BLIND CHANNEL EQUALIZATION BASED ON SECOND ORDER STATISTICS

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ABSTRACT

In this paper, we present new approaches to blind channel estimation and equalization based on Second Order Statistics (SOS). We first consider the case of minimum phase channels where the equalizer is designed based on the criterion of autocorrelation matching. We cast the problem as a convex optimization program that can be efficiently solved using interior point methods. Then we consider the equalization of single-input multiple-output SIMO channels. Due to oversampling, the equivalent channel matrix possesses a particular structure which enables us to estimate the channel based only on the information contained in the covariance matrix at zero delay. Simulation examples are provided to demonstrate the performance advantage of the proposed algorithms compared to existing techniques.

1. INTRODUCTION

Blind channel estimation and equalization plays a key rule in modern communication systems. It is employed in systems where an unknown signal is transmitted through an unknown multipath channel. Although training-sequence based equalization is simple and exhibits fast convergence, it suffers from the trade off between the sequence length and the capacity of the link. On the other hand, Higher Order Statistics HOS can be effectively utilized to estimate/equalize the channel, yet they require a large channel output data record. Therefore, for time-varying channels this method can fail to estimate/equalize the channel.

Recent work has been directed towards the use of second-order statistics as it requires less data samples and is computationally less expensive than HOS. The algorithm presented in [1] is one of the first second order statistical methods exploiting the multiple channel nature of fractionally sampled channel outputs. The methods presented in [1], [2] and [4] rely on the noise and signal subspaces separation which requires prior knowledge on the channel order. The methods presented in [3] and [5] overcome this difficulty. However, these methods utilize multiple covariance matrices computed at different delays which may result in error accumulation.

In this paper we first present a convex optimization formulation to estimate the equalizer coefficients in the case of minimum phase channel where the variable of interest in the formulation is the autocorrelation sequence of the equalizer. After solving the optimization problem the equalizer can be obtained by applying the spectral factorization technique. In addition, we present two methods for SIMO channel equalization. Unlike [1] and [2], our proposed methods are less sensitive to the over estimation of the channel order and are computationally less expensive than [3] and [5]. Our methods make use of the channel matrix structure and require only one covariance matrix in estimation. Moreover, the proposed methods are robust when the channel matrix approaches singularity. As a by-product, we will show that, the covariance matrix at different delays can be obtained directly from the covariance matrix at delay zero.

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2. PROBLEM FORMULATION

In a linear time invariant system, the received signal is given by,

$$x(t) = \sum_m s_m h(t - mT) + n(t), \tag{1}$$

where s_m is transmitted data sequence, T is the symbol baud duration, $h(t)$ is the channel impulse response, and $n(t)$ is an additive white noise independent of the input sequence. In the fractionally spaced scenario the received signal is sampled p times its original baud rate. The resulting i-th subchannel $h_i(n) = h(nT - (i-1)\Delta)$, $i = 1, \dots, p$ and $\Delta = T/p$, is modelled as an FIR filter of order L. The fractionally spaced system is shown in Fig. 1.

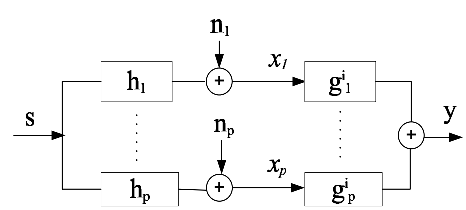

Fig. 1. Multichannel model

The $p \times 1$ received vector $\boldsymbol{x}(k)$ can be expressed as,

$$\boldsymbol{x}(k) = \sum_m s_m \mathbf{h}(k - m) + \boldsymbol{n}(k), \tag{2}$$

where $\mathbf{h}(k) = [h_1(k) \cdots h_p(k)]^T$ and $\boldsymbol{n}(k) = [n_1(k), \dots, n_p(k)]^T$ is the oversampled noise vector. Collecting M received vectors (where M is the length of each subequalizer) the model can be expressed in a matrix form as follows,

