A CODED MODULATION TECHNIQUE FOR COOPERATIVE DIVERSITY IN WIRELESS NETWORKS

Santosh Nagaraj and Mark Bell

School of Electrical and Computer Engineering
Purdue University, West Lafayette, IN-47907
E-mail: {nagarajs,mrb}@ecn.purdue.edu

ABSTRACT

In this paper, we propose a novel coded modulation (CM) protocol for user cooperation diversity in wireless networks. Unlike most existing protocols, the proposed scheme does not involve repetition. This translates to significant gains in terms of achievable rates. Hence, it is suitable for higher spectral efficiencies too. Each user transmits its own bits, along with bits of cooperating users, in every allotted symbol at appropriate positions on a specially labelled signal constellation. The scheme achieves full diversity order in the number of cooperating relays and obtains additional coding gains compared to repetition protocols. We show gains ranging from 2 dB to 3 dB over repetition protocols for cases involving two and three cooperating users. In its simplest form, the proposed scheme has a complexity at the relay equal to that of repetition protocols. Complexity at the base station, however, is increased.

1. INTRODUCTION

User cooperation diversity [1, 2] was introduced as a way to obtain multiple antenna gains even when each user has only one antenna. It is applicable to sensor or mobile communication networks, where, individual sensors, mobiles, or PDAs communicate with a common base station (BS) or access point (AP). In all these cases, physical constraints may preclude multiple antennas on a single device.

Laneman *et al.* [2] have proposed repetition based and space time code based cooperation. A downside of these protocols is the inherent repetition of symbols (to varying degrees), which reduces achievable rates. However, these protocols have the advantage that complexity at the relays (mobiles or sensor nodes) is kept at a minimum. Hunter *et. al* [3] have proposed a coded cooperation scheme that does not repeat symbols. The source encodes its data bits with a punctured convolutional code and transmits code bits to the destination and a relay. The relay decodes the received codeword, generates additional parity bits for the decoded data bits, and transmits them to the destination. Problems with this scheme include additional complexity and latency at the relays in decoding the received punctured code words.

In this paper, we propose a cooperation protocol based on coded modulation that is simple to implement at the relays and has no latency problems. It also avoids repetition by virtue of its coded nature. The complexity at the BS, however, is now increased and is similar to that of [3]. We, however, assume that complexity at the BS is not restrictive.

2. SYSTEM MODEL

Consider a time division multiplexed (TDM) system involving two cooperating mobiles — U1 and U2 — communicating with a common BS. The TDM system allows each user to transmit on alternate slots. If the symbol rate is R symbols/s, each user effectively has a channel of $R/2$ symbols/s to the BS. Suppose that each user generates 2 information bits for every transmission slot allotted to it. This translates to an effective throughput of R bits/s for each mobile. We make a key simplifying assumption in this paper. We assume that the inter-user channel is perfect to compare the performances of three different transmission strategies.

2.1. Direct Transmission (DT)

This is the case where users do not cooperate. Even-numbered slots $\mathtt{t}(2i)$ are assigned to U1 and odd-numbered slots $\mathtt{t}(2i+1)$ are assigned to U2. During their slots, users transmit 2 bits using a QPSK constellation. This scheme achieves first order diversity.

2.2. Repetition Protocol [2]

Users cooperate using the *Amplify and Forward* (AF) or symbol-by-symbol *Decode and Forward* (DF) protocol from [2]. Since the inter-user channel is assumed to be perfect, both AF and DF have the same performance. Users U1 and U2 transmit their data during slots $\mathtt{t}(4i)$ and $\mathtt{t}(4i+1)$, respectively. User U1 repeats U2's data during slot $\mathtt{t}(4i+2)$ and U2 repeats U1's data during slot $\mathtt{t}(4i+3)$. Clearly,

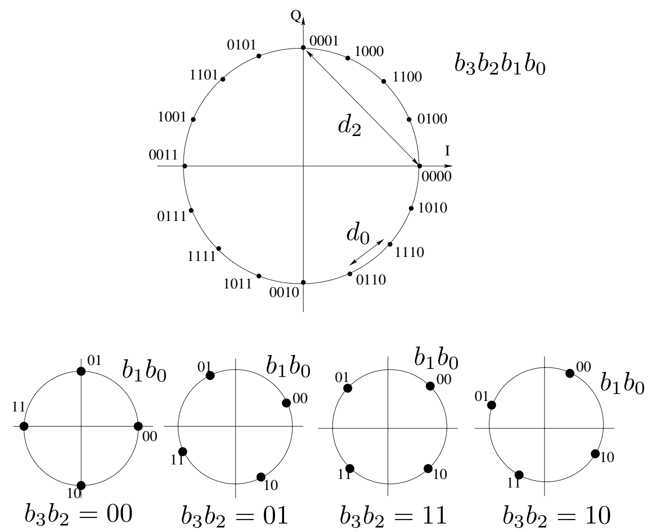

Fig. 1. Gray-Ungerboeck mapping.

this protocol achieves second order diversity. Since users transmit new data only once every two allotted slots, each symbol is now a 16-PSK symbol to match the throughput of DT. The inter-user channel should therefore be able to support 4 bits per channel use.

