CAPACITY AND LINEAR PRECODING FOR PACKET RETRANSMISSIONS

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ABSTRACT

This paper presents a method for enhancing wireless ARQ system performance by combining packet retransmissions through frequency-selective channels. As a measure of system performance, we derive the total mutual information (or channel capacity) provided by multiple transmissions of a packet through different channels. Retransmissions are uniquely linear precoded and/or truncated in relation to the original transmission. With the objective of maximizing total throughput, optimal linear precoders are developed for zero-padded packet transmissions. These precoders effectively implement power loading with the right singular vectors of the original transmission channel serving as basis vectors. Simulation results validate the increase the overall mutual information and throughput from optimized precoding.

1. INTRODUCTION

In practical packet switched wireless systems, critical figures of merit for communication effectiveness often include low frame error rates (FER) and high data throughputs to its end users. The effective handling and reduction of packet retransmissions is vital in meeting these needs. Generally for systems with retransmission capabilities, if errors remain (possibly after error correction) in demodulating a data packet, a request for retransmission is made to the transmitter. As a result, the development of Automatic Repeat reQuest (ARQ) protocols has been the subject of much research at the network and physical layers. A general discussion of ARQ systems from a coding perspective is provided in [1].

In most ARQ protocol designs, improvements in bit error rates and throughputs are the primary goals. A related objective to minimizing bit error rate is the maximization of mutual information, or channel capacity. We now take a different perspective on packet retransmissions, namely the gain in mutual information provided by precoding retransmissions through ISI channels. Maximizing the mutual information of single block transmissions through ISI channels has been the objective of several works. Dhahir and Cioffi developed realizable precoding filters using this criterion [2]. Scaglione *et al.* similarly studied the use of transmitter and receiver filterbanks to maximize information rates [3]. Dhahir and Diggavi later investigated optimizing the guard sequences that separate consecutive information blocks [4].

In this paper, we seek to maximize precoding gains, in terms of input/output mutual information obtained, when packet retransmissions are made. We begin by first modelling multiple transmissions of a packet through ISI channels (an FIR filter) with zero-padding between consecutive packets. Unique linear precoding of each retransmitted packet is incorporated into the model. Next, the total channel capacity over all M transmissions of a packet is formulated. A discussion of a procedure to find the optimal precoder for each retransmission follows. We conclude with a set of simulations and a presentation of results, indicating that precoding each retransmission produces a significant increase in data throughput.

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2. SYSTEM MODEL

We begin with a packet of N symbols denoted by the vector $\mathbf{s} = [s_1, \ldots, s_N]^T$. This packet is transmitted over a linear channel $\mathbf{H}$ and corrupted by noise $\mathbf{w}$, with the receiver obtaining a vector $\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{w}$. One frequent example of a linear channel uses our first model of a frequency-selective channel, with an FIR filter whose coefficients are $\mathbf{h} = [h_1, \ldots, h_L]^T$. The matrix $\mathbf{H}$ is a $(N + L - 1) \times N$ Toeplitz matrix defined as

$$\mathbf{H} = \begin{bmatrix} h_1 & 0 & \ldots & \ldots & \ldots & 0 \\ & \ddots & \ddots & & \ddots & \\ h_L & \ldots & h_1 & 0 & \ldots & \vdots \\ 0 & h_L & \ldots & h_1 & \ddots & \\ \vdots & & \ddots & & \ddots & 0 \\ 0 & \ldots & 0 & h_L & \ldots & h_1 \\ \vdots & & & \ddots & \ddots & \\ 0 & \ldots & \ldots & \ldots & 0 & h_L \end{bmatrix}. \tag{1}$$

Each transmitted packet $\mathbf{s}$ is followed by a guard sequence of at least $L - 1$ zeros (zero-padding) to eliminate the effect of inter-packet interference. Given this definition, both $\mathbf{y}$ and $\mathbf{w}$ are vectors of length $N + L - 1$. The noise vector $\mathbf{w}$ is assumed to be white and Gaussian with $\mathcal{CN}(\mathbf{0}_{(N+L-1)\times 1}, \sigma_w^2 \mathbf{I}_{N+L-1})$.

