ANALYSIS OF A SYNCHRONIZATION ALGORITHM FOR NON-COHERENT UWB RECEIVERS

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ABSTRACT

Fast and low-complexity synchronization constitutes a big challenge for ultra-wideband (UWB) radio systems. Due to the ultra-fine time-resolvability of the UWB signal and the difficulty of channel estimation, it is difficult to achieve synchronization with accuracy on the order of tens of picoseconds, which is required by coherent UWB receivers. Non-coherent UWB receivers, however, have less stringent requirement for the synchronization accuracy [1]. Thus, synchronization algorithms for non-coherent UWB receivers can be developed to achieve synchronization with higher inaccuracy but much lower implementation complexity than coherent receivers. This paper analyzes the performance of a non-coherent synchronization algorithm proposed in [2]. Based on the analysis, parameter optimization to minimize the average acquisition time is pursued. Simulations corroborate our results.

1. INTRODUCTION

UWB radio transmits information through sub-nanosecond pulses with a low duty-cycle. Some popular signaling schemes have been proposed for UWB radio systems, such as the traditional scheme [3], the transmitted reference (TR) scheme [4], and the differential (DF) scheme [5]. At the receive end, either coherent or non-coherent receivers can be adopted, depending on whether or not the receiver has the channel state information (CSI). The main type of non-coherent UWB receiver is the autocorrelation receiver, which autocorrelates the received signal at a specific time lag, circumventing the problem of channel estimation. TR and DF schemes are two primary schemes in the UWB literature adopting autocorrelation receivers.

Three synchronization levels can be defined depending on the synchronization accuracy, which are symbol-level, frame-level, and pulse-level when the synchronization inaccuracy is on the order of one frame, one multipath delay-spread, and one pulse duration, respectively. Unlike the coherent receiver, non-coherent autocorrelation receivers do not need pulse-level synchronization (PLS) to align the received signal with the locally generated template signal. They use synchronization information mainly for deciding the integration region of the autocorrelator. Thus, non-coherent UWB receivers are robust to the synchronization inaccuracy as shown in [1], which is a very attractive feature due to the difficulty of achieving very accurate synchronization in UWB systems.

A frame-level synchronization (FLS) algorithm is developed in [2] for non-coherent UWB receivers. Based on autocorrelation detection, the algorithm requires no channel estimation, and can achieve synchronization with higher inaccuracy but much lower implementation complexity than synchronization algorithms for coherent detection. Similar ideas are also independently proposed in [6] and the patent [7].

However, some important problems are not yet addressed in the above documents. Due to the low-power nature of the UWB signal, the false alarm rate (FAR) of the algorithm is usually very high, which may greatly decrease the synchronization speed. Thus, analysis of the algorithm needs to be carried out to help develop a strategy to combat the high FAR. Secondly, synchronization performance of the algorithm in terms of the average acquisition time is not yet analyzed, which is needed for parameter setting to achieve the best synchronization performance. All the above problems will be addressed in this paper.

The paper is organized as follows. We will first introduce the UWB signaling scheme and non-coherent receivers in Section 2. Then we will describe and analyze the synchronization algorithm in Section 3. Simulation results will be given in Section 4, followed by the conclusion in Section 5.

2. SYSTEM MODEL

For simplicity but without loss of generality only the DF scheme will be discussed, noting that the algorithm can be applied to the TR scheme as well.

The transmitted signal of the DF scheme can be expressed as:

$$s\left(t\right)=\sum_{i=0}^{\infty}\sum_{j=0}^{N_f-1}L_i\sqrt{E_p}p\left(t-(iN_f+j)T_f\right), \qquad (1)$$

where N_f is the number of frames in one symbol, T_f is the frame duration, $\{L_i = \pm 1\}$ is the differentially encoded bit sequence, E_p is the energy of a single pulse, and $p(t)$ is the normalized pulse waveform with duration T_p. One symbol with duration $T_s = N_f T_f$ represents a bit in binary signalling. Notice that both pseudo random time hopping and direct sequence spreading codes can be applied to the scheme, but are ignored without loss of generality in this paper.

