AN AXIOMATIC APPROACH TO RESOURCE ALLOCATION AND INTERFERENCE BALANCING

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ABSTRACT

We propose a general framework for joint resource allocation and interference control for classes of interference functions based on an axiomatic model. This model holds for a wide range of multiuser channels and even allows the incorporation of interference suppression techniques. The quality of service (QoS) of each user is modeled as a function of the interference. It is shown that for certain mappings between interference and QoS, the resulting achievable region is a convex set. Furthermore, we show that the problem of optimizing the sum of weighted QoS over the transmit powers is convex for certain classes of QoS functions. The choice of the weights determines the trade-off between fairness and overall efficiency. The problem can be solved efficiently by standard convex optimization techniques.

1. INTRODUCTION

The control of multiuser interference plays a fundamental role in wireless communications. Interference depends on the power allocation, as well as on signal processing and coding techniques used at the physical layer. Traditionally, power control and resource allocation are associated with higher layers, so that the signal processing is performed more or less independently. However, the physical layer design may have a drastic effect on the effective interference situation, thus much better results can be expected from cross-layer designs, which jointly optimize the resource allocation with the signal processing.

In this paper we propose a general framework for resource allocation based on weighted quality-of-service (QoS) functions. Following the approach in [1], we assume an axiomatic interference model, which incorporates many linear and non-linear processing strategies, e.g. the multiuser beamforming problem [2] or CDMA receiver designs. For a certain class of QoS functions, we study properties of the achievable region. Finally, the results are used to develop an algorithmic solution. Proofs are sketched in the appendix.

Some notational conventions are: Matrices and vectors are set in boldface. Let $\boldsymbol{y}$ be a vector, then $\boldsymbol{y} > 0$ means that $\boldsymbol{y}_l := [\boldsymbol{y}]_l > 0$ for the lth component.

Interference Model. Consider a multiuser system with K users subject to mutual interference. The amount of interference experienced by the individual users can be controlled by properly allocating the transmit powers, which are collected in a vector $\boldsymbol{p} = [p_1, p_2, \ldots, p_K]^T$. We also consider the effect of receiver noise with power σ^2. Both quantities are stacked in an extended power vector $\bar{\boldsymbol{p}} = \left[\begin{smallmatrix}\boldsymbol{p}\\ \sigma^2\end{smallmatrix}\right]$. The link of the kth user is corrupted by interference $\mathcal{I}_k$, being a function of the power allocation $\bar{\boldsymbol{p}}$. Thus, an adequate performance measure for each user is the ratio between the desired signal power and interference+noise power

$$\mathrm{SINR}_k(\bar{\boldsymbol{p}}) = \frac{p_k}{\mathcal{I}_k(\bar{\boldsymbol{p}})} \qquad \forall k \in \{1, 2, \ldots, K\} \; .$$

In order to keep the results as general as possible, we will not focus on a particular system design. Instead, we assume that $\mathcal{I}_k(\bar{\boldsymbol{p}})$ is characterized by the following properties.

Definition 1. A function $\mathcal{I}_k : \mathbb{R}_+^{K+1} \mapsto \mathbb{R}_+$ is called *interference function* if the following properties hold.

A1: $\mathcal{I}_k(\bar{\boldsymbol{p}})$ is continuous on $\mathbb{R}_+^{K+1}$

A2: $\mathcal{I}_k(\mu\bar{\boldsymbol{p}}) = \mu\mathcal{I}_k(\bar{\boldsymbol{p}})$ for all $\bar{\boldsymbol{p}} \in \mathbb{R}_+^{K+1}$ and $\mu > 0$.

A3: $\mathcal{I}_k\big(\left[\begin{smallmatrix}\boldsymbol{p}_1\\ \sigma^2\end{smallmatrix}\right]\big) \geq \mathcal{I}_k\big(\left[\begin{smallmatrix}\boldsymbol{p}_2\\ \sigma^2\end{smallmatrix}\right]\big)$ if $\boldsymbol{p}_1 \geq \boldsymbol{p}_2$.

A4: $\mathcal{I}_k\big(\left[\begin{smallmatrix}\boldsymbol{p}\\ \sigma_1^2\end{smallmatrix}\right]\big) > \mathcal{I}_k\big(\left[\begin{smallmatrix}\boldsymbol{p}\\ \sigma_2^2\end{smallmatrix}\right]\big)$ if $\sigma_1^2 > \sigma_2^2$.

