During refinement iterations, the V-CG method generates a rank-m approximation to the full-rank Wiener filter using the following adaptation steps:

$$\begin{aligned} \gamma_{m-1} &= ||\mathbf{r}_{m-1}||^2/(\mathbf{d}_{m-1}^T\mathbf{R}_{yy}\mathbf{d}_{m-1}), && \text{(8a)}\\ \mathbf{w}_m &= \mathbf{w}_{m-1} + \gamma_{m-1}\mathbf{d}_{m-1}, && \text{(8b)}\\ \mathbf{r}_m &= \mathbf{c}_1 - \mathbf{R}_{yy}\mathbf{w}_m, && \text{(8c)}\\ \mathbf{d}_m &= \mathbf{r}_m + \mathbf{d}_{m-1}(||\mathbf{r}_m||^2/||\mathbf{r}_{m-1}||^2). && \text{(8d)} \end{aligned}$$

It has been proven in [10] that the reduced-rank V-CG converges to the full-rank Wiener filter in at most M steps, where $M \le K$ is the number of *distinct* eigenvalues of $\mathbf{R}_{yy}$. Using strategies such as power control and good spreading code designs [9], a multiple access system can have $M \ll K$, leading to *warp convergence* of the V-CG Wiener filter.

To demonstrate the warp convergence of the V-CG method for UWB-MA, we consider a power-controlled DS UWB system with a set of K length-N_f Gold codes used as spreading codes for all active users. Under power control, $\mathcal{E}_1 = \mathcal{E}_k, \forall k$. However, the energy-capture gains $\{\alpha_k\}$ introduce additional power imbalances. Nevertheless, because channels of different users are independent, $\{\alpha_k\}_{k\neq 1}$ are typically very small and have approximately the same value level with respect to α_1. Let us assume $\alpha_k = \alpha$ for any $k \neq 1$, where α is much smaller than the energy capture gain α_1 of the desired user. When a good set of Gold codes is chosen, the Gram matrix of the signal mode matrix is

$$\begin{aligned} \mathbf{G} := (\mathbf{CA})^T\mathbf{CA} &= \frac{N_f+1}{N_f}\mathcal{E}_1\alpha^2\mathbf{I} - \frac{\mathcal{E}_1\alpha^2}{N_f}\mathbf{1}\mathbf{1}^T \\ &+ \mathcal{E}_1\begin{bmatrix} \beta_1 & \beta & \cdots & \beta \\ \beta & 0 & \cdots & 0 \\ \vdots & \vdots & 0 & \vdots \\ \beta & 0 & \cdots & 0 \end{bmatrix}, \end{aligned} \tag{9}$$

where $\beta_1 = \alpha_1^2 - \alpha^2$, $\beta = (\alpha_1 - \alpha)\alpha/N_f$, and $\mathbf{1}$ stands for a $K \times 1$ vector of all ones. The second term (with $\mathbf{1}\mathbf{1}^T$) on the right hand side of (9) is rank one, while the third term is rank 2. As a result, the V-CG method is expected to converge in 3 steps.

The analysis in (9) is based on a clean template $\hat{h}(t)$ constructed via either (5) or (6) with $\xi(t) = 0$. Even when employing the practical noisy template in (6) ($\xi(t) \neq 0$) that we recommend for multipath energy capture, we can prove that the warp convergence condition still holds. Indeed, it can be deduced that the Gram matrix $\tilde{\mathbf{G}}$ under noisy template is related to $\mathbf{G}$ in the clean-template case by

$$\tilde{\mathbf{G}} = \mathbf{G} + \sigma_\xi^2\left(\alpha_1^2 - \alpha^2/N_f - \alpha_1\alpha/N_f + \sigma_w^2 BT_f\right)\mathbf{I}. \tag{10}$$

The number of distinctive eigenvalues of $\tilde{\mathbf{G}}$ remains the same as that of $\mathbf{G}$.

5. SIMULATIONS

Computer simulations are carried out for a $K = 15$ UWB MA system operating in a typical indoor environment. In all test cases, the transmission parameters are selected as $T_p = T_c = 1$ns, $T_f = 100$ns and $N_f = 31$. Random channel realizations are generated according to the CM1 model from the IEEE 802.15.3a working group [15], with channel parameters $\Gamma = 30$ ns, $\gamma = 5$ns, $1/\Lambda = 2$ns and $1/\lambda = 0.5$ns. All 15 users employ distinct Gold codes with length N_f. The correlation template of the desired user is generated by averaging the received waveform of $N = 5$ training symbols, as a by-product during synchronization [13].