Symbol-by-symbol DF can be generalized to *Block* DF where the relay fully decodes a source codeword before repeating it. This provides coding gain to the inter-relay channel but suffers from high relay complexity and latency.

2.3. Proposed CM Protocol

Users use a 16-PSK constellation with labelling as shown in fig. 1. The mapping is such that if $b_3 \dots b_0$ are the 4 bits transmitted as one 16-PSK symbol, bits b_3b_2 select a particular rotated QPSK constellation and bits b_1b_0 select a point on the QPSK constellation. Bits b_1b_0 are mapped onto the selected QPSK constellation by Gray mapping. The mapping is such that if b_3b_2 are known, detection of b_1b_0 has the complexity and performance of a QPSK constellation. This is similar to the Set Partitioning idea of Ungerboeck [4]. The rotated QPSK constellations form the lowest level of partition. We call this mapping the *Gray-Ungerboeck Mapping*. We call bits b_3b_2 the *modulator state* and bits b_1b_0 the *modulated bits*.

Users U1 and U2 transmit in even- and odd-numbered slots respectively, as earlier. The difference is that, now, two new data bits are transmitted every slot. In slot $\mathbf{t}(2i)$, U1 transmits two new data bits $d_{2i+1}d_{2i}$ and uses for the modulator state, the two data bits transmitted by U2 during slot $\mathbf{t}(2i-1)$. The transmitted symbol is therefore one of the 16 signal points on a 16-PSK constellation. The algorithm is better explained by the pseudo-code below. Here, U[k] refers to the user that transmits during the slot $\mathbf{t}(k)$. The TDM scheme ensures that U[k] is U1 if k is even and is U2 if k is odd. Note that U[k] = U[k-2].

S1 Set k = 0; Initialize U1 in the modulator state 00 and transmit two bits d_1d_0 using the 16-PSK with Gray-Ungerboeck labelling.

S2 Set k = k + 1;

- U[k] knows the modulator state used by U[k-1] during slot $\mathbf{t}(k-1)$. This is because, the modulator state is either 00 (if k = 1) or the two data bits user U[k] (also U[k-2]) transmitted during $\mathbf{t}(k-2)$.
- U[k] demodulates the received symbol during $\mathbf{t}(k-1)$ to determine modulated bits $d_{2k-1}d_{2k-2}$.
- U[k] uses $d_{2k-1}d_{2k-2}$ as the modulator state for slot $\mathbf{t}(k)$ and transmits two new bits $d_{2k+1}d_{2k}$.

S3 Return to Step S2.

The CM protocol is hence just as simple as the symbol-by-symbol DF protocol at the relays. Further, there are some advantages of CM over symbol-by-symbol DF.

First, the inter-user channel now needs to support only 2 bits per channel use. The modulation and labelling schemes ensure that the two new bits a user tries to detect every symbol are transmitted as the four signal points of a (rotated) QPSK constellation. This translates to $10\log_{10}(d_2^2/d_0^2) \approx 9$ dB gain over the 16-PSK constellation. The coding gain Block DF [2] provides for the inter-user channel is now obtained easily. Simulations confirm the above hypothesis.

Second, repetition involved in the CM protocol is different from that in [2]. In [2], the repetition is equivalent to using a rate-$1/2$ repetition code. In the CM protocol, new data bits transmitted by a user during its slot are repeated in the following slot as the modulation state. This imparts a trellis (memory) structure to the transmitted sequence. The destination (BS) sees a Trellis Coded Modulation (TCM) sequence that can be decoded with a Viterbi decoder. As mentioned earlier, the complexity at the BS is now higher than that of [2]. In [2], the BS needs only perform maximum ratio combining to decode the data. Effectively, the repetition code in [2] is replaced by a well structured TCM scheme in this paper.

3. TRELLIS STRUCTURE AND PROPERTIES

Since the memory involved is 2 bits (modulator state), the trellis has 4 states. Fig. 2 shows a part of the trellis. Since the modulator state can change from any 2-bit label to any other in just one symbol (trellis stage), the trellis is fully connected. The following are some of the properties of the code trellis:

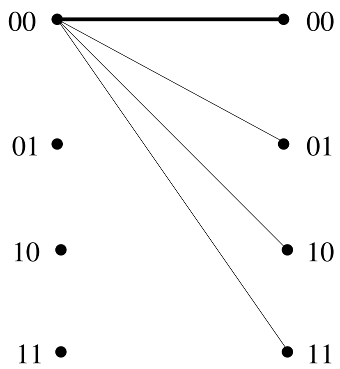

Fig. 2. Part of trellis.

