PERFORMANCE OF POR MULTIUSER DETECTION FOR UWB COMMUNICATIONS

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ABSTRACT

A power of R (POR) technique has been proposed to blindly estimate multipath parameters of the desired user in a multiple access ultra wideband (UWB) system. In this paper, we first analyze performance of the POR method in terms of both channel estimation mean-square-error (MSE) and receiver's output signal to interference plus noise ratio (SINR) and bit error rate (BER). Then we compare this method with both subspace and maximum likelihood methods based on practical UWB channels. Simulation results verify our analysis. They also show that the proposed POR method outperforms the subspace method for heavily loaded systems, and is superior to the maximum likelihood approach in all examined examples.

1. INTRODUCTION

Time-hopping (TH) ultra-wideband (UWB) modulation technology has attracted considerable research attention recently, due to its appealing features and recent release of the spectral mask from the Federal Communications Commission [1].

In a UWB system, a RAKE receiver is typically employed to detect information symbols. To fully capture signal energy spread over multiple paths, channel parameters are necessary to construct a RAKE receiver [2]. However, channel information is not known *a priori*, especially in a dense multipath wireless environment. Although maximum likelihood (ML) channel estimation methods with/without aid of training sequences [3], [4] have been proposed to blindly estimate channel, they approximate multiuser interference (MUI) as a Gaussian process, leading to degraded performance. Recently, blind multiuser detection schemes have been proposed for UWB communications [5], [6]. Among them, the POR technique [6], which approximates the noise subspace component from the power of the inversed data covariance matrix $\boldsymbol{R}^{-p}$ with p as a positive integer, has shown satisfactory performance in Gaussian channels [6].

In this paper, we first conduct performance analysis of the POR method. Channel estimation mean-square-error (MSE) is obtained. Signal to interference plus noise ratio (SINR) of a RAKE receiver is studied jointly with channel estimation. Those results are further used to predict detection performance. Then, we compare the POR method with subspace and ML methods for a set of realistic UWB channels. Simulation results show that the POR method is superior to the subspace method in a system with medium to heavy loading, and outperforms the ML method in all situations.

Throughout the paper, Kronecker product is denoted by $\otimes$, complex conjugate (*) transpose (^{T}) by ^{H}, inverse by $^{-1}$, pseudo-inverse by †, trace of a matrix by $tr()$, determinant by $det()$. $Re\{\cdot\}$ represents real part, $E\{\cdot\}$ expectation, $\boldsymbol{I}_a$ an identity matrix of degree a whose ith column is denoted by $\boldsymbol{e}_{a,i}$. $\mathbf{1}_a$ is a vector of length a with all elements equal to one. Integer floor is denoted as $\lfloor\ \rfloor$. A Q function is defined as $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{\frac{-t^2}{2}}dt$.

Prepared through collaborative participation in the Communications and Networks Consortium sponsored by the U. S. Army Research Laboratory under the Collaborative Technology Alliance Program, Cooperative Agreement DAAD19-01-2-0011. The U. S. Government is authorized to reproduce and distribute reprints for Government purposes notwithstanding any copyright notation thereon.

2. POR BASED CHANNEL ESTIMATION AND SYMBOL DETECTION

2.1. Discrete-Time UWB System Model

Let us consider an M-ary, K-user PPM TH-UWB system. Each user's information bit $I_k(n)$ is transmitted over N_f frames. Suppose each frame has N_c chips. If each chip duration can accommodate M pulses, a discrete-time UWB system sampled at a pulse rate has been derived in [5], [6] following the work of [7],

$$\boldsymbol{y}_n = \sum_{k,m,l} \boldsymbol{C}_{k,m,l}\boldsymbol{g}_k s_{k,m}(n+l) + \boldsymbol{v}_n \quad (1)$$

where for user k we have defined $s_{k,m}(n+l) = \delta\Big(I_k(n+l) - m\Big)$ with $m = 0, \ldots, M-1$ as its M virtual inputs at delay l, $\boldsymbol{C}_{k,m,l}$ constructed from user k's TH codes as a code filtering matrix for its mth virtual input at delay l, and $\boldsymbol{g}_k$ as its unknown channel vector containing channel coefficients at the pulse rate and power factor. Eq. (1) resembles a model very similar to a multi-code multirate CDMA system [8]. It can be further expressed in a compact form

$$\boldsymbol{y}_n = \sum_{k,l} \boldsymbol{H}_{k,l}\boldsymbol{s}_{k,n,l} + \boldsymbol{v}_n = \boldsymbol{H}\boldsymbol{s}_n + \boldsymbol{v}_n \quad (2)$$

where $\boldsymbol{s}_{k,n,l} = [s_{k,0}(n+l), \cdots, s_{k,M-1}(n+l)]^T$ and

$$\boldsymbol{H}_{k,l} = [\boldsymbol{C}_{k,0,l}\boldsymbol{g}_k, \cdots, \boldsymbol{C}_{k,M-1,l}\boldsymbol{g}_k]. \quad (3)$$