$$\mathbf{x}(k) = \mathbf{H}\mathbf{s}(k) + \mathbf{n}(k), \tag{3}$$

where $\mathbf{x}(k) = [\boldsymbol{x}(k)^T, \cdots, \boldsymbol{x}(k-M+1)^T]^T$ is $pM \times 1$ received vector, $\mathbf{s}(k) = [s_k, \cdots, s_{k-M-L+1}]^T$ is $(M+L) \times 1$ transmitted data sequence of (i.i.d) zero mean symbols with covariance matrix $E\{\mathbf{s}(k)\mathbf{s}^H(k)\} = \mathbf{I}_s$, $\mathbf{n}(k) = [\boldsymbol{n}(k)^T, \cdots, \boldsymbol{n}(k-M+1)^T]^T$ is $pM \times 1$ white Gaussian noise with zero mean and covariance matrix, $E\{\mathbf{n}(k)\mathbf{n}^H(k)\} = \sigma^2\mathbf{I}_n$, and $\mathbf{H}$ is $pM \times (M+L)$ Toeplitz channel matrix,

$$\mathbf{H} = \begin{bmatrix} \mathbf{h}(0) & \cdots & \mathbf{h}(L) & \cdots & 0 \\ \vdots & \ddots & & \ddots & \vdots \\ 0 & \cdots & \mathbf{h}(0) & \cdots & \mathbf{h}(L) \end{bmatrix}.$$

The condition for identifying the channel matrix $\mathbf{H}$ was given in [1], [2], [3] and [4] that is the channel matrix $\mathbf{H}$ should be full column rank or equivalently, the subchannels h_i have no common zeros. In the following we will assume that $\mathbf{H}$ meets this condition.

3. MINIMUM PHASE CHANNEL EQUALIZER

In this section, we propose an approach to minimum phase channels equalization (T-spaced equalizer). The assumption that the transmitted signal is white will lead to a convex optimization formulation in the autocorrelation sequence of the equalizer. The equalizer output is,

$$\mathbf{y}(k) = \sum_j \mathbf{g}(k-j)\mathbf{x}(j), \tag{4}$$

where $\mathbf{g}(z)$ is the equalizer. The output autocorrelation sequence can be expressed as follows,

$$\begin{aligned}\mathbf{r}_y(m) &= E\{\mathbf{y}^*(k)\ \mathbf{y}(k+m)\},\\ &= E\left\{\sum_j \mathbf{g}^*(k-j)\mathbf{x}^*(j) \sum_\ell \mathbf{g}(k+m-\ell)\mathbf{x}(\ell)\right\},\\ &= \sum_i \mathbf{r}_g(m-i)\ \mathbf{r}_x(i),\end{aligned}$$

or simply $\mathbf{r}_y = \mathbf{R}_x\mathbf{r}_g$, where $\mathbf{r}_x$ and $\mathbf{r}_g$ are the autocorrelation sequences of the received signal and the equalizer respectively and $\mathbf{R}_x$ is a Toeplitz convolution matrix constructed from $\mathbf{r}_x$. Ideally $\mathbf{r}_y = (0, \cdots, 1, \cdots, 0)$ which leads to the ℓ_2-norm equalizer design criterion as follows,

$$\min_{\mathbf{r}_g} \quad \mathbf{r}_g^T R_x^T R_x \mathbf{r}_g \qquad \text{s.t} \quad \mathbf{a}_x^T\mathbf{r}_g = 1,\ \mathbf{r}_g(e^{j\omega}) \geq 0\ \forall\omega,$$

where $\mathbf{a}_x^T$ is the m-th row of $\mathbf{R}_x$ and the output autocorrelation length is $2m-1$. This results in a semi-definite programming SDP (using Schur complement) with linear equality and inequality constraints over the variable $\mathbf{r}_g$,

$$\begin{aligned}&\min_{\mathbf{r}_g, t} \quad t\\ &\text{s.t} \quad \mathbf{a}_x^T\mathbf{r}_g = 1,\ \mathbf{r}_g(e^{j\omega}) \geq 0\ \forall\omega, \begin{bmatrix} t & \mathbf{r}_g^T \\ \mathbf{r}_g & (\mathbf{R}_x^T\mathbf{R}_x)^{-1} \end{bmatrix} \succeq 0.\end{aligned}$$