3. PRECODING PACKET RETRANSMISSIONS

We expand our original model to include packet retransmissions, concentrating specifically on zero-padded transmissions. In enhancing the existing diversity over all transmissions of the packet, pre-processing or linear precoding of retransmissions is included. Moreover, truncation (puncturing) is incorporated, where the size of the retransmitted packet is smaller than that of the original transmission. Fig. 1 illustrates several precoded transmissions of $\mathbf{s}$, a packet of N message symbols, where the m^{th} received transmission is $\mathbf{y}_m = \mathbf{H}_m \mathbf{Q}_m \mathbf{s} + \mathbf{w}_m$. The matrix $\mathbf{Q}_m$ is a $N_m \times N$

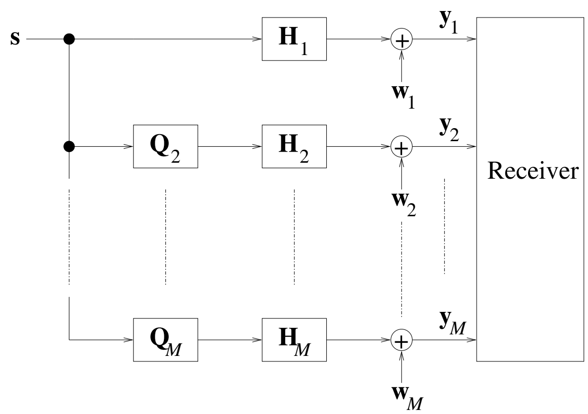

Fig. 1. Block diagram of multiple precoded transmissions of a packet through frequency-selective channels.

precoder where $N_m \leq N$ for bandwidth conservation in retransmission. Thus, $\mathbf{Q}_m$ represents the pre-processing and/or truncation of the m^{th} transmission. By default, $N_1 = N$ and $\mathbf{Q}_1 = \mathbf{I}_N$ for the initial transmission. The precoder $\mathbf{Q}_m$ is constrained by $\text{tr}\{\mathbf{Q}_m\mathbf{Q}_m^H\} = N_m$, so as to not amplify the transmit power. The $(N_m + L - 1) \times N_m$ channel matrix $\mathbf{H}_m$ represents the channel $\mathbf{h}_m$. All M channels are assumed to have length L and unit energy, but can be distinct (i.e. $\mathbf{h}_1 \neq \mathbf{h}_2 \neq \cdots \neq \mathbf{h}_M$). Finally, the vectors $\mathbf{y}_m$ and $\mathbf{w}_m$ have length $N_m + L - 1$, with $\mathbf{w}_1, \ldots, \mathbf{w}_M$ independent noise vectors with variance σ_w^2. Overall, the vector of received samples $\mathbf{y}$ is defined as

$$\mathbf{y} = \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_M \end{bmatrix} = \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2\mathbf{Q}_2 \\ \vdots \\ \mathbf{H}_M\mathbf{Q}_M \end{bmatrix} \mathbf{s} + \begin{bmatrix} \mathbf{w}_1 \\ \mathbf{w}_2 \\ \vdots \\ \mathbf{w}_M \end{bmatrix}, \quad (2)$$
$$= \mathbb{H}\mathbf{s} + \mathbf{w}.$$

With the above model, two coupled issues naturally arise. First, the optimal precoders $\mathbf{Q}_2, \ldots, \mathbf{Q}_M$ must be determined. Second, a criterion for optimality has to be defined. While traditional measures like BER or receiver SNR are valid, we intend to discuss a different, information-theoretic approach. We define our criterion to be the channel capacity (mutual information) provided over all M transmissions, with the goal of maximization. In other words, optimality is defined as selecting $\mathbf{Q}_2, \ldots, \mathbf{Q}_M$ to maximize the mutual information $I(\mathbf{y}; \mathbf{s})$ between $\mathbf{y}$ and $\mathbf{s}$.