2.2. Blind Channel Estimation and Symbol Detection

The POR method requires covariance of $\boldsymbol{y}_n$ [6]. For this goal, we first obtain zero-mean data $\boldsymbol{z}_n = \boldsymbol{y}_n - E\{\boldsymbol{y}_n\}$ [6] as the following, after applying (1) and the definition of $\boldsymbol{s}_{k,m}(n+l)$

$$\boldsymbol{z}_n = \sum_{k,l} [\boldsymbol{C}_{k,0,l}\boldsymbol{g}_k, \ldots, \boldsymbol{C}_{k,M-1,l}\boldsymbol{g}_k]\boldsymbol{a}_{k,n,l} + \boldsymbol{v}_n \quad (4)$$

where $\boldsymbol{a}_{k,n,l} = \boldsymbol{s}_{k,n,l} - \frac{1}{M}\mathbf{1}_M$. The vector $\boldsymbol{a}_{k,n,l}$ has been shown to be correlated with covariance $\boldsymbol{\Phi} = \frac{1}{M}(\boldsymbol{I}_M - \frac{1}{M}\mathbf{1}_M\mathbf{1}_M^H)$ at the rank of $M-1$. To facilitate channel estimation, we decompose $\boldsymbol{\Phi}$ as $\boldsymbol{\Phi} = \boldsymbol{B}_a\boldsymbol{\Lambda}_a^2\boldsymbol{B}_a^H$ and define a new code matrix $\boldsymbol{S}_{k,j,l} = \sum_{i=1}^{M} b_{i,j}\widetilde{\lambda}_j\boldsymbol{C}_{k,i-1,l}$ for $j = 1,\ldots,M-1$ with $b_{i,j}$ and $\widetilde{\lambda}_j$ as the (i,j)th element of $\boldsymbol{B}_a$ and the jth diagonal element of $\boldsymbol{\Lambda}_a$. Consequently, the covariance of $\boldsymbol{z}_n$ can be shown to be

$$\boldsymbol{R} = \sum_{k,l,j}\boldsymbol{S}_{k,j,l}\boldsymbol{g}_k\boldsymbol{g}_k^H\boldsymbol{S}_{k,j,l}^H + \sigma_v^2\boldsymbol{I}_\nu \tag{5}$$

where σ_v^2 is the noise power, ν is the size of data vector. Express eigen-decomposition of $\boldsymbol{R}$ as

$$\boldsymbol{R} = \begin{bmatrix} \boldsymbol{U}_s & \boldsymbol{U}_n \end{bmatrix}\begin{bmatrix} \boldsymbol{\Lambda}_s + \sigma_v^2\boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \sigma_v^2\boldsymbol{I} \end{bmatrix}\begin{bmatrix} \boldsymbol{U}_s^H \\ \boldsymbol{U}_n^H \end{bmatrix}. \tag{6}$$

Suppose user 1 is the desired user and define $\mathcal{S}_j \triangleq \boldsymbol{S}_{1,j,0}$ for a simple notation. The POR based channel estimation method is obtained as

$$\boldsymbol{g}_{por} = arg \min_{||\boldsymbol{g}||=1}\sum_{j=1}^{M-1}\boldsymbol{g}^H\mathcal{S}_j^H\boldsymbol{R}^{-p}\mathcal{S}_j\boldsymbol{g} \tag{7}$$

where p is a positive integer. Solution to (7) is immediately obtained as the eigenvector of the matrix $\sum_j \mathcal{S}_j^H\boldsymbol{R}^{-p}\mathcal{S}_j$ corresponding to its minimum eigenvalue γ_{por} (for convenience, we call it minimum eigenvector).