As an example, consider a multi-access channel with a link gain matrix $\boldsymbol{\Psi}(\boldsymbol{z}) \in \mathbb{R}_+^{K\times K}$, which possibly depends on

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some equalization strategy $\boldsymbol{z}$. Then, $\mathcal{I}_k$ could take on the non-linear form

$$\mathcal{I}_k^{(\Psi)}(\bar{\boldsymbol{p}}) = \min_{\boldsymbol{z}} \sum_{l \neq k} p_l \, [\boldsymbol{\Psi}(\boldsymbol{z})]_{kl} + \sigma^2, \quad 1 \leq k \leq K \ . \quad (1)$$

Thus, the proposed axiomatic definition of interference functions implicitly contains known problems in signal processing, like the joint optimization of beamformers and transmit powers [2].

2. QoS ACHIEVABLE REGION

Interference has a direct impact on the QoS of each user. We assume that the QoS is related to the SINR by a bijective mapping f, i.e., we have

$$Q_k = f(\mathrm{SINR}_k), \quad 1 \leq k \leq K \ . \quad (2)$$

Let γ be the inverse function of f, then the QoS requirement Q_k translates into the SINR requirement $\gamma_k := \gamma(Q_k)$.

Theorem 1. *The targets $\boldsymbol{Q} = [Q_1, \ldots, Q_K]^T$ are feasible, i.e., there exists a power allocation $\bar{\boldsymbol{p}}$ such that $\mathrm{SINR}_k(\bar{\boldsymbol{p}}) \geq \gamma(Q_k)$, $\forall k \in \{1, 2, \ldots, K\}$, if and only if $C_\gamma(\boldsymbol{Q}) < 1$, where*

$$C_\gamma(\boldsymbol{Q}) = \inf_{\boldsymbol{p} > 0} \Big(\max_{1 \leq k \leq K} \frac{\gamma(Q_k)\mathcal{I}_k(\bar{\boldsymbol{p}})}{p_k} \Big) \ . \quad (3)$$

If the powers $\boldsymbol{p}$ are unconstrained, then the achievable QoS region

$$\mathcal{Q} = \{\boldsymbol{Q} : C_\gamma(\boldsymbol{Q}) < 1\} \quad (4)$$

is only limited by mutual interference.

The region may be further restricted by additional power constraints. To this end, suppose that targets $Q_1, \ldots, Q_K$ are feasible, i.e., $\boldsymbol{Q} \in \mathcal{Q}$. Then we can minimize the total transmit power subject to QoS constraints:

$$P_{min}(\boldsymbol{Q}) = \min \sum_{k=1}^{K} p_k \quad \text{s.t.} \quad \mathrm{SINR}_k(\bar{\boldsymbol{p}}) \geq \gamma(Q_k) \ . \quad (5)$$

Problem (5) has a unique optimizer $\boldsymbol{p}^{opt}(\boldsymbol{Q})$, which is characterized by

$$\frac{p_k^{opt}}{\mathcal{I}_k(\bar{\boldsymbol{p}}^{opt})} = \gamma_k, \quad 1 \leq k \leq K \ . \quad (6)$$

Thus, the QoS region under a total power constraint is defined as

$$\mathcal{Q}(P_{max}) = \{\boldsymbol{Q} : \sum_k p_k^{opt}(\boldsymbol{Q}) \leq P_{max}\} \ .$$

The unconstrained region (4) is the union over all constrained regions

$$\mathcal{Q} = \bigcup_{P_{max}} \mathcal{Q}(P_{max}) \ . \quad (7)$$

Similarly, the QoS region under individual power constraints $\tilde{p} = [p_1^{max}, \ldots, p_K^{max}]$ is

$$\mathcal{Q}(\tilde{\boldsymbol{p}}) = \{\boldsymbol{Q} : \boldsymbol{p}^{opt}(\boldsymbol{Q}) \leq \tilde{\boldsymbol{p}}\} \ . \quad (8)$$

An arbitrary point $\boldsymbol{Q}$ in the respective achievable region is reached by the following algorithm, which monotonically converges to the unique allocation with optimal sum power efficiency [1]:

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initialize: n := 0, p̄^(0) := [0, ..., 0, σ²]^T
repeat
    n := n + 1
    p_k^(n) := γ(Q_k) I_k(p̄^(n−1)), ∀k ∈ {1, 2, ..., K}
    p̄^(n) := [p_1^(n), ..., p_K^(n), σ²]^T
until convergence
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It can be observed that $\sigma^2 > 0$ is required. The result extends previous work on decentralized power control [3, 4], where a specific choice of interference functions $\mathcal{I}_k$ is assumed.

