COOPERATIVE RANDOM ACCESS WITH LONG PN SPREADING CODES*

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ABSTRACT

Cooperative wireless communication systems have attracted much attention in recent years, due to the diversity advantage they can afford. Existing cooperative transmission modalities have been developed in conjunction with fixed-rate multiplexing based on TDMA, CDMA or FDMA. We advocate user cooperation as the method of choice for enabling diversity in wireless random access networks. The specific protocol developed herein exploits the fact that user cooperation can be viewed as a form of multipath, and capitalizes on the suitability of long pseudo-noise (PN) spreading codes for dealing with multipath channels. Analysis and numerical results confirm that throughput increases considerably when random access via spread-spectrum slotted Aloha protocols is aided by user collaboration.

1. INTRODUCTION

User cooperation receives increasing attention as a diversity enabler, whereby single-antenna distributed terminals collaborate to form a virtual antenna array enjoying benefits analogous to those of collocated antennas. The advantages of user cooperation in point-to-point and multiuser links with *fixed* multi-access protocols are by now well appreciated [4, 7, 9]. In this paper, we consider collaborating communicators over a *random* access channel and establish that user cooperation can increase throughput markedly.

On the one hand, we draw from Spread Spectrum Random Access (SSRA) protocols considered in e.g., [1, 3, 5]; while on the other hand, we capitalize on the observation that user cooperation can be seen as a form of multipath, capable of providing diversity gains. As PN sequences can effectively deal with multipath, Code Division Multiple Access (CDMA) with long PN spreading codes have been used for implementing cooperative protocols for fixed access [7].

Starting with a summary of results we put forth in [8], we first recognize that diversity offers the potential to increase throughput in SSRA networks (Section 2). Having made the case for diversity, we subsequently argue that user cooperation is a convenient diversity enabler for random access networks, and develop our cooperative SSRA protocol (Section 3). Throughput analysis reveals that the novel protocol can capture part –though not all – of the available diversity (Section 4); nevertheless, throughput increases commensurately as we also confirm by numerical results (Section 5). We conclude the paper in Section 6.

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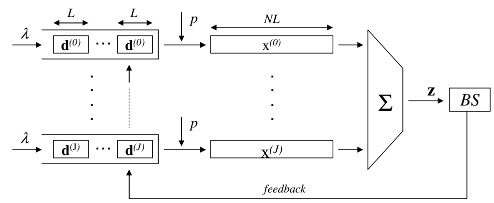

Fig. 1. Non-Cooperative SSRA System

2. NON-COOPERATIVE SS RANDOM ACCESS

Figure 1 outlines the SSRA system we put forth in [8]. Each of the $J+1$ users has an infinitely long buffer for storing packets of fixed length L that arrive with rate λ (packets per packet duration). These packets are to be transmitted to a certain access point (call it from now on base-station (BS)). The arrival processes are assumed independent and identically distributed (i.i.d.) across users; hence, the total arrival rate is $(J+1)\lambda$ packets per packet duration. Before transmission, packets are spread using a long-PN sequence of period NL, where N is the spreading gain. Letting $\mathbf{d}^{(j)} := \{d^{(j)}(n)\}_{n=0}^{L-1}$ denote a data packet of user U_j, the transmitted chip sequence is (for $l \in [0, L-1]$ and $n \in [0, N-1]$):

$$x^{(j)}(Nl+n) = d^{(j)}(l)c(Nl+n)\ , \qquad (1)$$

where $\mathbf{x}^{(j)} := \{x^{(j)}(n)\}_{n=0}^{NL-1}$ is a vector representing the transmitted block of the j^{th} user; $\mathbf{c} := \{c(n)\}_{n=0}^{NL-1}$ is the common long PN sequence *shared* by all users; and $c(\cdot)$ should be interpreted as the cyclic extension of $\{c(n)\}_{n=0}^{NL-1}$.

Transmission of queued packets obeys the following rules that describe the proposed protocol:

[R1] Time is slotted, and if users have a packet to transmit, they do so at the beginning of a slot.

[R2] Packets are spread before transmission according to (1). Each user chooses a random shift of the common PN sequence for spreading.