A popular method of handling the infinite linear constraints $\mathbf{r}_g(e^{j\omega}) \geq 0$ is through discretization on a finite grid of length N covering the interval $[0, \pi]$. The problem can be solved efficiently using the interior point methods to obtain $\mathbf{r}_g$, which can be spectrally factorized to obtain the equalizer $\mathbf{g}(z)$.

4. FRACTIONALLY SPACED EQUALIZER

4.1. Blind Equalizer

Consider the equalizers bank shown in Fig. 1 where $\mathbf{g}_j^i, j = 1, \cdots, p$ is the j-th subequalizer of order $M-1$ at delay i. The estimated symbol at delay i is,

$$\hat{s}_i = \tilde{\mathbf{g}}_i^H\mathbf{x}(k), \qquad 0 \leq i \leq M+L-1, \tag{5}$$

where $\tilde{\mathbf{g}}_i = \text{vec}([\mathbf{g}_1^i, \cdots, \mathbf{g}_p^i]^T)$ and 'vec' denotes the vector operator. Considering all the available delays, the equalization matrix can be written as $\tilde{\mathbf{G}} = [\tilde{\mathbf{g}}_0, \cdots, \tilde{\mathbf{g}}_{M+L-1}]^T$. For minimum mean square equalizer MMSE the equalization matrix is given by,

$$\begin{aligned}\tilde{\mathbf{G}} &= \arg\min_{\mathbf{G}} E\{||\,\mathbf{G}\,\mathbf{x}(k) - \hat{\mathbf{s}}(k)||^2\},\\ &= \mathbf{H}^H\mathbf{R}_0^\dagger,\end{aligned} \tag{6}$$

where $\hat{\mathbf{s}}(k)$ is the estimated sequence and '$\dagger$' denotes the pseudo inverse. Since $\mathbf{R}_0$ can be estimated from the received signal, it remains to estimate the columns of the channel matrix $\mathbf{H}$ corresponding to the equalizer at different delays.

4.2. Covariance Matrix

The covariance matrix $\mathbf{R}_0$ of the received vector $\mathbf{x}$ and its eigenvalue decomposition are given by,

$$\begin{aligned}\mathbf{R}_0 &= E\{\mathbf{x}(k)\mathbf{x}^H(k)\} = \mathbf{H}\mathbf{H}^H + \sigma^2\mathbf{I}_n\\ &= [\mathbf{U}_1\mathbf{U}_2] \begin{bmatrix} \boldsymbol{\Lambda} + \sigma^2\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \sigma^2\mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{U}_1^H \\ \mathbf{U}_2^H \end{bmatrix},\end{aligned} \tag{7}$$

where the transmitted data and noise are assumed to be white, independent of each other and with zero mean. The channel matrix $\mathbf{H}$ can be obtained up to an unknown orthonormal matrix $\mathbf{V}^H$ (where $\mathbf{V}$ is the $(M+L) \times (M+L)$ right singular matrix of $\mathbf{H}$),

$$\begin{aligned}\mathbf{H}\mathbf{H}^H &= \mathbf{U}_1\boldsymbol{\Lambda}\mathbf{U}_1^H,\\ \mathbf{H} &= \mathbf{A}\mathbf{V}^H.\end{aligned} \tag{8}$$

where $\mathbf{A} = \mathbf{U}_1\boldsymbol{\Lambda}^{\frac{1}{2}}$ and $\mathbf{V}^H = [\boldsymbol{v}_1, \ldots, \boldsymbol{v}_{M+L}]$. The key step in the algorithm is to find $\boldsymbol{v}_1$ and then apply a recursive method to estimate the j-th vector $\boldsymbol{v}_j$ from which we can obtain the j-th column of the channel matrix that contains all the channel coefficients. Note that, it is not necessary to estimate the equalizer at all the $(M+L)$ delays. However, estimating a channel column vector that has all the channel coefficients results in a performance that is close to the best delay equalizer.