Interestingly, the uniqueness of the optimum power allocation [1] holds for all kind of interference functions $\mathcal{I}$ which fulfill the axioms A1-A4. Thus, it can be concluded that uniqueness also holds for the joint optimization of beamformers and transmit powers [2], where the interference functions have the non-linear form (1). Note, that uniqueness of the optimal powers is not obvious, since the optimal beamformers themselves are not unique. The same result has been shown recently by means of a completely different mathematical technique [5].

3. GEOMETRICAL PROPERTIES FOR SPECIAL CLASSES OF QoS FUNCTIONS

In order to optimize the QoS over the achievable region, it is desirable to know the geometrical properties of the region. In this section we restrict our attention to certain classes of QoS functions. Namely, we assume that $\gamma(Q)$ is log-convex[1]. Some examples for mappings f with log-convex inverse are: $\log(\mathrm{SINR})$ (capacity in the high SNR regime) or $1/\mathrm{SINR}^d$ (bit error approximation for diversity order d [6]). We further restrict our attention to classes of interference functions $\mathcal{I}_k(\mathrm{e}^{\boldsymbol{s}})$ which are log-convex with respect to $\boldsymbol{s}$. Here we substitute $\boldsymbol{p} := \mathrm{e}^{\boldsymbol{s}}$ for the power vector. It should be emphasized that this property is fulfilled, e.g. by all interference functions with the common linear form $\sum_l p_l \nu_l + \sigma_n^2$ for some path attenuations ν_l and receiver noise σ_n^2.

Theorem 2. *$C_\gamma(\boldsymbol{Q})$, as defined in (3), is log-convex with respect to $\boldsymbol{Q}$.*

Moreover, it can be shown that 1) If $\gamma(Q)$ is strictly log-convex, then $C_\gamma(\boldsymbol{Q})$ is strictly log-convex. 2) If the functions $\mathcal{I}_k(\boldsymbol{p}(\lambda))$, with $\boldsymbol{p}(\lambda) = \mathrm{e}^{\boldsymbol{s}(\lambda)}$, $\boldsymbol{s}(\lambda) = (1-\lambda)\boldsymbol{s}^{(0)} +$

[1]A function f is called log-convex if $\log f$ is convex. Log-convexity is stronger than convexity since each log-convex function is convex, but the converse is not true.

$\lambda\boldsymbol{s}^{(1)}$, $\boldsymbol{s}^{(0)} \neq \boldsymbol{s}^{(1)}$, are strictly log-convex, then $C_\gamma(\boldsymbol{Q})$ is strictly log-convex.

Theorem 3. *Let $\boldsymbol{p}^{opt}(\boldsymbol{Q})$ be the optimizer of the power minimization problem (5) with target SINR's $\gamma(Q_k)$, $1 \leq k \leq K$, where $\gamma(Q_k)$ is log-convex, then the functions $p_k^{opt}(\boldsymbol{Q})$, $1 \leq k \leq K$, are log-convex with respect to $\boldsymbol{Q}$.*

Theorem 4. *The minimum total power $P_{min}(\boldsymbol{Q})$, as defined in (5), is log-convex with respect to $\boldsymbol{Q}$.*

For the scenario under consideration (axioms A1-A4 plus the above assumptions), it can even be shown that the feasibility region $\mathcal{Q}$ is strictly convex. Thus, if $\boldsymbol{Q}^{(1)}$ and $\boldsymbol{Q}^{(2)}$ are boundary points of the QoS feasibility region, then all points on the interconnecting line lie in the interior of the region. Furthermore, if $\gamma(Q)$ is log-convex, then the set $\mathcal{Q}(\tilde{\boldsymbol{p}})$, as defined in (8), is convex, and the achievable region $\mathcal{Q}(P_{max})$ is strictly convex.

4. RESOURCE ALLOCATION BY WEIGHTED QoS OPTIMIZATION

In this section we focus on another class of QoS functions. We assume that the SINR is mapped on the QoS region by a function $f(x) = g(1/x)$, thus

$$Q = g(1/\text{SINR}) \ .$$

The function g is assumed to be monotonically increasing and $g(\mathrm{e}^x)$ shall be convex with respect to x. Examples for such functions are $g(x) = x$ or $g(x) = \log x$. Thus, the QoS is proportional to the inverse SINR, which can be interpreted as the bit error rate approximation, as discussed in Section 3.