The channel can be described as a tapped-delay-line model as in the UWB literature: $h(t) = \sum_{l=0}^{N_m-1}\alpha_l\delta(t-\tau_l)$, where N_m is the number of resolvable paths and τ_l is the delay of path l. We define the effective delay spread as $T_m = \tau_{N_m-1} - \tau_0 + T_p$. By setting $T_f \gg T_m$ as in the UWB literature, the inter-frame and inter-symbol interferences can be ignored. The received signal can then be expressed as:

$$r\left(t\right)=\sum_{i=0}^{\infty}\sum_{j=0}^{N_f-1}L_i\sqrt{E_p}g\left(t-(iN_f+j)T_f\right)+n(t), \qquad (2)$$

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where $g(t) = p(t) * h(t)$, "$*$" denotes convolution, and $n(t)$ is the additive white Gaussian noise with zero mean and two-sided power spectral density $N_0/2$.

The received signal is passed through an ideal bandpass filter (BPF) with one-sided bandwidth W and center frequency f_0, where W is the bandwidth of the UWB signal. The resultant noise term $\hat{n}(t)$ is non-white Gaussian with an auto-correlation function: $R_{\hat{n}}(\tau) = WN_0 \frac{\sin(\pi W\tau)}{\pi W\tau} \cos(2\pi f_0 \tau)$.

The filtered signal is then delayed for T_s and correlated with itself. Since $T_f \gg T_m$, the effective signal region (SR) occupies only a small part of each frame. All the remaining parts where no signal exists at the output of the correlator are referred to as noise-only-regions (NORs). For non-coherent UWB receivers, the synchronization accuracy directly decides the amount of undesired NORs being included into the integration. When only symbol-level synchronization (SLS) is available, the receiver can not tell the SR from the NORs, and the integration length during each frame is as large as one frame duration. By the FLS algorithm in [2], the integrator can reduce the integration region during each frame to a length on the order of T_m, thus being able to exclude most of the NORs from the integration. In [1], BER performance of non-coherent DF receivers is analyzed for various synchronization levels. The result shows that with FLS information, the non-coherent UWB receiver can achieve a performance much better than the scenario when only SLS is available, and slightly worse than the scenario when PLS is achieved. This motivated the proposition of the FLS algorithm in [2], which will be discussed in detail in what follows.

3. FLS ALGORITHM AND ITS ANALYSIS

3.1. Algorithm Description

We now proceed to describe the algorithm in [2], which reduces the synchronization inaccuracy from symbol-level to frame-level.

The basic idea of the algorithm is to divide the original integration region (also referred to as the search region) during each frame into M sub-integration-windows (SIWs), and find out by comparing the autocorrelator outputs which SIW includes the desired SR. The selected SIW will be the new search region, and the algorithm goes to the next step to further divide it into M smaller SIWs, until an end condition is met. The end condition occurs when the integration region during each frame reaches a length less than a pre-set threshold T_L. Typically $T_L = T_m \sim 3T_m$.

Assuming that the integration length during each frame at step l is $T_w^{(l)}$, the k^{th} SIW will produce an autocorrelator output as:

$$G_{l,k} = \sum_{j=0}^{N_f-1} \int_{jT_f+A_{l,k}}^{jT_f+B_{l,k}} \hat{r}(t)\hat{r}(t-T_s)dt,\ k = 1, 2 \cdots M \quad (3)$$

where $[A_{l,k}, B_{l,k}]$ is the range of SIW k in the first frame at step l, $\hat{r}(t)$ is the received signal output from the BPF, and $T_w^{(l)} = B_{l,k} - A_{l,k}$ is the SIW length in one frame. We refer to the SIW completely containing the SR at each step as the "desired SIW". Since the noise at the integrator output has a zero mean [1], by comparing M integrator outputs, we can find the desired SIW and set it as the new search region for the next step. Note that herein the pilot bits are assumed to be all 1s.