In Fig. 1, all users have the same transmission SNR ($= \mathcal{E}/\sigma_\zeta^2$). Predicted by (9), the V-CG method is expected to converge in 3 steps, which is verified by simulations. The output SINR converges to the steady state in 3 iterations (Fig. 1(a)), and the BER vs. SNR curve at the 4th iteration overlaps with that of the full-rank MMSE filter (Fig. 1(b)).

Fig. 2 illustrates the near-far resistance (NFR) of Wiener filters. All 14 interfering users are power-controlled to have $\mathrm{SNR}_k = \mathrm{SNR}_1 + \mathrm{NFR}, \forall k \neq 1$, where NFR ranges from -20 to 20 dB. The V-CG MUD converges to full-rank MMSE in 4-5 steps, even under strong power imbalances.

6. CONCLUDING SUMMARY

MUD techniques are developed in this paper for UWB multiple access, with reception capability of sufficient energy capture in dense multipath. Bypassing chip-rate sampling, we establish a frame-rate reception model to effect MUD at practical complexity. We develop a reduced-rank conjugate gradient Wiener filter for frame-rate MUD in UWB MA, and demonstrate its warp convergence to the full-rank MMSE MUD.

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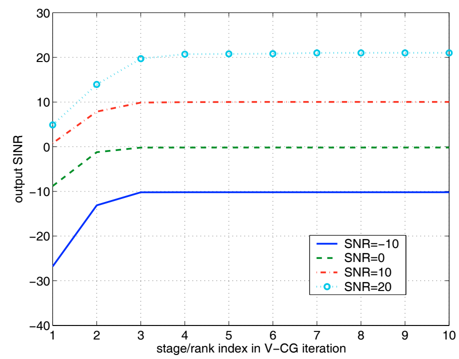
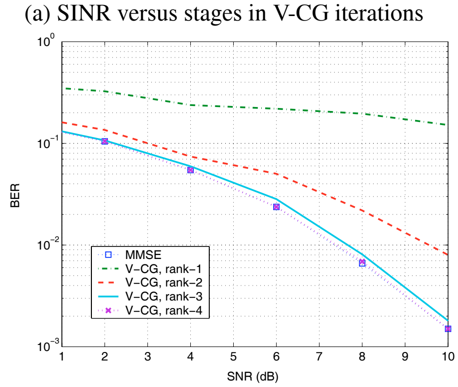
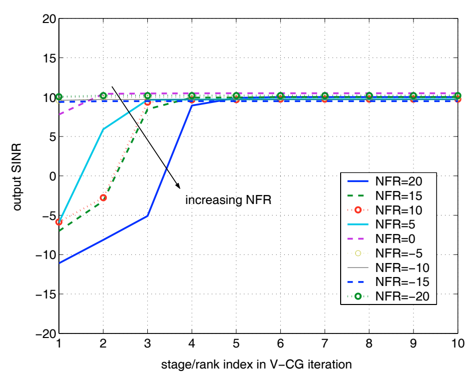


(a) SINR versus stages in V-CG iterations

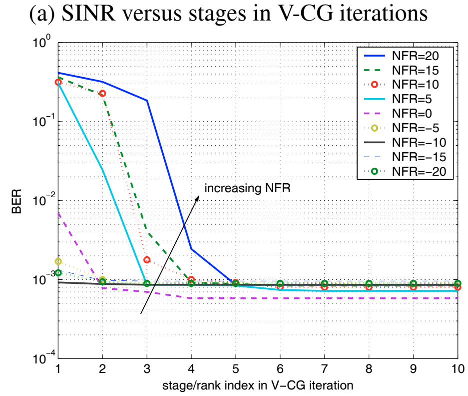


(b) BER versus SNR

Fig. 1. Performance of the V-CG MUD for equal-powered users.



(a) SINR versus stages in V-CG iterations



(b) BER versus stages in VCG iterations

Fig. 2. Performance of the V-CG MUD under power imbalances.