4.3. Channel Estimation

4.3.1. Method A

We define a matrix $\mathbf{H}_1$ of size $p(M-1) \times (M+L)$, containing the last $p(M-1)$ rows of the original matrix $\mathbf{H}$ with the following structure,

$$\mathbf{H}_1 = \begin{bmatrix} 0 & \mathbf{h}(0) & \cdots & \mathbf{h}(L) & \cdots & 0 \\ \vdots & & \ddots & & \ddots & \vdots \\ 0 & & & \mathbf{h}(0) & \cdots & \mathbf{h}(L) \end{bmatrix}.$$

Since $\mathbf{H}$ is a full column rank matrix with $\text{rank}(\mathbf{H}) = M+L$ then it is clear that $\text{rank}(\mathbf{H}_1) = M+L-1$ where the first zero column in $\mathbf{H}_1$ induces the rank deficiency. Partitioning $\mathbf{H}$ into two sub-matrices $\tilde{\mathbf{H}}_1$ and $\mathbf{H}_1$, (8) can be written as,

$$\begin{bmatrix} \tilde{\mathbf{H}}_1 \\ \mathbf{H}_1 \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{A}}_1 \\ \mathbf{A}_1 \end{bmatrix} \mathbf{V}^H, \tag{9}$$

and it follows that,

$$\mathbf{H}_1 = \mathbf{A}_1 \mathbf{V}^H. \tag{10}$$

Since $\mathbf{V}^H$ is a full rank matrix then, $\mathrm{rank}(\mathbf{A}_1) = M + L - 1$. Comparing the first column in both sides of (10) we get,

$$\mathbf{A}_1 \boldsymbol{v}_1 = \mathbf{0}. \tag{11}$$

Since $\mathbf{A}_1^H \mathbf{A}_1$ has only one zero eigenvalue, and $\boldsymbol{v}_1$ is a unit norm vector then, $\boldsymbol{v}_1$ is the solution to the following problem,

$$\min_{\boldsymbol{v}_1} \quad \boldsymbol{v}_1^H \mathbf{A}_1^H \mathbf{A}_1 \boldsymbol{v}_1 \qquad \text{s.t.} \quad \|\boldsymbol{v}_1\| = 1. \tag{12}$$

Therefore, $\boldsymbol{v}_1$ is the eigenvector corresponding to the unique zero eigenvalue of $\mathbf{A}_1^H \mathbf{A}_1$. To find the j-th column vector of $\mathbf{V}^H$ ($j > 1$), we define a matrix $\mathbf{H}_j$ of size $p(M-j) \times (M+L)$ that has the last $p(M-j)$ rows of the channel matrix $\mathbf{H}$ with the condition $(M+L) < p(M-j)$. This condition is necessary to ensure that $\mathbf{H}_j$ is a tall matrix and the rank deficiency arises from the first j zero columns. Following the same steps and considering the last $p(M-j)$ rows of the channel matrix $\mathbf{H}$ we obtain,

$$\mathbf{H}_j = \mathbf{A}_j \mathbf{V}^H, \tag{13}$$

where $\mathrm{rank}(\mathbf{A}_j) = M + L - j$ and the matrix $\mathbf{A}_j^H \mathbf{A}_j$ has j zero eigenvalues. Comparing the j-th column in each side we obtain, $\mathbf{A}_j \boldsymbol{v}_j = \mathbf{0}$ and consequently,

$$\boldsymbol{v}_j^H \mathbf{A}_j^H \mathbf{A}_j \boldsymbol{v}_j = \mathbf{0}. \tag{14}$$