Since the information symbol is reflected by the position of the only maximum value in $\boldsymbol{a}_{1,n,0}$, we need to design M receivers $\boldsymbol{f}_m$ $(m = 1,\cdots,M)$, each detecting one element in $\boldsymbol{a}_{1,n,0}$. Then symbol $I_1(n)$ is estimated from the index of the receiver yielding the maximum output

$$\widehat{I}_1(n) = \arg\max_{m\in\{1,\cdots,M\}} Re\{\boldsymbol{f}_m^H\boldsymbol{z}_n\} - 1.$$

According to (4), the mth RAKE receiver in detecting the corresponding element in $\boldsymbol{a}_{1,n,0}$ can be found to be $\boldsymbol{f}_m = \boldsymbol{C}_{1,m,0}\boldsymbol{g}_1$. Invoking (3), the M RAKE receivers can be shown to be $\boldsymbol{F}_{1,RAKE} = [\boldsymbol{f}_1,\ldots,\boldsymbol{f}_M] = \boldsymbol{H}_{1,0}$.

3. PERFORMANCE STUDY

3.1. Channel Estimation Performance

3.1.1. Asymptotic Performance

Let the data covariance matrix be estimated as

$$\widehat{\boldsymbol{R}} = \frac{1}{N}\sum_{n=1}^{N}(\boldsymbol{y}_n - \frac{1}{N}\sum\boldsymbol{y}_n)(\boldsymbol{y}_n - \frac{1}{N}\sum\boldsymbol{y}_n)^H. \tag{8}$$

According to (6), we have

$$\sigma_v^{2p}\boldsymbol{R}^{-p} = \boldsymbol{U}_n\boldsymbol{U}_n^H + \boldsymbol{U}_s diag\{(\frac{\sigma_v^2}{\lambda_i^2+\sigma_v^2})^p\}\boldsymbol{U}_s^H \tag{9}$$

Considering (9), it is straightforward to see that if either $\sigma_v^2 \to 0$ or $p \to \infty$, the POR method will yield a perfect channel estimate under some channel identifiability conditions imposed by the subspace method.

3.1.2. Noise Induced Channel Estimation Error

Let us define $\boldsymbol{A} \triangleq \sum_{j=1}^{M-1}\mathcal{S}_j^H\boldsymbol{R}^{-p}\mathcal{S}_j$. Applying (9), we can express $\boldsymbol{A}$ into two terms as $\boldsymbol{A} = \boldsymbol{A}_0 + \delta\boldsymbol{A}_0$ where $\boldsymbol{A}_0 \triangleq \sum_{j=1}^{M-1}\mathcal{S}_j^H\boldsymbol{U}_n\boldsymbol{U}_n^H\mathcal{S}_j$ and $\delta\boldsymbol{A}_0 \triangleq \sum_{j=1}^{M-1}\mathcal{S}_j^H\boldsymbol{U}_s diag\{(\frac{\sigma_v^2}{\lambda_i^2+\sigma_v^2})^p\}$ $\boldsymbol{U}_s^H\mathcal{S}_j$. Note that the minimum eigenvector of $\boldsymbol{A}_0$ is exactly the desired channel. Treating $\delta\boldsymbol{A}_0$ as a perturbation to $\boldsymbol{A}_0$, the perturbation to the minimum eigenvector of $\boldsymbol{A}_0$ which is the channel vector $\boldsymbol{g}_1$ is termed as the channel estimation error induced by noise. It is given by [10]

$$\delta\boldsymbol{g}_{noise} \approx -\boldsymbol{A}_0^\dagger\sum_{j=1}^{M-1}\mathcal{S}_j^H\boldsymbol{U}_s diag\{(\frac{\sigma_v^2}{\lambda_i^2+\sigma_v^2})^p\}\boldsymbol{U}_s^H\mathcal{S}_j\boldsymbol{g}_1.$$

Each element of the diagonal matrix in the above equation is a fractional number. Thus, good channel estimation performance can be achieved for sufficiently large p, irrespective of noise power. At high SNR, with the assumption that $\sigma_v^2 \ll \lambda_i$ for each i, the factional term $(\frac{\sigma_v^2}{\lambda_i^2+\sigma_v^2})^p$ can be expanded into a Taylor series of σ_v^2, resulting in the following simplified channel estimation error

$$\delta\boldsymbol{g}_{noise} \approx -\sigma_v^{2p}\boldsymbol{A}_0^\dagger\boldsymbol{A}_p\boldsymbol{g}_1 + \boldsymbol{O}(\sigma_v^{2p+2}). \tag{10}$$

This result implies that $\delta\boldsymbol{g}_1$ is at the order of $O(\sigma_v^{2p})$.