A general strategy for network resource allocation is

$$\min_{\boldsymbol{s}\in\mathbb{R}^K} \sum_{k=1}^{K} \alpha_k \ g\big(\mathcal{I}_k(\mathrm{e}^{\boldsymbol{s}})/\mathrm{e}^{\boldsymbol{s}_k}\big) \quad \text{s.t.} \ \ \|\mathrm{e}^{\boldsymbol{s}}\|_1 \leq P_{max} \ , \qquad (9)$$

where the interference function $\mathcal{I}_k$ fulfills the axioms A1-A4. In addition, $\mathcal{I}_k(\mathrm{e}^{\boldsymbol{s}})$ is log-convex in $\boldsymbol{s}$. The weights $\boldsymbol{\alpha} = [\alpha_1, \ldots, \alpha_K]$ model individual user requirements and possibly depend on system parameters like priorities, queue lengths, etc. By appropriately choosing $\boldsymbol{\alpha}$ it is possible to trade off throughput against fairness (illustrated in Fig. 1). The optimum can be found with the following result [7].

Theorem 5. *The problem (9) is convex if and only if $g(\mathrm{e}^x)$ is convex with respect to x.*

Thus, with the results in Section 3 we have two different definitions of QoS functions, namely f and g, whose special properties ensure that the resulting QoS regions are convex. For the special choice of g we can even solve the general problem of weighted QoS optimization of the form (9).

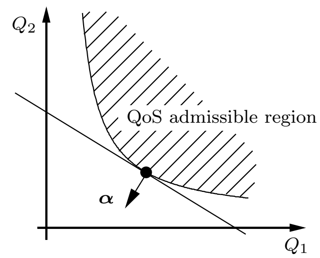

Fig. 1. The resource allocation problem: weighted minimization over the boundary of the QoS achievable region

Note, that the optimization is over the non-compact set $\mathbb{R}^K$, thus even if the problem is convex, it is not obvious that the optimum is achieved (e.g. $\boldsymbol{s} \to -\infty$ might be necessary to achieve a valid QoS point). However, this case is excluded by axiom A4 and the fact that $\sigma_n^2 > 0$. For a practical system with receiver noise $\sigma_n^2 > 0$, we always have strictly positive interference $\mathcal{I}(\bar{\boldsymbol{p}}) > 0$. Thus, $\mathrm{e}^{\boldsymbol{s}_k} \to 0$ can be ruled out, since otherwise the objective would tend to infinity, away from the minimum.

Such a behavior is desired for systems, where all users are active at the same time. Then, (9) could be used to control certain performance aspects, like bit error rates. For any choice of parameters $\boldsymbol{\alpha}$, the optimization problem (9) can be solved by standard convex optimization techniques.

5. CONCLUSIONS

The complex problem of joint interference equalization and resource allocation can be handled by using an axiomatic approach. Many properties known in power control theory [3, 4] and network optimization [7] can be extended to a more general framework.

In this paper we show for certain classes of QoS functions, that the resulting achievable region has suitable geometrical properties. This helps to better understand the performance tradeoff between users in a network. The convexity of the region allows for efficient algorithmic solutions. The results may prove useful for a wide range of problems in the field of interference management and resource allocation. By optimizing over the boundary of the achievable region it is possible to trade-off fairness against efficiency.

Sketches of the Proofs

Thm. 1. Suppose that there exists $\bar{\boldsymbol{p}}'$ such that $\boldsymbol{Q}$ is feasible. Let $\bar{\boldsymbol{p}}'(\lambda) := \left[\begin{smallmatrix}\lambda\boldsymbol{p}'\\ 1\end{smallmatrix}\right]$. The function $f_k(\lambda) := \frac{\lambda p_k'}{\mathcal{I}_k\left(\bar{\boldsymbol{p}}'(\lambda)\right)} = \frac{p_k'}{\mathcal{I}_k\left(\left[\begin{smallmatrix}\boldsymbol{p}'\\ 1/\lambda\end{smallmatrix}\right]\right)}$ is strictly monotonically increasing in λ (axiom A4). Combining axioms A3 and A4 and defining $\underline{\mathcal{I}}_k(\boldsymbol{p}) := \mathcal{I}_k\left(\left[\begin{smallmatrix}\boldsymbol{p}\\ 0\end{smallmatrix}\right]\right)$, it follows that for all $\lambda > 1$ we have

$$\frac{p'_k}{\underline{\mathcal{I}}_k(\boldsymbol{p}')} > \frac{p'_k}{\mathcal{I}_k\big(\big[\begin{smallmatrix}\boldsymbol{p}'\\1/\lambda\end{smallmatrix}\big]\big)} > \frac{p'_k}{\mathcal{I}_k(\bar{\boldsymbol{p}}')} \geq \gamma_k, \quad \forall k\ .$$

Thus, $\min_{1\leq k\leq K}\frac{p'_k}{\gamma_k \underline{\mathcal{I}}_k(\boldsymbol{p}')} > 1$ and $C(\boldsymbol{\Gamma}) > 1$.