In finding the optimal precoders, consideration of the ARQ mechanism is required. When the m^{th} transmission is made, using the precoder $\mathbf{Q}_m$, it is made with the intention that future transmissions should not be necessary. Thus, the design of $\mathbf{Q}_m$ should not consider $\mathbf{y}_{m+1}, \ldots, \mathbf{y}_M$. An iterative approach to finding precoders is preferred, where $\mathbf{Q}_2$ is first designed to maximize capacity for $M = 2$, then $\mathbf{Q}_3$ is chosen to maximize capacity for $M = 3$ with $\mathbf{Q}_2$ already designed, etc. Before discussing precoder design, we first formulate the channel capacity over M transmissions.

4. OVERALL CHANNEL CAPACITY

Dhahir and Cioffi developed mutual information expressions for block or packet transmissions through ISI channels [2]. We extend some of their results to the model outlined in (2). Given that the noise $\mathbf{w}$ and data $\mathbf{s}$ are Gaussian, the mutual information is

$$I(\mathbf{y}; \mathbf{s}) = \log\det(\mathbf{R}_{ww}^{-\frac{1}{2}}\mathbf{R}_{yy}\mathbf{R}_{ww}^{-\frac{1}{2}}),$$

where $\mathbf{R}_{yy} = E\{\mathbf{y}\mathbf{y}^H\}$ and $\mathbf{R}_{ww} = E\{\mathbf{w}\mathbf{w}^H\}$ are the autocorrelation matrices of the received signal and noise, respectively. The base of the logarithm is arbitrary; a base of two is typically used to express the mutual information in bits. Using (2), the received autocorrelation becomes

$$\mathbf{R}_{yy} = \mathbb{H}\mathbf{R}_{ss}\mathbb{H}^H + \mathbf{R}_{ww},$$

with $\mathbf{R}_{ss} = E\{\mathbf{s}\mathbf{s}^H\}$. We assume the signal and noise are white, with correlation matrices $\mathbf{R}_{ss} = \sigma_s^2\mathbf{I}_N$, $\mathbf{R}_{w_m w_m} = \sigma_w^2\mathbf{I}_{N_m+L-1}$, and $\mathbf{R}_{w_m w_k} = \mathbf{0}_{(N_m+L-1)\times(N_k+L-1)}$ for $m \neq k$. We also define the SNR $\gamma = \sigma_s^2/\sigma_w^2$. Thus,

$$I(\mathbf{y}; \mathbf{s}) = \log\det(\gamma\mathbb{H}\mathbb{H}^H + \mathbf{I}), \quad (3)$$
$$= \log\det(\gamma\mathbb{H}^H\mathbb{H} + \mathbf{I}_N). \quad (4)$$

Expanding the matrix $\mathbb{H}$ leads to an expanded form of $I(\mathbf{y}; \mathbf{s})$,

$$\log\det\left(\mathbf{I}_N + \gamma\mathbf{H}_1^H\mathbf{H}_1 + \gamma\sum_{m=2}^{M}\mathbf{Q}_m^H\mathbf{H}_m^H\mathbf{H}_m\mathbf{Q}_m\right). \quad (5)$$

5. OPTIMAL RETRANSMISSION PRECODING

In developing a method for obtaining the optimal precoder(s), we start with $M = 2$ and work toward obtaining the optimal $\mathbf{Q}_2$. Using the singular value decomposition (SVD), we define $\mathbf{H}_m = \mathbf{U}_m\mathbf{\Sigma}_m\mathbf{V}_m^H$ with $\mathbf{U}_m$ a $(N_m + L - 1) \times N_m$ unitary matrix and $\mathbf{V}_m$ being a $N_m \times N_m$ unitary matrix. Generally $\mathbf{U}_m$ is a $(N_m + L - 1) \times (N_m + L - 1)$ matrix, but the rank (and the number of singular values) of $\mathbf{H}_m$ is N_m. Hence, the last $L - 1$ columns of $\mathbf{U}_m$ become unnecessary. The matrix $\mathbf{\Sigma}_m$ is a $N_m \times N_m$ diagonal matrix whose diagonal elements are the N_m singular values $\sigma_m[1], \ldots, \sigma_m[N_m]$ of $\mathbf{H}_m$. We assume that the singular values are real numbers that are ordered from largest to smallest, and the singular vectors of $\mathbf{U}_m$ and $\mathbf{V}_m$ are ordered accordingly.