3.1.3. Perturbation Error from Finite Data Length

When data length N is finite, a perturbation $\delta\boldsymbol{R} = \widehat{\boldsymbol{R}} - \boldsymbol{R}$ occurs. It will cause $\boldsymbol{A}$ perturbed as $\delta\boldsymbol{A} = -\sum_{j=1}^{M-1}\mathcal{S}_j^H\sum_{k=1}^{p}\boldsymbol{R}^{-k}\delta\boldsymbol{R}$ $\boldsymbol{R}^{-(p-k)}\boldsymbol{R}^{-1}\mathcal{S}_j$. Due to $\delta\boldsymbol{A}$, $\boldsymbol{g}_{por}$ is perturbed as $\widetilde{\boldsymbol{g}}_{por}$, causing a perturbation error $\delta\boldsymbol{g}_N \triangleq \widetilde{\boldsymbol{g}}_{por} - \boldsymbol{g}_{por}$ as [10]

$$\delta\boldsymbol{g}_N \approx -(\boldsymbol{A} - \gamma_{por}\boldsymbol{I})^\dagger\delta\boldsymbol{A}\boldsymbol{g}_{por} \approx \sum_{j=1}^{M-1}\sum_{k=1}^{p}\boldsymbol{T}_{j,k}\delta\boldsymbol{R}\boldsymbol{t}_{j,k} \tag{11}$$

where $\boldsymbol{T}_{j,k}$ and $\boldsymbol{t}_{j,k}$ are deterministic quantities given by $\boldsymbol{T}_{j,k} = (\boldsymbol{A}-\gamma_{por}\boldsymbol{I})^\dagger\mathcal{S}_j^H\boldsymbol{R}^{-k}$ and $\boldsymbol{t}_{j,k} = \boldsymbol{R}^{-(p-k)}\boldsymbol{R}^{-1}\mathcal{S}_j\boldsymbol{g}_{por}$. Observe that $\delta\boldsymbol{g}_N$ is a random quantity due to randomness of $\delta\boldsymbol{R}$. Therefore the covariance of $\delta\boldsymbol{g}_N$ becomes

$$\mathrm{Cov}\boldsymbol{g}_N \approx \sum_{j=1}^{M-1}\sum_{k_1,k_2=1}^{p}\boldsymbol{T}_{j,k_1}E\{\delta\boldsymbol{R}\boldsymbol{t}_{j,k_1}\boldsymbol{t}_{j,k_2}^H\delta\boldsymbol{R}\}\boldsymbol{T}_{j,k_2}^H, \tag{12}$$

and the mean-square-error is equal to the trace of $\mathrm{Cov}\boldsymbol{g}_N$. To compute (12), it is sufficient to determine a general term $\boldsymbol{\Psi}(\boldsymbol{\Theta}) = E\{\delta\boldsymbol{R}\boldsymbol{\Theta}\delta\boldsymbol{R}\}$. Following similar steps in [9], one can verify that for a real system,

$$\begin{aligned}\boldsymbol{\Psi}(\boldsymbol{\Theta}) &= \frac{(N-1)^2}{N^3}\sum_{l=1}^{L}\frac{1}{M}\sum_{j=1}^{M}(\widetilde{\boldsymbol{h}}_{l,j}^T\boldsymbol{\Theta}\widetilde{\boldsymbol{h}}_{l,j})\widetilde{\boldsymbol{h}}_{l,j}\widetilde{\boldsymbol{h}}_{l,j}^T \\ &-\frac{(N-1)^2}{N^3}\sum_{l=1}^{L}tr(\boldsymbol{H}_l\boldsymbol{\Phi}\boldsymbol{H}_l^T\boldsymbol{\Theta})\boldsymbol{H}_l\boldsymbol{\Phi}\boldsymbol{H}_l^T \\ &-\frac{(N-1)^2}{N^3}\sum_{l=1}^{L}\boldsymbol{H}_l\boldsymbol{\Phi}\boldsymbol{H}_l^T(\boldsymbol{\Theta}+\boldsymbol{\Theta}^T)\boldsymbol{H}_l\boldsymbol{\Phi}\boldsymbol{H}_l^T \\ &+\frac{N-1}{N^2}[tr(\boldsymbol{R}\boldsymbol{\Theta})\boldsymbol{R} + \boldsymbol{R}\boldsymbol{\Theta}^T\boldsymbol{R}] + \frac{1}{N^2}\boldsymbol{R}\boldsymbol{\Theta}\boldsymbol{R}\end{aligned} \tag{13}$$