Since the null space $\mathcal{N}(\mathbf{A}_j^H \mathbf{A}_j)$ has dimension j then $\boldsymbol{v}_j$ can not be obtained directly from (14). Let $\mathcal{N}(\mathbf{A}_j^H \mathbf{A}_j) = \mathrm{Range}(\mathbf{U}_j) = \mathrm{span}(\boldsymbol{u}_1, \cdots, \boldsymbol{u}_j)$. Since $\boldsymbol{v}_j$ is a linear combination of these vectors, it can be expressed as,

$$\boldsymbol{v}_j = \mathbf{U}_j \mathbf{a}, \tag{15}$$

where $\boldsymbol{a} = (a_1, \cdots, a_j)^T$ and $\|\boldsymbol{a}\| = 1$. This condition is necessary for $\boldsymbol{v}_j$ to be a unit norm. Moreover, $\boldsymbol{v}_j$ should belong to the null space of the matrix $\mathbf{W}_j$ whose columns are the first $(j-1)$-th columns of $\mathbf{V}^H$ defined as, $\mathbf{W}_j = [\boldsymbol{v}_1, \cdots, \boldsymbol{v}_{j-1}]$. So we have,

$$\mathbf{W}_j^H \boldsymbol{v}_j = \mathbf{W}^H \mathbf{U}_j \mathbf{a} = \mathbf{B}\mathbf{a} = \mathbf{0}. \tag{16}$$

Note that the matrix $\mathbf{B}$ is $j - 1 \times j$ full row rank matrix. Therefore, $\mathbf{a}$ is the eigenvector corresponding to the unique zero eigenvalue of $\mathbf{B}^H \mathbf{B}$. Substituting the vector $\mathbf{a}$ in (15) we obtain $\boldsymbol{v}_j$. Starting from $\boldsymbol{v}_1$ and applying this recursive algorithm we can obtain the first j columns of $\mathbf{V}^H$. The advantage of method-A is evident in the estimation of $\boldsymbol{v}_j$ where it is obtained as the intersection between two sets $\mathcal{N}(\mathbf{W}_j)$ (orthogonal to previous $\boldsymbol{v}_i$, $i < j$) and $\mathcal{N}(\mathbf{A}_j^H \mathbf{A}_j)$ (due to the channel matrix structure). Therefore, making use of the orthogonality property and the channel structure leads to a better estimation than [1], [2], [3] and [4].

4.3.2. Method B

In this method the equalizer is estimated using only the vector $\boldsymbol{v}_1$ obtained in the previous section and the covariance matrix at delay $d = i$. We will show that this covariance matrix can be obtained directly from the covariance matrix at delay zero. The covariance matrix at delay $d = i$ is given as,

$$\mathbf{R}_i = E\{\mathbf{x}(k)\mathbf{x}^H(k+i)\} = \mathbf{H}\mathbf{J}_s^i \mathbf{H}^H + \sigma^2 \mathbf{J}_n^i, \tag{17}$$

where $\mathbf{J}_s$ is $(M+L) \times (M+L)$ and $\mathbf{J}_n$ is $pM \times pM$ Jordan matrices (matrix with ones in the subdiagonal below the main diagonal). Substituting the channel matrix (8) we obtain,

$$\mathbf{R}_i - \sigma^2 \mathbf{J}_n^i = \mathbf{H}\mathbf{J}_s^i \mathbf{H}^H = \mathbf{U}_1 \boldsymbol{\Lambda}^{\frac{1}{2}} \mathbf{V}^H \mathbf{J}_s^i \mathbf{V} \boldsymbol{\Lambda}^{\frac{1}{2}} \mathbf{U}_1^H.$$

Multiplying by $\boldsymbol{\Lambda}^{-\frac{1}{2}} \mathbf{U}_1^H$ and its hermitian transpose from left and right respectively we get,

$$\boldsymbol{\Lambda}^{-\frac{1}{2}} \mathbf{U}_1^H (\mathbf{R}_i - \sigma^2 \mathbf{J}_n^i) \mathbf{U}_1 \boldsymbol{\Lambda}^{-\frac{1}{2}} = \mathbf{V}^H \mathbf{J}_s^i \mathbf{V}. \tag{18}$$