The channel capacity with $M = 2$ then becomes

$$\log\det\left(\mathbf{I}_N + \gamma\mathbf{V}_1\mathbf{\Sigma}_1^2\mathbf{V}_1^H + \gamma\mathbf{Q}_2^H\mathbf{V}_2\mathbf{\Sigma}_2^2\mathbf{V}_2^H\mathbf{Q}_2\right). \quad (6)$$

With $\mathbf{V}_1\mathbf{V}_1^H = \mathbf{I_N}$ and $\det(\mathbf{V}_1) = \det(\mathbf{V}_1^H) = 1$, (6) becomes

$$\log\det\left(\mathbf{I}_N + \gamma\mathbf{\Sigma}_1^2 + \gamma\mathbf{V}_1^H\mathbf{Q}_2^H\mathbf{V}_2\mathbf{\Sigma}_2^2\mathbf{V}_2^H\mathbf{Q}_2\mathbf{V}_1\right). \quad (7)$$

Maximizing (7) is fairly simple as Hadamard's matrix inequality indicates that the determinant is maximized when its matrix argument is diagonalized [5]. However, we have a power constraint $\text{tr}\{\mathbf{Q}_2\mathbf{Q}_2^H\} = N_m$, and one can construct simple cases where the maximizing matrix argument is not diagonal. Fortunately, we have constructed a proof that returns us to the intuitive solution of diagonalizing the matrix argument [6].

The matrix of interest in (7) is thus diagonalized when

$$\mathbf{Q}_2 = \mathbf{V}_2\mathbf{\Sigma}_{Q_2}\mathbf{V}_1^H, \quad (8)$$

where $\mathbf{\Sigma}_{Q_2}$ is a $N_2 \times N$ diagonal matrix, with the singular values $\sigma_{Q_2}[1], \ldots, \sigma_{Q_2}[N_2]$ denoting the diagonal elements. Note that this definition of $\mathbf{Q}_2$ is the singular value decomposition of $\mathbf{Q}_2$. The channel capacity then simplifies to

$$\sum_{n=1}^{N} \log\left(1+\gamma\sigma_1^2[n]\right) + \sum_{n=1}^{N_2} \log\left(1+\frac{\sigma_2^2[n]\sigma_{Q_2}^2[n]}{\frac{1}{\gamma}+\sigma_1^2[n]}\right). \quad (9)$$

The channel capacity is maximized when the second summation in (9) is maximized. The energy constraint reduces to

$$\mathrm{tr}\{\mathbf{Q}_2\mathbf{Q}_2^H\} = \sum_{n=1}^{N_2} \sigma_{Q_2}^2[n] = N_2,$$

maintaining the proper SNR. The above mentioned proof also indicates the proper pairing of the N_2 singular values of $\mathbf{H}_2$ with N_2 of the N singular values of $\mathbf{H}_1$. The smallest singular value of $\mathbf{H}_1$ is paired with the largest singular value of $\mathbf{H}_2$, the second smallest with the second largest, etc.

This singular value pairing aligns the weakest singular vector of $\mathbf{H}_1$ with the strongest singular vector of $\mathbf{H}_2$, which is intuitively gratifying. The pairing of singular vectors and values is obtained through the function $p_2[n]$, which assigns the n^{th} singular vector/value of $\mathbf{H}_2$ to $(p_2[n])^{\text{th}}$ singular vector/value of $\mathbf{H}_1$. In this case, $p_2[1] = N_2, p_2[2] = N_2 - 1, \ldots, p_2[N_2] = 1$. Additionally, $\mathbf{P}_2$ is an $N_2 \times N$ matrix representation of $p_2[n]$; in each row n, column element $p_2[n]$ is unity and the remaining elements are zero. The optimal precoder is now