Conversely, assume that $C(\boldsymbol{\Gamma}) > 1$, which implies the existence of a vector $\hat{\boldsymbol{p}}$ such that $\frac{\hat{p}_k}{\underline{\mathcal{I}}_k(\hat{\boldsymbol{p}})} > \gamma_k, \quad 1 \leq k \leq K$. The function $\mathcal{I}_k\big(\big[\begin{smallmatrix}\hat{\boldsymbol{p}}\\1/\lambda\end{smallmatrix}\big]\big)$ is strictly monotonically increasing in λ. Because of the continuity of $\mathcal{I}_k$ (axiom A1) we have

$$\lim_{\lambda\to\infty}\mathcal{I}_k\big(\big[\begin{smallmatrix}\hat{\boldsymbol{p}}\\1/\lambda\end{smallmatrix}\big]\big) = \underline{\mathcal{I}}_k(\hat{\boldsymbol{p}}) < \frac{\hat{p}_k}{\gamma_k}\ .$$

Thus, there exists a λ such that $\frac{p_k(\lambda)}{\mathcal{I}_k\big(\bar{\boldsymbol{p}}(\lambda)\big)} > \gamma_k$, $1 \leq k \leq K$, which proves feasibility. ■

Thm 2. Let $\boldsymbol{Q}(\lambda) = (1-\lambda)\boldsymbol{Q}^{(1)} + \lambda\boldsymbol{Q}^{(2)}$, $\lambda \in [0,1]$, where $\boldsymbol{Q}^{(1)}, \boldsymbol{Q}^{(2)} \in \mathcal{Q}$. Also, $\gamma(\lambda) := \gamma(Q_k(\lambda))$ and $\gamma_k^{(l)} = \gamma(Q_k^{(l)})$, $l = 1,2$. There exists an $\epsilon > 0$ and vectors $\boldsymbol{p}_\epsilon^{(1)}$, and $\boldsymbol{p}_\epsilon^{(2)}$, such that

$$\max_{1\leq k\leq K}\log\frac{\gamma_k^{(l)}\mathcal{I}_k(\boldsymbol{p}_\epsilon^{(l)})}{[\boldsymbol{p}_\epsilon^{(l)}]_k} \leq \log C_\gamma\big(\boldsymbol{Q}^{(l)}\big) + \epsilon, \quad l = 1,2\ . \tag{10}$$

Substituting $\boldsymbol{s}_\epsilon^{(l)} = \log\boldsymbol{p}_\epsilon^{(l)}$, we define $\boldsymbol{s}(\lambda) = (1-\lambda)\boldsymbol{s}_\epsilon^{(1)} + \lambda\boldsymbol{s}_\epsilon^{(2)}$ and $\boldsymbol{p}(\lambda) = \mathrm{e}^{\boldsymbol{s}(\lambda)}$. It can be shown that

$$\begin{aligned}&\log\Big(\gamma_k(\lambda)\frac{\mathcal{I}_k\big(\mathrm{e}^{\boldsymbol{s}(\lambda)}\big)}{[\mathrm{e}^{\boldsymbol{s}(\lambda)}]_k}\Big)\\ &\quad\leq (1-\lambda)\log C_\gamma\big(\boldsymbol{Q}^{(1)}\big) + \lambda\log C_\gamma\big(\boldsymbol{Q}^{(2)}\big) + \epsilon\end{aligned}$$

for all $k \in \{1,2,\ldots,K\}$. Here we have used (10) and the assumption that $\log\gamma(Q_k)$ is convex with respect to Q_k and $\mathcal{I}_k\big(\mathrm{e}^{\boldsymbol{s}}\big)/[\mathrm{e}^{\boldsymbol{s}}]_k$ is log-convex with respect to $\boldsymbol{s}$. It follows that