and for a complex system

$$\begin{aligned}\boldsymbol{\Psi}(\boldsymbol{\Theta}) &= \frac{(N-1)^2}{N^3}\sum_{l=1}^{L}\frac{1}{M}\sum_{j=1}^{M}(\widetilde{\boldsymbol{h}}_{l,j}^H\boldsymbol{\Theta}\widetilde{\boldsymbol{h}}_{l,j})\widetilde{\boldsymbol{h}}_{l,j}\widetilde{\boldsymbol{h}}_{l,j}^H \\ &-\frac{(N-1)^2}{N^3}\sum_{l=1}^{L}tr(\boldsymbol{H}_l\boldsymbol{\Phi}\boldsymbol{H}_l^H\boldsymbol{\Theta})\boldsymbol{H}_l\boldsymbol{\Phi}\boldsymbol{H}_l^H + \frac{1}{N^2}\boldsymbol{R}\boldsymbol{\Theta}\boldsymbol{R}\end{aligned}$$

$$-\frac{(N-1)^2}{N^3}\sum_{l=1}^{L}\boldsymbol{H}_l\boldsymbol{\Phi}[\boldsymbol{H}_l^H\boldsymbol{\Theta}\boldsymbol{H}_l+(\boldsymbol{H}_l^H\boldsymbol{\Theta}\boldsymbol{H}_l)^T]\boldsymbol{\Phi}\boldsymbol{H}_l^H$$
$$+\frac{N-1}{N^2}[tr(\boldsymbol{R}\boldsymbol{\Theta})\boldsymbol{R}+\boldsymbol{H}\mathcal{A}(\boldsymbol{H}^H\boldsymbol{\Theta}\boldsymbol{H})^T\mathcal{A}\boldsymbol{H}^H] \quad (14)$$

where $\boldsymbol{H}$ in (2) is partitioned into L sub-blocks as $\boldsymbol{H}=[\boldsymbol{H}_1,\cdots,\boldsymbol{H}_L]$ with each sub-block corresponding to one symbol irrespective of user, $\widetilde{\boldsymbol{h}}_{l,j}=\boldsymbol{H}_l\widetilde{\boldsymbol{e}}_{M,j}, \widetilde{\boldsymbol{e}}_{M,j}=\boldsymbol{e}_{M,j}-\frac{1}{M}\boldsymbol{I}_M, \mathcal{A}=\boldsymbol{I}_L\otimes\boldsymbol{\Phi}$ for shorter notations. In the special case of $M=2$, we have $\widetilde{\boldsymbol{h}}_{l,1}=-\widetilde{\boldsymbol{h}}_{l,2}$. Then $\boldsymbol{\Phi}$ can be simplified for a real system as

$$\boldsymbol{\Psi}(\boldsymbol{\Theta})=-\frac{(N-1)^2}{N^3}\sum_{l=1}^{L}\boldsymbol{H}_l\boldsymbol{\Phi}\boldsymbol{H}_l^T(\boldsymbol{\Theta}+\boldsymbol{\Theta}^T)\boldsymbol{H}_l\boldsymbol{\Phi}\boldsymbol{H}_l^T$$
$$+\frac{N-1}{N^2}[tr(\boldsymbol{R}\boldsymbol{\Theta})\boldsymbol{R}+\boldsymbol{R}\boldsymbol{\Theta}^T\boldsymbol{R}]+\frac{1}{N^2}\boldsymbol{R}\boldsymbol{\Theta}\boldsymbol{R}$$

while for complex channel and noise as

$$\boldsymbol{\Psi}(\boldsymbol{\Theta})=\frac{1}{N^2}\boldsymbol{R}\boldsymbol{\Theta}\boldsymbol{R}$$
$$-\frac{(N-1)^2}{N^3}\sum_{l=1}^{L}\boldsymbol{H}_l\boldsymbol{\Phi}[\boldsymbol{H}_l^H\boldsymbol{\Theta}\boldsymbol{H}_l+(\boldsymbol{H}_l^H\boldsymbol{\Theta}\boldsymbol{H}_l)^T]\boldsymbol{\Phi}\boldsymbol{H}_l^H$$
$$+\frac{N-1}{N^2}[tr(\boldsymbol{R}\boldsymbol{\Theta})\boldsymbol{R}+\boldsymbol{H}\mathcal{A}(\boldsymbol{H}^H\boldsymbol{\Theta}\boldsymbol{H})^T\mathcal{A}\boldsymbol{H}^H].$$