$$\mathbf{Q}_2 = \mathbf{V}_2\mathbf{\Sigma}_{Q_2}\mathbf{P}_2\mathbf{V}_1^H,$$

and the resulting channel capacity is

$$\log\det\left(\mathbf{I}_N + \gamma\mathbf{\Sigma}_1^2 + \gamma\mathbf{P}_2^T\mathbf{\Sigma}_{Q_2}^T\mathbf{\Sigma}_2^2\mathbf{\Sigma}_{Q_2}\mathbf{P}_2\right),$$

or

$$\sum_{n=1}^{N} \log\left(1+\gamma\sigma_1^2[n]\right) + \sum_{n=1}^{N_2} \log\left(1+\frac{\sigma_2^2[n]\sigma_{Q_2}^2[n]}{\frac{1}{\gamma}+\sigma_1^2[p_2[n]]}\right).$$

Our optimization criterion is

$$\max_{\sigma_{Q_2}[1],\ldots,\sigma_{Q_2}[N_2]} \sum_{n=1}^{N_2} \log\left(1+\frac{\sigma_2^2[n]\sigma_{Q_2}^2[n]}{\frac{1}{\gamma}+\sigma_1^2[p_2[n]]}\right), \quad (10)$$

subject to the constraints

$$\sum_{n=1}^{N_2} \sigma_{Q_2}^2[n] = N_2, \quad \sigma_{Q_2}[n] \in \mathcal{R}, \sigma_{Q_2}[n] \geq 0, \quad \forall n. \quad (11)$$

This is a highly nonlinear optimization, particularly due to the requirement that the singular values be real. Fortunately, various quadratic programming algorithms may be used to solve (10). As an alternative, we propose relaxing the non-negativity constraints and using a Lagrange multiplier,

$$\sum_{n=1}^{N_2} \log\left(1+\frac{\sigma_2^2[n]\sigma_{Q_2}^2[n]}{\frac{1}{\gamma}+\sigma_1^2[p_2[n]]}\right) + \lambda\left(\sum_{n=1}^{N_2}\sigma_{Q_2}^2[n] - N_2\right). \quad (12)$$

Solving (12) leads to

$$\sigma_{Q_2}^2[n] = 1 - \frac{\frac{1}{\gamma}+\sigma_1^2[p_2[n]]}{\sigma_2^2[n]} + \frac{1}{N_2}\sum_{k=1}^{N_2}\frac{\frac{1}{\gamma}+\sigma_1^2[p_2[k]]}{\sigma_2^2[k]},$$

This solution generally produces some negative values for $\sigma_{Q_2}^2[n]$, which violates the real-valued constraint. Let $\mathcal{S}$ denote the indices of these negative values, with $\mathcal{S}_0$ denoting the indices of the non-negative singular values. By setting $\sigma_{Q_2}^2[n] = 0, n \in \mathcal{S}$, the real-valued constraint is satisfied, but the energy constraint is violated. To satisfy the energy constraint, we recompute $\alpha = \sum_{n\in\mathcal{S}_0}\sigma_{Q_2}^2[n]$ and set $\sigma_{Q_2}^2[n] = N_2\sigma_{Q_2}^2[n]/\alpha$ for $n \in \mathcal{S}_0$. Using this simpler solution provides results very near those produced by quadratic programming.

Overall, the optimal precoder effectively aligns the strongest N_2 right singular vectors of $\mathbf{H}_2$ to the weakest N_2 right singular vectors of $\mathbf{H}_1$. Additionally, the matrix $\mathbf{\Sigma}_{Q_2}$ is a power loading mechanism that attempts to alleviate the smaller singular values of $\mathbf{H}_1$ by using the larger singular values of $\mathbf{H}_2$. This is similar to "water-filling" or power loading for discrete multitone (DMT) modulations [5, 7]. One can relate DMT subcarriers with the right singular vectors of $\mathbf{H}_1$.