$$\begin{aligned}\log C_\gamma\big(\boldsymbol{Q}(\lambda)\big) &= \inf_{\boldsymbol{s}\in\mathbb{R}^K_+}\Big(\max_{1\leq k\leq K}\frac{\gamma_k(\lambda)\mathcal{I}_k\big(\mathrm{e}^{\boldsymbol{s}(\lambda)}\big)}{\big[\mathrm{e}^{\boldsymbol{s}(\lambda)}\big]_k}\Big)\\ &\leq (1-\lambda)\log C_\gamma(\boldsymbol{Q}^{(1)}) + \lambda\log C_\gamma(\boldsymbol{Q}^{(2)}) + \epsilon.\end{aligned} \tag{11}$$

This holds for any $\epsilon > 0$ and the left-hand side of (11) does not depend on ϵ. Thus $C_\gamma(\boldsymbol{Q})$ is log-convex. ■

Thm 3. Let $\boldsymbol{Q}^{(0)}$ and $\boldsymbol{Q}^{(1)}$ be feasible points. From Theorem 2 it is clear that $1 \geq C_\gamma\big(\boldsymbol{Q}(\lambda)\big)$, thus, all $\boldsymbol{Q}(\lambda)$ are feasible and achieved by optimal power allocations $\boldsymbol{p}(\lambda) := \boldsymbol{p}^{opt}(\boldsymbol{Q}(\lambda))$ characterized by $p_k(\lambda) = \gamma_k(\lambda)\mathcal{I}_k\big(\bar{\boldsymbol{p}}(\lambda)\big), \quad 1 \leq k \leq K$. Let $\gamma_k(\lambda) := \gamma\big(Q_k(\lambda)\big)$. Exploiting the log-convexity of γ and $\mathcal{I}_k(\bar{\boldsymbol{p}})$ we have

$$\begin{aligned}p_k(\lambda) &= \gamma_k(\lambda)\mathcal{I}_k\big(\bar{\boldsymbol{p}}(\lambda)\big)\\ &\leq \big(\gamma_k(0)\big)^{(1-\lambda)}\big(\gamma_k(1)\big)^{\lambda}\big(\mathcal{I}_k\big(\bar{\boldsymbol{p}}(0)\big)\big)^{(1-\lambda)}\big(\mathcal{I}_k\big(\bar{\boldsymbol{p}}(1)\big)\big)^{\lambda}\\ &= \big(\gamma_k(0)\mathcal{I}_k\big(\bar{\boldsymbol{p}}(0)\big)\big)^{(1-\lambda)}\big(\gamma_k(1)\mathcal{I}_k\big(\bar{\boldsymbol{p}}(1)\big)\big)^{\lambda}\\ &= \big(p_k(0)\big)^{(1-\lambda)}\big(p_k(1)\big)^{\lambda}\ ,\end{aligned}$$

and thus $\log p_k(\lambda) \leq (1-\lambda)\log p_k(0) + \lambda\log p_k(1)$. ■

Thm 4. Let $\boldsymbol{Q}^{(0)}$ and $\boldsymbol{Q}^{(1)}$ be feasible points and let $\boldsymbol{Q}(\lambda)$ and the associated optimizers $\boldsymbol{p}(\lambda)$ be defined as before. Consider vectors $\boldsymbol{p}'(\lambda)$ which fulfill $\log p'_k(\lambda) = (1-\lambda)\log p_k(0) + \lambda\log p_k(1)$. We have $P_{min}\big(\boldsymbol{Q}(\lambda)\big) = \sum_k p_k(\lambda)$, thus

$$\begin{aligned}P_{min}\big(\boldsymbol{Q}(\lambda)\big) &\leq \sum_k p'_k(\lambda) = \sum_k \big(p_k(0)\big)^{(1-\lambda)}\big(p_k(1)\big)^{\lambda}\\ &\leq \big(\sum_k p_k(0)\big)^{(1-\lambda)}\big(\sum_k p_k(1)\big)^{\lambda}\ ,\end{aligned}$$

where the last step follows from the Hölder inequality. Thus,

$$\log P_{min}\big(\boldsymbol{Q}(\lambda)\big) \leq (1-\lambda)\log P_{min}(\boldsymbol{Q}^{(0)}) + \lambda\log P_{min}(\boldsymbol{Q}^{(1)})\ .$$

In [8] it was shown that for functions $\gamma(Q) = \mathrm{e}^Q$ and $\gamma(Q) = 1/Q$, the inequality is strict. ■

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