[R3] When queued packets are available, each user node transmits them with probability p.

Notice that [R3] controls the transmission rate that will be adjusted to maximize throughput; [R2] provides statistical user separation; and [R1] will turn out to be helpful when analyzing throughput.

To grasp how long PN codes separate users, let T_j be the random shift (measured in chip intervals) specific to user U_j and re-write (1) as $x^{(j)}(Nl+n) = d^{(j)}(l)c(Nl+n-T_j)$. The block $\mathbf{z} := \{z(n)\}_{n=0}^{NL-1}$ received at the BS comprises the superposition of (up to) $J+1$ transmissions and has entries

$$z(Nl+n) = \sum_{j=0}^{J} h^{(j)}(l)d^{(j)}(l)c(Nl+n-T_j) + n(Nl+n)$$

where $h^{(j)}(l-T_j)$ is the channel from user U_j to the BS, and $n(Nl+n)$ denotes zero mean Additive White Gaussian Noise (AWGN) with variance $\mathrm{E}[n^2(Nl+n)] = N_0$. To recover packets from the user of interest (here U_0), we despread $\{z(Nl+n)\}$ using a properly delayed version of the long PN $\mathbf{c}$. The resultant decision vector $\mathbf{r}^{(0)} := \{r^{(0)}(l)\}_{l=0}^{L-1}$ has entries

$$\begin{aligned} r^{(0)}(l) &= \frac{1}{N}\sum_{n=0}^{N-1} z(Nl+n)c(Nl+n-T_0) \qquad (2) \\ &= h^{(0)}(l)d^{(0)}(l) + i^{(0)}(l) + \tilde{n}(l), \end{aligned}$$

where the AWGN $\tilde{n}(Nl+n)$ has $\mathrm{E}[\tilde{n}^2(l)] = N_0$; and the interference term $i^{(0)}(l)$ caused by users $\{U_j\}_{j=1}^{J}$ is given by

$$\begin{aligned} i^{(0)}(l) =& \frac{1}{N}\sum_{j=1}^{J} h^{(j)}(l)d^{(j)}(l) \qquad (3) \\ &\times \sum_{n=0}^{N-1}\left[\ c(Nl+n-T_j)c(Nl+n-T_0)\ \right] . \end{aligned}$$

Since long PN sequences are approximately white, we have $\mathrm{E}[c(Nl+n-T_j)c(Nl+n-T_0)] \approx 0$, for $T_j \neq T_0$. This implies that through [R2] our protocol effects statistical separation of different users' packets whose probability of error is determined by the interference. Notice though that there is also a chance to have $T_j = T_0$ for some j(s). Both this and the interference term will determine the throughput of the proposed protocol that we analyze next.

2.1. Throughput Analysis

Consider a symmetric system where users transmit with the same power and share the same channel statistics. The throughput of the protocol defined by [R1]-[R3] can be bounded by that of an associated dominant system, where users queue a dummy packet whenever their queues are empty, rendering the system stationary [2, 10]. By analyzing the throughput of this dominant system, we have established the following result [8]:

Proposition 1 *For the protocol adhering to [R1]-[R3], the packet loss probability is given by*

$$P_l = P_c + P_e(1-P_c)\ , \qquad (4)$$

where $P_c = 1 - [1 - p/(NL)]^J$ is the probability of packet collision, and P_e is given by

$$P_e = \sum_{n=0}^{J}\binom{J}{n} p^n(1-p)^{J-n} P_e(N/n), \qquad (5)$$

with $P_e(N/n)$ being the packet error probability for signal-to-interference-plus-noise-ratio (SINR) N/n. The throughput is found as

$$\eta := (J+1)p(1-P_l)\ . \qquad (6)$$

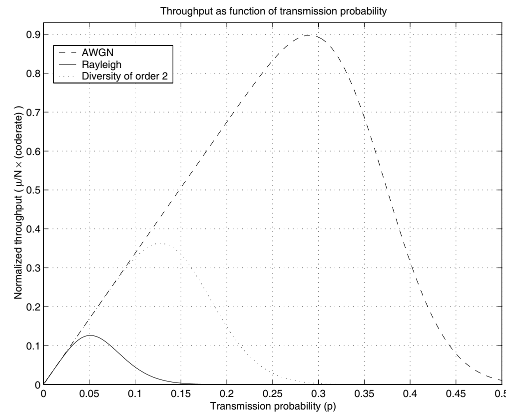

Fig. 2. Diversity enhances throughput over wireless channels ($J = 128$, $N = 32$, $L = 1024$, $215/255$ BCH code capable of correcting $t = 5$ errors)