Either serial or parallel search may be adopted for the SIW containing the SR. In serial search, only one integrator is employed at the receiver. Thus, each step the receiver requires M symbol durations to calculate the integrator outputs for the M SIWs. The parallel search, however, employs M integrators, which reduces the search time by a factor of M, but also leads to a higher receiver complexity. For simplicity we will only discuss the serial search in this paper.

To ensure that the SR lies completely in at least one of the SIWs, two neighboring SIWs need to have an overlapping area of a length T_m, as shown in Figure 1. Note that in the serial search, different SIWs correspond to different symbols. Thus, the so-called "overlapping area" only means the same located area in different symbols, so that $A_{l,k+1} - B_{l,k} = T_s - T_m$.

Notice that it is possible for the algorithm to falsely select an undesired SIW as the desired one due to noise, called a false alarm (FA). When a FA occurs, the algorithm needs to detect it, return to a previous step, and re-search the former region. The false alarm rate (FAR) is usually high due to the low power nature of the UWB signal, especially at the beginning of the algorithm when the SNR is low. A FA is highly undesirable. To reduce the FAR, one way is to repeat each step for a specific number of times and average the integrator outputs before comparison. We will analyze the FAR in the next subsection and choose the repetition times at each step to keep the FAR under a pre-set threshold.

Some modifications of the algorithm in [2] can be made to further decrease the FAR. Assume that the SR sits completely in SIW k, as shown in Figure 1. We note that when the SR is partially contained in a neighboring SIW, the probability of falsely choosing that neighboring SIW will be high. To reduce the FAR only by increasing the number of repetition times during each step will result in a much longer synchronization time, as will be shown in Section 4. Here we propose to expand the selected SIW for T_{ex} on both its sides before it enters the next step to be the new search region. For example, if the SIW length at step l is $T_w^{(l)}$, then the new search region at step $l+1$ will have a length of $T_w^{(l)} + 2T_{ex}$. Thus, when the SR is partially in a falsely selected SIW, it is possible for it to be completely included in the new SIW after expansion. Generally $T_{ex} \in [0, T_m]$, and the larger T_{ex} is, the more the FAR will be decreased.

Considering the overlapping and expanding effects of the SIWs, the minimum number of steps that the algorithm needs to take to reach the final integration length T_L, when there is no FA, can be derived as:

$$N = \left\lceil \log_M \frac{(M-1)(T_f - T_m) - 2MT_{ex}}{(M-1)(T_L - T_m) - 2T_{ex}} \right\rceil. \quad (4)$$

And the SIW length at step l can be expressed as:

$$T_w^{(l)} = T_m + \frac{1}{M}\left(2T_{ex} - T_m + T_w^{(l-1)}\right),\ l = 1, 2, \cdots N, \quad (5)$$

where $T_w^{(0)} = T_f - T_m$.

3.2. Average False Alarm Rate

Let us consider the SR in the first frame of symbol 0, which starts at τ_0 and ends at $\tau_0 + T_m$, as shown in Figure 1. Assume that the current step is step l, and the SR lies completely in SIW k. The range of SIW k can be assumed without loss of generality to be $[0, T_w^{(l)}]$. Denoting the probability of falsely choosing the m^{th} ($m \neq k$) SIW as the desired SIW as $P_{k\rightarrow m}^{(l)}$, the pairwise error probabilities can be derived in a similar way as the derivation of the BER performance of non-coherent UWB receivers in [1]:

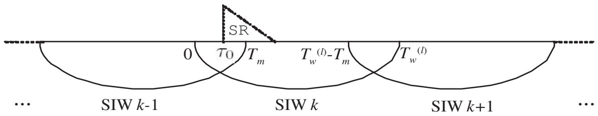

Fig. 1. SIWs and the SR

$$P_{k\to k-1}^{(l)} = Q\left(\sqrt{\frac{\frac{1}{2}R_l H_0(T_m,\tau)}{E_b[H_1(\tau_0,\tau)+H_1(\tau_0,T_m)]+N_f H_2(0,T_w^{(l)})}}\right) \quad (6)$$

$$P_{k\to k+1}^{(l)} = Q\left(\sqrt{\frac{\frac{1}{2}R_l H_0(\tau_0,T^{(l)})}{E_b[H_1(\tau_0,\tau)+H_1(T^{(l)},\tau)]+N_f H_2(0,T_w^{(l)})}}\right) \quad (7)$$

$$P_{k\to m}^{(l)} = Q\left(\sqrt{\frac{\frac{1}{2}R_l H_0(\tau_0,\tau)}{E_b H_1(\tau_0,\tau)+N_f H_2(0,T_w^{(l)})}}\right), \ |m-k|>1 \quad (8)$$

where $E_b = N_f E_p$, $\tau = \tau_0 + T_m$, $T^{(l)} = T_w^{(l)} - T_m$, R_l is the repetition times at step l, and

$$H_0(x,y) := \left[E_b \int_x^y g^2(t)dt\right]^2 \quad (9)$$

$$H_1(x,y) := \int_x^y \int_x^y g(t)g(\tau)R_{\hat{n}}(t-\tau)dtd\tau \quad (10)$$

$$H_2(x,y) := \int_x^y \int_x^y R_{\hat{n}}^2(t-\tau)dtd\tau. \quad (11)$$

Notice that the pairwise probabilities are functions of τ_0. Since the SR could be at any place within a frame with equal probability, it is reasonable to assume uniform distribution of τ_0 over $[0, T_w^{(l)} - T_m]$. Thus, $P_{k\to m}^{(l)}$ can be averaged over τ_0, and the union bound of the average FAR is given as:

$$U_{FAR}^{(l)} = \frac{1}{T^{(l)}}\int_0^{T^{(l)}}\left(\sum_{m=1,|m-k|>1}^{M} P_{k\to m}^{(l)}\right)d\tau_0 + \frac{1}{T^{(l)}}\left[\int_{T_{ex}}^{T^{(l)}} P_{k\to k-1}^{(l)}d\tau_0 + \int_0^{T^{(l)}-T_{ex}} P_{k\to k+1}^{(l)}d\tau_0\right] \quad (12)$$

Note that in (12), integration regions for $P_{k\to k\pm 1}^{(l)}$ are smaller than $[0, T_w^{(l)} - T_m]$. This is because some of the false alarms due to falsely choosing the neighboring SIW can be corrected by expanding the SIW for T_{ex} at both sides, and thus are not taken into consideration to calculate the average FAR. Using (6) to (12), we can choose the minimum R_l to ensure the FAR at each step to be under a pre-set threshold, say, 10^{-5}.

3.3. Flow Graph Description and Average Acquisition Time

The synchronization algorithm can be viewed as a Markov chain with a finite number of states, as described by the flow graph in Figure 2. There are altogether $(N+1)(N+2)/2$ states, among which $X_0, X_1, \cdots, X_N$ are referred to as right decision states, i.e., when the algorithm correctly chooses the desired SIW. All the other states ($X_{N+1} \sim X_{N(N+3)/2}$) are associated with FAs, and are referred to as "FA states". Each FA state has its way back to a right decision state by a FA detection strategy. It is also possible that a right decision state is falsely detected as a FA state, resulting in a return to the previous state, which is called a false false-alarm (FFA). A good FA detection strategy leads to a successful FA detection rate close to 1 and a FFA rate close to 0. The FA detection strategy will not be discussed in this paper due to the limit of space.