3.1.4. *Channel Estimation Error Due to Noise and Finite N*

Based on the above analysis, total channel estimation error due to combined effects of noise and finite N can be approximated by

$$E\{||\widetilde{\boldsymbol{g}}-\boldsymbol{g}_1||^2\}\approx||\boldsymbol{g}_{por}-\boldsymbol{g}_1||^2+E\{||\widetilde{\boldsymbol{g}}-\boldsymbol{g}_{por}||^2\} \quad (15)$$

where the first term is obtained from (10) and the second term by the trace of (12). The cross term can be neglected because $E\{\delta\boldsymbol{R}\}=\boldsymbol{0}$ leads to $E\{\widetilde{\boldsymbol{g}}-\boldsymbol{g}_{por}\}\approx\boldsymbol{0}$ according to (11).

3.2. Detection Performance

We first study performance of the ideal RAKE receiver when channel and data covariance are perfectly known. Then we investigate its sensitivity to sample size. Express $\boldsymbol{z}_n$ as $\boldsymbol{z}_n=\boldsymbol{H}_1\boldsymbol{a}_{1,n,0}+\boldsymbol{u}_n$, where $\boldsymbol{u}_n$ includes intersymbol interference and MAI, and is approximated as a Gaussian process for convenience of analysis. Assume information symbol 0 is transmitted. According to our data model, $\boldsymbol{a}_{1,n,0}=\boldsymbol{e}_{M,1}-\frac{1}{M}\boldsymbol{I}_M=\widetilde{\boldsymbol{e}}_{M,1}$ and then $\boldsymbol{z}_n=\widetilde{\boldsymbol{h}}_{1,1}+\boldsymbol{u}_n$. Denote M RAKE receivers simply by $\boldsymbol{f}_j$ for $j=1,\cdots,M$. Then a correct detection event is equivalent to $\{\boldsymbol{f}_1^H\boldsymbol{z}_n>\boldsymbol{f}_j^H\boldsymbol{z}_n, j=2,\cdots,M\}=\{\Delta\boldsymbol{f}_j^H\boldsymbol{z}_n>0\}$, where $\Delta\boldsymbol{f}_j=\boldsymbol{f}_1-\boldsymbol{f}_j$. Define a $(M-1)$ dimensional random vector $\boldsymbol{x}_n=\Delta\boldsymbol{F}^H\boldsymbol{z}_n$ where $\Delta\boldsymbol{F}$ contains all $\Delta\boldsymbol{f}_j$ as columns. Since $\boldsymbol{z}_n$ is assumed Gaussian distributed, $\boldsymbol{x}_n$ is also Gaussian with probability density function as the following

$$f\boldsymbol{x}=\frac{e^{-\frac{1}{2}(\boldsymbol{x}_n-\Delta\boldsymbol{F}^H\widetilde{\boldsymbol{h}}_{1,1})^H(\mathrm{COV}(\boldsymbol{x}))^{-1}(\boldsymbol{x}_n-\Delta\boldsymbol{F}^H\widetilde{\boldsymbol{h}}_{1,1})}}{\sqrt{(2\pi)^{M-1}det(\mathrm{COV}(\boldsymbol{x}))}}$$

where $\mathrm{COV}(\boldsymbol{x})=\Delta\boldsymbol{F}^H\boldsymbol{R}_{int}\Delta\boldsymbol{F}$, $\boldsymbol{R}_{int}=\boldsymbol{R}-\widetilde{\boldsymbol{h}}_{1,1}\widetilde{\boldsymbol{h}}_{1,1}^H$. In the special case of $M=2$, it is straightforward to show that $BER_0=Q(\frac{\Delta\boldsymbol{f}_1^H\widetilde{\boldsymbol{h}}_{1,1}}{\sigma_1})$. Similarly $BER_1=Q(\frac{\Delta\boldsymbol{f}_2^H\widetilde{\boldsymbol{h}}_{1,2}}{\sigma_2})$, where one can verify that $\Delta\boldsymbol{f}_2=-\Delta\boldsymbol{f}_1, \sigma_1^2=\sigma_2^2=\Delta\boldsymbol{f}_1^H(\boldsymbol{R}-\widetilde{\boldsymbol{h}}_{1,1}\widetilde{\boldsymbol{h}}_{1,1}^H)\Delta\boldsymbol{f}_1$. Therefore, $BER=BER_0=BER_1$. Moreover, one can verify that $\Delta\boldsymbol{f}_1=2\widetilde{\boldsymbol{h}}_{1,1}$. After examining BER_0, it is found that BER depends on SINR of the receiver $\Delta\boldsymbol{f}_1$. The output SINR can be obtained as

$$\mathrm{SINR}=\frac{|\Delta\boldsymbol{f}_1^H\widetilde{\boldsymbol{h}}_{1,1}|^2}{\sigma_1^2}=\frac{\Delta\boldsymbol{f}_1^H\widetilde{\boldsymbol{h}}_{1,1}\widetilde{\boldsymbol{h}}_{1,1}^H\Delta\boldsymbol{f}_1}{\Delta\boldsymbol{f}_1^H\boldsymbol{R}_{int}\Delta\boldsymbol{f}_1}. \quad (16)$$