Note that here we neglect the AWGN term in SINR because this is primarily a collision- and interference-limited system. What determines $P_e(N/n)$ is the channel model and the Forward Error Correcting (FEC) code used. For simplicity, let us consider block codes capable of correcting t errors; e.g., BCH codes [6, p.437]. In this case, if $q(N/n)$ is the code-bit error probability, we have

$$P_e(N/n) = 1 - \sum_{k=0}^{t}\binom{L}{k} q(N/n)^k[1-q(N/n)]^{(L-k)}. \qquad (7)$$

It is interesting to compare the throughput as determined by (6) for different channels. The best possible scenario is for $h^{(j)}(l)$ constant (AWGN channel), in which case

$$q(N/n) = Q(\sqrt{2N/n})\ , \qquad (8)$$

where $Q(x) := (1/\sqrt{2\pi})\int_x^\infty e^{x^2/2}\,dx$. More appropriate for the wireless environment however, is a Rayleigh fading channel where $h^{(j)}(l)$ is random Rayleigh distributed. In this case, [6, sec. 14.4]

$$q(N/n) = \frac{1}{2}\left[1 - \rho(N/n)\right]\ , \qquad (9)$$

where $\rho(x) := \sqrt{x/(1+x)}$. Finally, let us consider a Rayleigh channel with two paths having equal power $P_0/2$, which provides diversity order two and

$$q(N/n) = \frac{1}{2}\left[1-\rho\left(\frac{N}{n+1/2}\right)\right]^2\left[2+\rho\left(\frac{N}{n+1/2}\right)\right], \qquad (10)$$

where the factor $1/2$ comes from the self interference between the two diversity paths.

For each of the three channels under consideration, Fig. 2 depicts normalized throughput when BCH coding is used. It can be seen that the throughput for a Rayleigh channel is very poor (even for the relatively powerful code considered), particularly

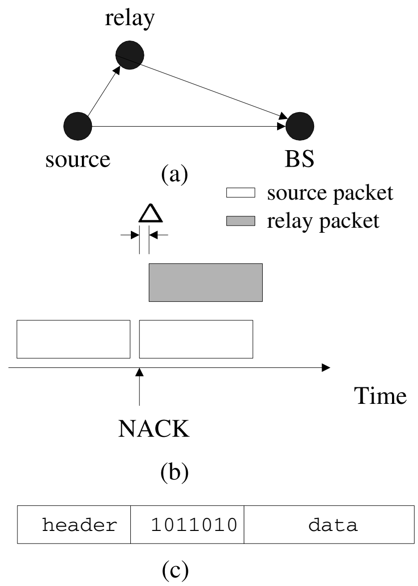

Fig. 3. (a) Each active user (source) receives cooperation from an idle user (relay); (b) When a packet is lost both source and relay transmit a copy of the packet; (c) Retransmissions occur at pre-specified randomly chosen time slots.

when compared with the throughput over an AWGN channel. The second-order diversity channel, on the other hand, exhibits significant improvement. This points to a diversity-enhanced approach for ameliorating fading effects even in RA protocols, and motivates nicely the introduction of user cooperation we pursue next.

3. COOPERATIVE SS RANDOM ACCESS

A form of diversity, well suited for wireless random access networks, is enabled through user cooperation. Indeed, since many users' queues are empty when traffic is bursty and the overall load is not heavy, many such idle users can serve as cooperators for active users. Consider the motivating example depicted in Figs. 3-(a) and 3-(b), where a source transmits a packet in the first time slot. Due to the broadcast nature of the wireless channel, a close-by idle user willing to serve as a cooperator receives the packet, and decodes it successfully. If the packet is lost at the destination, it will be retransmitted by both the source and the relay using the common PN code with the same time shift. Collision is avoided at the BS due to the time delay Δ between these two transmissions. This allows the BS to distinguish the source from the cooperator's signal. In practice, this time delay can be intentionally introduced by the relay according to a common time reference, or caused by the random propagation time difference between the two paths from the source and the relay, respectively.