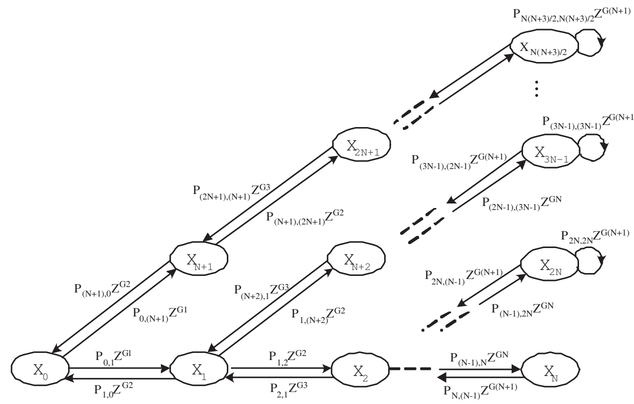

Fig. 2. Signal Flow Graph

In the graph, $P_{x,y}$ $(x, y = 1, 2\cdots, (N+1)(N+2)/2)$ are transition probabilities corresponding to branches $x \to y$, and Z^{G_l} $(l = 1, 2\cdots, N)$ are the corresponding weights with $G_l = R_l M$ indicating the number of symbol durations consumed during the transition. Notice that $P_{l,l-1}, l \in \{1, 2, \cdots, N\}$ is the FFA rate mentioned above, and $P_{l,l+N+1}, l \in \{1, 2, \cdots, N\}$ is the FAR discussed in Section 3.2.

Also, notice that a state always corresponds to a smaller SIW length than states on its left side, and states in the same column (aligned vertically in Figure 2) correspond to a same SIW length, which leads to the same R_l value according to (6) to (12).

The average acquisition time $\bar{T}$, which measures the average number of symbol durations needed for the algorithm to terminate, can be calculated by the flow graph theory [8]. The transfer function $H(Z)$ from X_0 to X_N can be calculated as a rational function of Z. Then, the average acquisition time is given by $\bar{T} = \left[\frac{d}{dZ}H(Z)\right]_{Z=1}$. As an example, we calculate the state transfer function for $N = 2$ as:

$$H(Z) = \frac{P_{0,1}P_{1,2}Z^{G_1+G_2}[A1 + A5]}{[A2 + A3][A1 + A4] - P_{0,1}P_{1,0}Z^{G_1+G_2}A1 \cdot A2}$$

where

$$A1 = 1 - P_{5,5}Z^{G_3} - P_{3,5}P_{5,3}Z^{G_2+G_3} \quad (13)$$

$$A2 = 1 - P_{4,4}Z^{G_3} \quad (14)$$

$$A3 = P_{1,2}P_{2,1}P_{4,4}Z^{G_2+2G_3} - (P_{1,2}P_{2,1}+P_{1,4}P_{4,1})Z^{G_2+G_3} \quad (15)$$

$$A4 = -P_{0,3}P_{3,0}Z^{G_1+G_2} + P_{0,3}P_{3,0}P_{5,5}Z^{G_1+G_2+G_3} \quad (16)$$

$$A5 = -P_{4,4}Z^{G_3} + P_{4,4}P_{5,5}Z^{2G_3} + P_{4,4}P_{3,5}P_{5,3}Z^{G_2+2G_3} \quad (17)$$

The result can be simplified by some practical assumptions. First of all, by a good FA detection strategy, we can assume that the successful FA detection rate is close to 1, i.e., $P_{3,0}, P_{5,3}, P_{4,1} \approx 1$ and $P_{3,5}, P_{5,5}, P_{4,4} \approx 0$. Secondly, since the FAR at each step is ensured to be lower than a pre-set low threshold (see Section 3.2), and the FFA rate can be kept low, the right decision probabilities, $P_{l,(l+1)}$ $(l = 0, 1, \cdots, N-1)$, can be approximated by 1. Thus, we can get an approximated value of $\bar{T}$ as:

$$\bar{T} = \left[\frac{d}{dZ}H(Z)\right]_{Z=1} \approx G_1 + G_2 \quad (18)$$

In fact, the result can be generalized to the N step scenario, as long as the above assumptions are satisfied. The average acquisition time for the N step scenario can be derived as:

$$\begin{aligned} \bar{T} &= \left[\frac{d}{dZ}H(Z)\right]_{Z=1} \approx G_1 + G_2 + \cdots + G_N \\ &= M(R_1 + R_2 + \cdots + R_N) \end{aligned} \tag{19}$$

Notice that even if some of the above assumptions are not satisfied, the exact average acquisition time can still be calculated using the state transfer function, which, though, has a very complicated form when N is large.