In practice, channel estimation error $\delta\boldsymbol{g}_{noise}+\delta\boldsymbol{g}_N$ causes the perturbation in $\Delta\boldsymbol{f}_1$ as $\widetilde{\Delta\boldsymbol{f}}_1=2\mathcal{S}_1\boldsymbol{g}_{por}+2\mathcal{S}_1\delta\boldsymbol{g}_N$. Note that $\boldsymbol{g}_{por}$ is a deterministic quantity containing noise induced error while $\delta\boldsymbol{g}_N$ is a random quantity. If we denote $2\mathcal{S}_1\boldsymbol{g}_{por}$ as $\boldsymbol{m}$ and $2\mathcal{S}_1\delta\boldsymbol{g}_N$ as $\delta\boldsymbol{m}$ for simple notations, then the perturbed SINR can be evaluated as

$$\widehat{\mathrm{SINR}}\approx\frac{\boldsymbol{m}^H\widetilde{\boldsymbol{h}}_{1,1}\widetilde{\boldsymbol{h}}_{1,1}^H\boldsymbol{m}+E\{\delta\boldsymbol{m}^H\widetilde{\boldsymbol{h}}_{1,1}\widetilde{\boldsymbol{h}}_{1,1}^H\delta\boldsymbol{m}\}}{\boldsymbol{m}^H\boldsymbol{R}_{int}\boldsymbol{m}+E\{\delta\boldsymbol{m}^H\boldsymbol{R}_{int}\delta\boldsymbol{m}\}}. \quad (17)$$

Each unperturbed term can be evaluated using covariance matrix and the desired user's codes. Each expectation is then computed as $E\{\delta\boldsymbol{m}^H\boldsymbol{X}\delta\boldsymbol{m}\}=4tr\{\mathcal{S}_1^H\boldsymbol{X}\mathcal{S}_1\mathrm{Cov}\boldsymbol{g}_{\mathrm{N}}\}$, where $\boldsymbol{X}$ may be replaced by $\widetilde{\boldsymbol{h}}_{1,1}\widetilde{\boldsymbol{h}}_{1,1}^H$ or $\boldsymbol{R}_{int}$ correspondingly, and $\mathrm{Cov}\boldsymbol{g}_{\mathrm{N}}$ can be evaluated by (12).

4. NUMERICAL EXAMPLES

We first verify analytical results based on 100 independent realizations. Consider a UWB system with $N_c=8$, $N_f=4$, $K=8$, and $M=2$. 16-path Gaussian channels spread over one frame are used. Fig. 1 illustrates channel MSE for different p's, where noise induced MSE is plotted in Fig. 1(a), perturbation error from finite N in the presence of 15dB noise is in Fig. 1(b), and total channel MSE in 15dB noise is in Fig. 1(c). We see that all experimental results converge to their analytical ones. Fig. 1 (c) also suggests that the POR method with $p=1$ has worst performance due to a dominant noise induced error. The MSE of $p=2$ is very similar to that of $p=4$, indicating that $p=2$ in practice can achieve a good performance and complexity tradeoff. The receiver's performance is presented in Fig. 2, with the output SINR in Fig. 2(a) and BER performance of RAKE receivers in Fig. 2(b). Similar conclusions can be drawn. Discrepancies between the analytical and experimental BER curves for $p=2,\ 4$ at low SNRs are caused by finite N, and at high SNRs are caused by possible violation of the Gaussian assumption on the MUI.