Precoders for $M > 2$ are found using the same method. As an example, we consider finding the precoder $\mathbf{Q}_3$. With $\mathbf{Q}_2$ optimized, the channel capacity for $M = 3$ is similar to (8),

$$\log\det\left(\mathbf{I}_N + \gamma\tilde{\mathbf{\Sigma}}_1^2 + \gamma\mathbf{V}_1^H\mathbf{Q}_3^H\mathbf{V}_3\mathbf{\Sigma}_3^2\mathbf{V}_3^H\mathbf{Q}_3\mathbf{V}_1\right),$$

with

$$\tilde{\mathbf{\Sigma}}_1^2 = \mathbf{\Sigma}_1^2 + \mathbf{P_2}^T\mathbf{\Sigma}_{Q_2}^T\mathbf{\Sigma}_{Q_2}\mathbf{\Sigma}_2^2\mathbf{P_2}.$$

The optimal precoder $\mathbf{Q}_3 = \mathbf{V}_3\mathbf{\Sigma}_{Q_3}\mathbf{P}_3\mathbf{V}_1$ has the same form as $\mathbf{Q}_2$, and its singular values are determined similarly, using pairing function $p_3[n]$ and $N_3 \times N$ pairing matrix $\mathbf{P}_3$. The pairing function assigns the largest singular value of $\mathbf{H}_3$ ($\sigma_3[N_3]$) to the smallest singular value in $\tilde{\mathbf{\Sigma}}_1$, the second largest to the second smallest, etc.

In general, the optimal precoder $\mathbf{Q}_M$ is

$$\mathbf{Q}_M = \mathbf{V}_M\mathbf{\Sigma}_{Q_M}\mathbf{P}_M\mathbf{V}_1,$$

with pairing function $p_M[n]$ and $N_M \times N$ pairing matrix P_M, and the singular values σ_{Q_M} are determined by

$$\max_{\sigma_{Q_M}[1],\ldots,\sigma_{Q_M}[N_M]} \sum_{n=1}^{N_M} \log\left(1+\frac{\sigma_M^2[n]\sigma_{Q_M}^2[n]}{\frac{1}{\gamma}+\tilde{\sigma}_1^2[p_M[n]]}\right),$$

with σ_{Q_M} constrained as σ_{Q_2} is in (11), and where $\tilde{\sigma}_1^2[n]$ are the diagonal elements of

$$\tilde{\mathbf{\Sigma}}_1^2 = \mathbf{\Sigma}_1^2 + \sum_{m=2}^{M-1}\mathbf{P}_m^T\mathbf{\Sigma}_{Q_m}^T\mathbf{\Sigma}_m^2\mathbf{\Sigma}_{Q_m}\mathbf{P}_m.$$

6. RESULTS

To illustrate the value of linear precoding, specifically the alignment of singular values, we present a simple example with $M = 2$ and $N = 50$ symbols. The channels of interest in this example are

$$\mathbf{h}_1 = [0.227\ 0.46\ 0.688\ 0.46\ 0.227]^T$$

$$\mathbf{h}_2 = [0.227\ 0.227\ 0.46\ 0.46\ 0.688]^T.$$

Fig. 2 contains the aligned sets of singular values σ_1 and σ_2 of the transmission channels and the set of precoder singular values σ_{Q_2} for $N_2 = 50$. It illustrates the pairing of σ_1 and σ_2 values and the

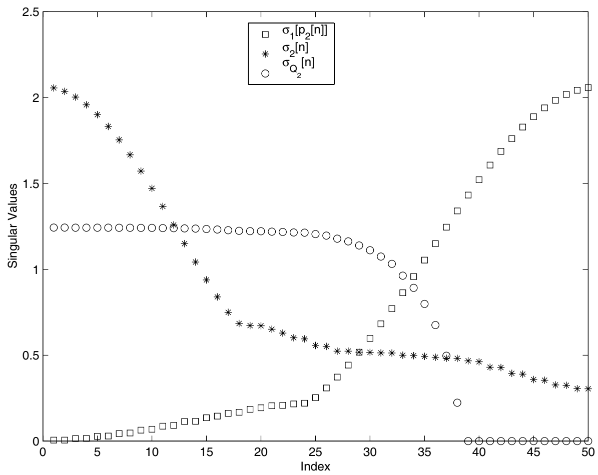

Fig. 2. Singular value alignments for precoding example.