Define a new matrix $\mathbf{C}_i$ associated with the delay $d = i$ as follows,

$$\mathbf{C}_i = \mathbf{V}^H \mathbf{J}_s^i \mathbf{V}. \tag{19}$$

Multiplying $\mathbf{C}_i$ by $\mathbf{V}$ and $\mathbf{V}^H$ from right and left respectively,

$$\mathbf{V}\mathbf{C}_i \mathbf{V}^H = \mathbf{J}_s^i = \begin{bmatrix} \mathbf{0_{i \times M+L-i}} & \mathbf{0} \\ \mathbf{I_{M+L-i}} & \mathbf{0} \end{bmatrix}.$$

By equating the $(i+1, 1)$ entry in both sides we obtain $\boldsymbol{v}_{i+1}^H \mathbf{C}_i \boldsymbol{v}_1 = 1$, and hence,

$$\boldsymbol{v}_{i+1} = \mathbf{C}_i \boldsymbol{v}_1. \tag{20}$$

We can directly obtain $\mathbf{R}_i$ from $\mathbf{R}_0$ in a recursive method. Divide the covariance matrix at delay j as $\mathbf{R}_j = [\mathbf{Z} \;\; \mathbf{P} \;\; \mathbf{Q}]$ where $\mathbf{Z}$ is $pM \times p$, $\mathbf{P}$ is $pM \times p(M-2)$ and $\mathbf{Q}$ is $pM \times p$. Using the channel matrix structure, the covariance matrix at delay $j+1$ can be obtained from the following relation,

$$\mathbf{R}_{j+1} = \mathcal{F}\{\mathbf{R}_j\} = [\mathbf{P} \;\; \mathbf{Q} \;\; \mathbf{J}^p \mathbf{Q}], \tag{21}$$

where $\mathcal{F}$ is an operator acting on the sub matrices $\mathbf{Z}, \mathbf{P}, \mathbf{Q}$. To illustrate this operation, by using the channel structure it can be shown that $\mathbf{H}\mathbf{J}_s \mathbf{H}^H = \mathcal{F}\{\mathbf{H}\mathbf{H}^H\}$. The algorithm is less sensitive to the over-estimation of the channel order. furthermore, it avoids the error propagation associated with the recursive channel estimation in [1], as well as the error accumulation due to the multiplication of estimated covariance matrices computed at different delays [3] and [5]. Obviously, the computational complexity in method-B is substantially less than method-A.

5. SIMULATION RESULTS

In this section four examples are presented to evaluate the performance of the proposed algorithms. The covariance matrix is estimated using 1000 samples.

In the first example, we consider the transmission of BPSK sequence. The channel impulse response is shown in Fig. 2(a) (minimum phase channel). The received autocorrelation sequence is estimated over 1000 symbols and the equalizer length $L_g = 10$. The combined channel/equalizer response is shown in Fig. 2(b). Simulation results demonstrate the potential of the proposed method in channel equalization.

In the second example, the performance of the two FSE methods to equalize the channel is evaluated. The channel impulse response $h(t) = (p(t,\alpha) - 0.6p(t - t_0, \alpha))W(t)$ is shown in Fig. 3(a) where $p(t,\alpha)$ is a raised cosine pulse, $\alpha = 0.1$ is the roll-off factor, $W(t)$ is a window of length $6T$ and $t_0 = T/4$. The subchannel order is $L = 5$. The delay $i = 6$, oversampling factor $p = 4$ and the subequalizer length $M = 12$. The input sequence is an i.i.d 16-QAM signal. Fig. 3(b) shows the received signal while Fig. 3(c) and (d) show the equalized signal constellation with method-A and method-B respectively at SNR= 25 dB.

In the third example, the robustness of the proposed algorithms when the channel matrix approaches singularity is compared with the subspace method presented in [2]. The channel impulse response is $h(z) = (1 + 0.2z^{-1})/(1 + 0.8z^{-1})$, $L = 2$, $d = 3$, and $M = 12$. The equalized signal constellation is shown in Fig. 4 at SNR=20 dB.