3.4. Parameter Setting

Two parameters need to be set in advance, which are T_{ex} and M. The other parameters such as N, R_l and $T_w^{(l)}$ $(l = 1, 2, \cdots, N)$ will be determined by them. We will see that for T_{ex}, a value close to T_m is preferred. While for M, its value needs to be selected to achieve the best synchronization performance, which is to minimize the average acquisition time $\bar{T}$.

Notice that when M decreases, N has to increase due to (4). At the same time, the SIW length $T_w^{(l)}$ during each step will increase according to (5). As a result, R_l has to increase to keep the FAR under the pre-set threshold by (6) to (12). Consequently, the average acquisition time $\bar{T}$, as a product of M and $\sum_{l=1}^{N} R_l$, may increase or decrease, depending on which factor contributes more to the product.

Thus, by varying M, we can find the minimum value of $\bar{T}$ and its corresponding M. The procedure can be concluded as follows:

1. Choose a value of M (starting from $M = 2$)
2. Calculate N and $T_w^{(l)}$ $(l = 1, \cdots, N)$ according to (4) and (5)
3. Choose the minimum values of R_l $(l = 1, 2, \cdots, N)$ according to Equations (6) to (12), to keep the FAR under a pre-set threshold, say, 10^{-5}
4. Calculate $\bar{T}$ according to (19)
5. Increase M and recalculate $\bar{T}$
6. Choose M corresponding to the minimum value of $\bar{T}$

4. SIMULATION RESULTS

A non-coherent DF receiver is simulated. Parameters are set as: $N_f = 10$, $T_f = 200$ns, $T_p = 0.7$ns, $T_m = 7$ns, $T_L = 2.5T_m$, $E_b/N_0 = 25$dB, and the FAR threshold equals 10^{-5}. Four T_{ex} values are simulated, and the results are shown in Figure 3. As we can see from the figure, a T_{ex} value close to T_m achieves the best performance. The minimum average acquisition time (in multiples of T_s) can be achieved by $T_{ex} \approx T_m$ and $M = 5$, and the algorithm can achieve FLS within only tens of symbol durations.

5. CONCLUSIONS

In this paper, an FLS algorithm proposed in [2] for non-coherent UWB receivers is analyzed. Some necessary modifications of the algorithm are made based on insight gained from the analysis. Expansion of the SIW combined with the repetition strategy to control the FAR is proposed. The union bound on the average FAR is derived. The average acquisition time $\bar{T}$ in multiples of T_s is calculated through flow graph theory. Finally, parameter setting to minimize $\bar{T}$ is pursued. Simulations are carried out to give a specific example of the optimization process, and show that $T_{ex} \approx T_m$ and a small value of M lead to the minimum average acquisition time. The algorithm is shown to achieve the FLS within only tens of symbol durations.

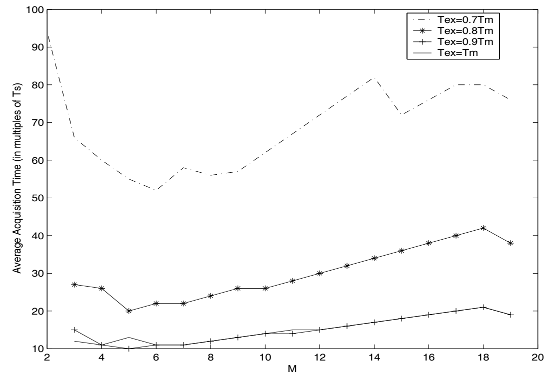

Fig. 3. M vs. $\bar{T}$

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