We then compare the POR method ($p=2$) with data-aided ML (DA-ML), non data-aided ML (NDA-ML) [4] and subspace [5] methods when $N=800$. Channels are generated by the IEEE 802.15 channel model CM2 [11] which is for non line-of-sight (NLOS) communication between 0 to 4 meters. The channel delay spread is truncated at $40ns$ for reduced complexity, yielding approximately 80% of the total energy on the average. $N_c=5$ and $N_f=3$. Fig. 3 (a) $\sim$ (c) plots channel MSE versus input SNR for cases of 2, 5 and 8 users. Clearly, the POR outperforms the DA-ML and NDA-ML in all situations. It also outperforms the subspace method when 8 users are in the system, indicating robustness to the system load variation on the same token as [9]. The BER performance of different receivers is demonstrated in Fig. 4 (a) $\sim$ (c) correspondingly. Similar conclusions can be drawn. [1]

[1]The views and conclusions contained in this document are those of the authors and should not be interpreted as representing the official policies, either expressed or implied, of the Army Research Laboratory or the U. S. Government.

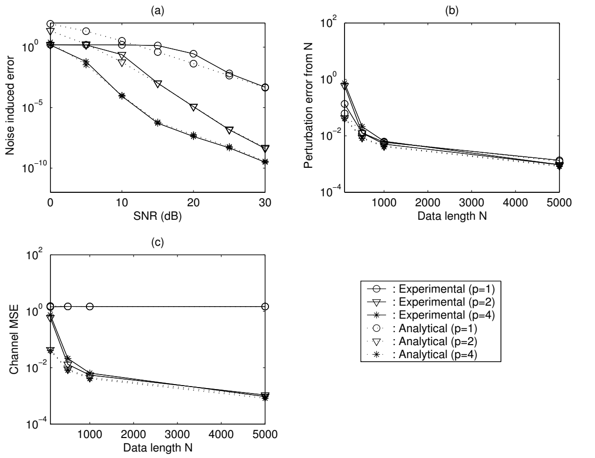

Fig. 1. Performance of channel estimation.

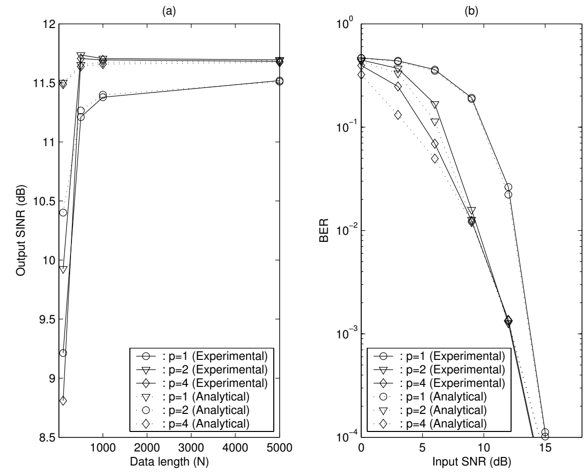

Fig. 2. Performance of symbol detection.

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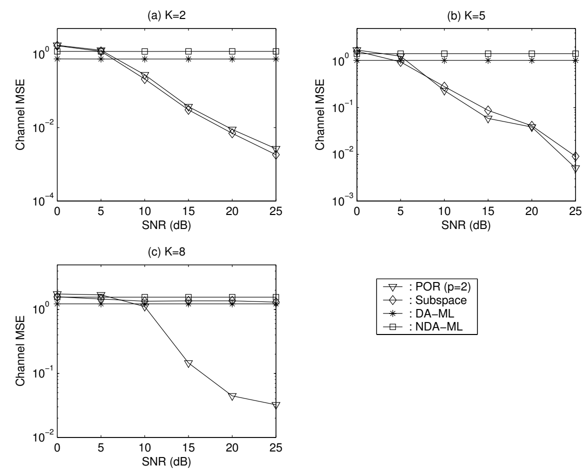

Fig. 3. Channel MSE comparison based on channel model CM2.

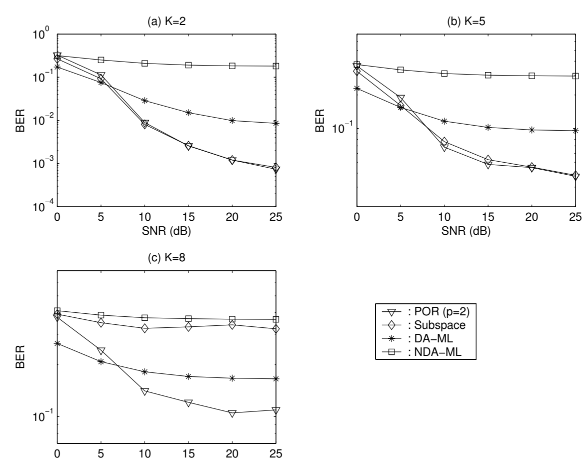

Fig. 4. BER comparison based on channel model CM2.