P1 The trellis does not have parallel transitions.

P2 Transitions diverging from the same state are associated with signals from the same lowest level of partition (QPSK). Hence, the minimum Eucledian distance between such signals is d_2 (refer fig. 1).

P3 Transitions merging at the same state have minimum Eucledian distance d_0, corresponding to the 16-PSK constellation.

4. PROTOCOL GENERALIZATION

The CM based cooperation protocol can be generalized based on TCM design. In its most general form, users use a convolutional encoder to generate the bits that select the modulator state. Suppose that the users use a rate-$2/4$ systematic convolutional encoder with memory m bits for cooperation. During slot $\mathtt{t}(k-1)$, user U[k-1] adds two new data bits. User U[k] generates two parity bits based on the last m transmitted bits (including the two bits in $\mathtt{t}(k-1)$) and uses the two parity bits as the modulator state for slot $\mathtt{t}(k)$. User U[k] then adds two new bits for transmission during slot $\mathtt{t}(k)$.

Eg 1. The example protocol in Section II can be realized by a rate-$2/4$ convolutional encoder with generator polynomials $[1,2,4,10]$ in octal. Generators $[1,2]$ represent the two systematic (current) bits. Generators $[4,10]$ generate the parity bits. Note that $[4,10]$ (in octal) correspond to just delay elements. Hence the modulator state is just the two bits transmitted in the previous slot.

Eg 2. A cooperation protocol for three users operating at 2 bits/s/Hz can be designed using a rate-$2/4$ convolutional encoder with generator polynomials $[1,2,34,70]$ in octal. The memory in this code is 4 bits. The BS uses a 16-state Viterbi decoder to jointly detect all three users' bits.

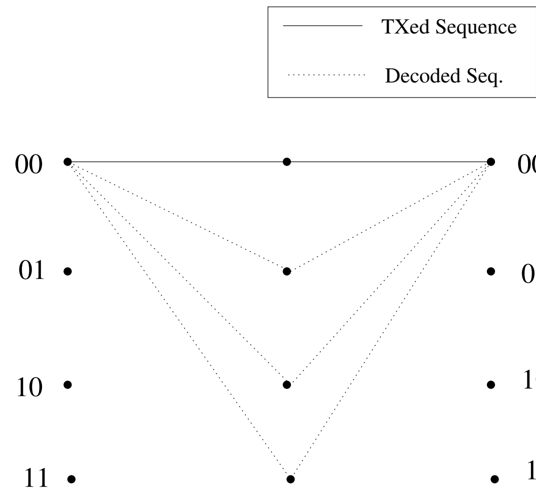

Fig. 3. Most likely error events.

5. PERFORMANCE ANALYSIS

Since the trellis does not have parallel transitions, any error event must be spread over at least two successive symbols. Hence, the code achieves second order diversity. TCM schemes, in general, are not linear codes. All possible codewords need to be considered for determining the probability of error.

A simple approximation to the performance at high SNRs can be obtained by considering only the most likely error events. Fig. 3 shows the most likely error events when the all-zero codeword is transmitted. Most likely error events are those that diverge from the transmitted codeword at some stage of the trellis and remerge with it two stages later. By property P2 of the trellis, Eucledian distance between two diverging branches is atleast $\sqrt{2E_s}$ if E_s is the symbol energy. By property P3, Eucledian distance between two remerging branches is atleast $0.39\sqrt{E_s}$.

If a_1 and a_2 are the channel gains of U1 and U2 respectively, the probability of error can be bounded above by the Chernoff bound as

$$P_2 \approx 3e^{-(2a_1^2E_s+0.16a_2^2Es)/4N_o}. \tag{1}$$

The factor 3 in front is because there are three most likely error events (refer fig. 3). The average probability of error can be determined by averaging the above equation over the distributions of a_1 and a_2. If a_1 and a_2 are i.i.d. Rayleigh variables with unit power, the average probability of error P_e in terms of the average SNR γ can be shown to be

$$P_e \approx 2.4/\gamma^2. \tag{2}$$

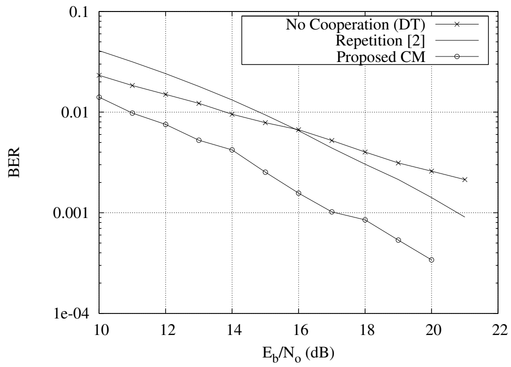

Fig. 4. Two cooperating users at 2 bits/s/Hz.