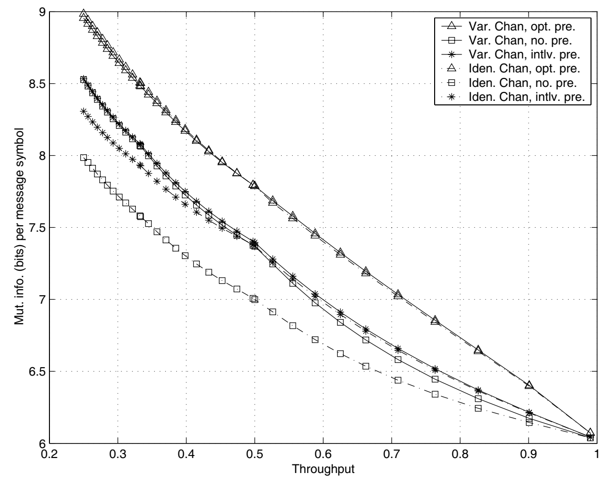

Fig. 3. Capacity per message symbol achieved using various precoding strategies at SNR of 20dB with channel lengths $L = 5$.

distribution of the precoder singular values. Clearly, the precoder energy tends to remedy the weakest singular vectors of $\mathbf{H}_1$.

We next present results from simulations of the effectiveness of precoding retransmissions. Three types of precoding are considered: the optimal precoder, no precoder, and an interleaver. The interleaver randomly chooses N_m of the N message symbols for transmission. For all test cases, we compare situations where the channel either varies ($\mathbf{h}_1 \neq \mathbf{h}_2 \neq \cdots \neq \mathbf{h}_M$), or remains identical ($\mathbf{h}_1 = \mathbf{h}_2 = \cdots = \mathbf{h}_M$) when retransmissions occur.

A maximum of four transmissions of a packet are allowed and we consider $M = 2, 3, 4$. When dealing with the M^{th} transmission, we increment the variable N_M until $N_M = N$ to evaluate the effects of truncating retransmissions. While considering the M^{th} transmission, the previous $M - 1$ transmissions are optimally precoded with $\mathbf{Q}_1, \ldots, \mathbf{Q}_{M-1}$ and un-truncated ($N_1 = \cdots = N_{M-1} = N$). With each packet, N_2 is incremented until $N_2 = N$, then N_3 is incremented until $N_3 = N$, and N_4 is incremented until $N_4 = N$. At each increment, the optimal precoder $\mathbf{Q}_M$ is determined and the mutual information produced from the three precoding strategies, with and without channel variation between retransmissions, are computed. For a given SNR and channel length L, we perform Monte Carlo simulations over 100 packets, with a unique channel $\mathbf{h}_1$ for each packet. Figs. 3 contain plots of the average mutual information per message symbol versus throughput at SNR of 20 dB and $L = 5$. We define the throughput at any instance of N_M as $N/((M-1)N + N_M)$.

The optimal precoders always provide substantial improvements in mutual information, providing the same improvements regardless of channel variation between retransmissions. Closer inspection of the results leads to some interesting observations. Interleaving is beneficial when the channel does not vary, but ineffective when the channel does vary. Also, channel length seems to have little effect. Additionally, the various throughput gains are independent of SNR. Finally, when no precoding is performed, channel variations provide an increase in mutual information. The independence of the retransmissions is a source of diversity.

7. CONCLUDING REMARKS

This paper presents a method for improving system performance when packet retransmissions are required in frequency-selective channels. Using total channel capacity as the performance objective, a process was developed to obtain linear precoders that maximize the mutual information available at the receiver. Using the singular vectors of the original transmission channel as basis vectors, optimal precoding bears many similarities to power loading. Simulation results indicate that significant gains in capacity and/or data throughput are provided with optimized precoding.

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