In the fourth example, the bit error rate BER versus the signal-to-noise ratios SNRs is considered. The channel impulse response is $h = [.04\ -0.05\ 0.07\ -.2\ -0.5\ 0.72\ 0.36\ 0.21\ 0.03\ 0.07\ .03\ -.01]$. The parameters used are $M = 6$, $p = 4$, $i = 2$ and $L = 2$. The input sequence is drawn from BPSK constellation and the signal subspace order is assumed to be known. The BER is averaged over 500 Monte Carlo runs. The performance is compared with [1] (TXK) and [3] (Direct) and is shown in Fig. 5. In method-A, the restriction that $\boldsymbol{v}_i$ lies in the intersection of $\mathcal{N}(\mathbf{W}_i)$ and $\mathcal{N}(\mathbf{A}_i^H\mathbf{A}_i)$ substantially improve the performance. While the improvement in method-B arises as the method avoids the recursive estimation error in [1] and the multiplication of the estimated covariance matrices in [3] and [5].

6. CONCLUSION

In this paper, we have addressed the problem of blind channel equalization. For minimum phase channels, we have proposed an algorithm based on autocorrelation matching and formulated the problem as a convex optimization SDP program. Moreover, we exploited the channel matrix structure induced by oversampling the received signal and have proposed two methods for SIMO channel equalization that outperform existing schemes.

7. REFERENCES

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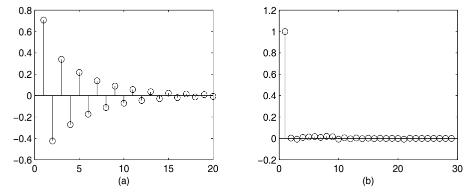

Fig. 2. (a) Channel response (b) Combined channel/equalizer

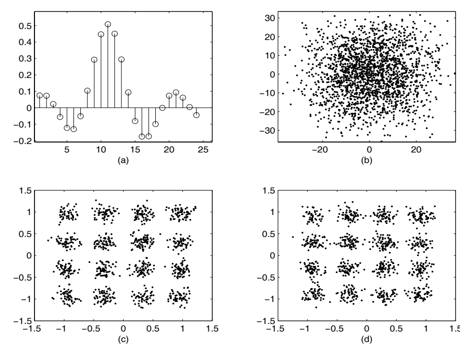

Fig. 3. (a) Channel coefficients (b) Received signal. Output signal constellation (c) method-A and (d) method-B

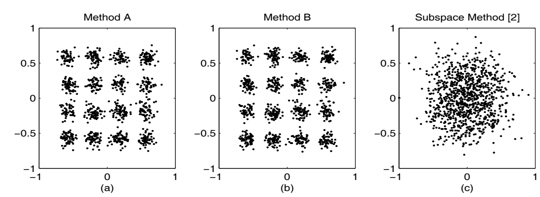

Fig. 4. Equalized signal constellation SNR=20 dB

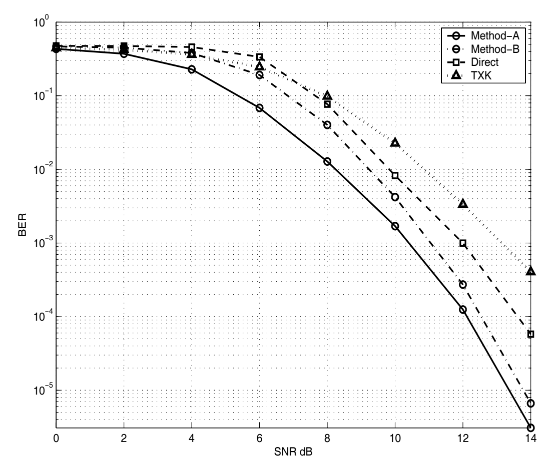

Fig. 5. BER v.s. SNR