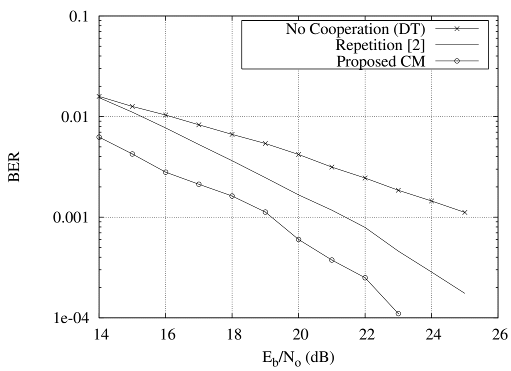

Fig. 5. Two cooperating users at 3 bits/s/Hz.

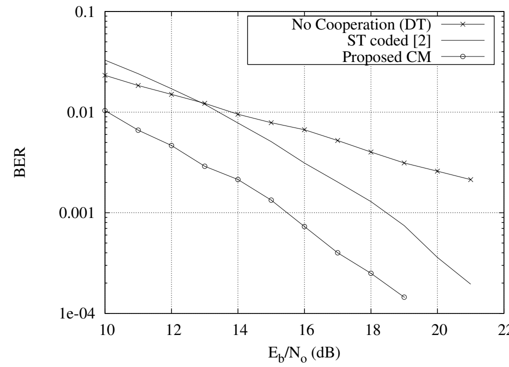

Fig. 6. Three cooperating users at 2 bits/s/Hz.

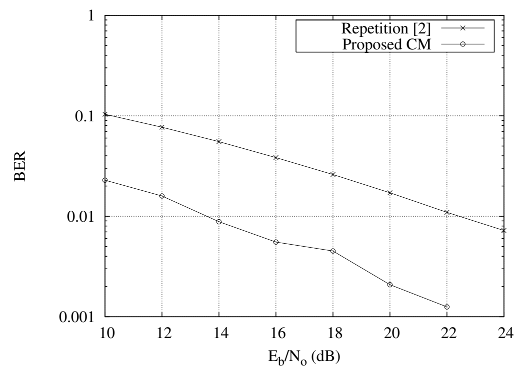

Fig. 7. Performance on the inter-user link.

6. SIMULATION RESULTS

Simulations were performed to compare DT, DF, and CM protocols. Figs. 4 and 5 compare performances of the three protocols for 2 and 3 bits/s/Hz, respectively, when there are two cooperating users. As expected, DT achieves first order diversity. Both DF and CM achieve second order diversity. CM outperforms DF by about 3 dB and 2 dB, respectively, using 16-PSK and 64-QAM constellations. The 64-QAM was partitioned based on ideas similar to 16-PSK. Fig. 6 shows performances with three cooperating users at 2 bits/s/Hz. The DF protocol uses an Alamouti ST code [2]. CM uses the convolutional encoder in Eg 2. of Section 4.

If the inter-user link is not perfect, both DF and CM suffer from error propagation if the relaying user makes an error in decoding the transmitting user's bits. Fig. 7 plots the probabilities that the relay makes this error. CM is nearly 8 dB better than DF, as explained in Section 2.

7. CONCLUSIONS

A novel protocol for cooperative diversity has been suggested. TCM forms the basis for the proposed protocol. The working of the protocol has been explained with examples. Simulations show gains of approx. 2–3 dB for the CM protocol over other existing techniques. Further, the proposed modulation and labelling scheme is a simple way to obtain significant coding gain (nearly 8 dB for 16-PSK) on the inter-relay channel.

8. REFERENCES

[1] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity — part i: System description," *IEEE Trans. Commun.*, pp. 1927–1938, Nov 2003.

[2] J. N. Laneman and G. W. Wornell, "Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Trans. Inform. Theory*, pp. 2415–2425, Oct 2003.

[3] T. E. Hunter and A. Nosratinia, "Cooperative diversity through coding," in *Proc. IEEE Int. Symp. Inform. Theory*, Lausanne, Switzerland, 2002, p. 220.

[4] G. Ungerboeck, "Channel coding with multilevel/phase signals," *IEEE Trans. Inform. Theory*, vol. IT-28, pp. 55–67, Jan 1982.