Although the same idea applies also to multi-hop wireless networks, in this paper we focus on the slotted SSRA system described in Section II. Since in such systems a lost packet is re-transmitted in the next slot with probability p, the relay must know when the source re-transmits. This can be made possible using a header specifying the randomly chosen re-transmission slots as shown in Fig. 3-(c). Before the first transmission of a packet, the source generates a finite-length binary Bernoulli sequence in which the outcomes 1 and 0 occur with probability p and $1-p$, respectively. This sequence determines in which slots the source will re-transmit, if the original transmission fails. For the example in Fig. 3-(c), and assuming that the first slot is slot 0, the source will retransmit the failed packet in slot 1. If retransmission fails again, the source will retransmit in the 3rd slot and so on, until the packet is correctly received. This sequence is shifted to the left in each slot with the rightmost bit replaced by 0, so that the relay can still cooperate if it fails decoding in the slot 0 but succeeds during the source's pre-specified retransmission slot.

Another scheme is to let the source and the relay employ a common binary Bernoulli sequence generator with a long period, and let the source specify the initial value of the generator in the header, which is updated after each slot. This way, transmission attempts of the source and the relay are well synchronized according to the output of the generator.

Note that the aforementioned cooperative transmission can be easily extended to multiple relays. This gives rise to packet replicas arriving at the BS in a fashion analogous to multipath propagation. Different from this natural multipath (NM) however, the "multi-relay paths" are introduced opportunistically as in [7].

4. THROUGHPUT ANALYSIS OF CSSRA

To compute the throughput of our cooperative SSRA protocol, we resort to the dominant system as in Section 2.1. To simplify analysis, we adopt the following assumptions (that do not restrict practical implementation):

- *AS1* Each user is paired with one idle user serving as relay.
- *AS2* The relay can always successfully decode the corresponding source's packet.
- *AS3* Packets are first transmitted with power P_0; and if needed, they are retransmitted with power $P_0/2$.

The reason for *AS1* is to ensure system symmetry. Albeit ignoring relay errors, $AS2$ is a reasonable approximation because the optimum cooperators are those closest to the source [7]; hence, decoding errors at the relay are very unlikely. Finally, *AS3* guarantees that the average interference level from a source-relay pair to other users is constant, no matter whether the source is receiving cooperation or not.

Notice that at any time a busy user node is in one of two states: When in state 0, the first packet in its queue is a new packet and the code-bit error probability is given by (9). When in state 1, the first packet in its queue is a retransmission packet and since the user is receiving cooperation, the code-bit error probability is given by (10). Averaging over the two states, we obtain the average packet error probability as

$$\overline{P_e} = s_0 P_e^{(0)} + (1-s_0) P_e^{(1)} \; , \tag{11}$$

where s_0 is the steady state probability of finding the user of interest in state 0, and $\{P_e^{(k)}\}_{k=0}^{1}$ is the packet error probability in state k. These are obtained, respectively, after substituting (9) and (10) into (7).

To find s_0, note that a user's state across time slots forms a Markov chain with state-transition diagram shown in Fig. 4. The pertinent transition matrix is

$$\mathbf{M} = \begin{bmatrix} 1-pP_e^{(0)} & pP_e^{(0)} \\ p(1-P_e^{(1)}) & 1-p(1-P_e^{(1)}) \end{bmatrix} , \tag{12}$$

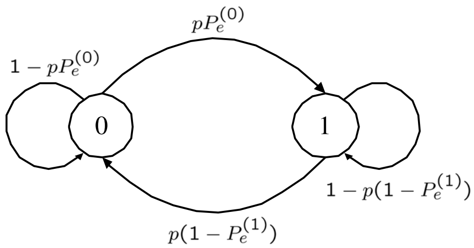

Fig. 4. Each user alternates between a non-cooperative state (state 0) and a cooperative one (state 1)

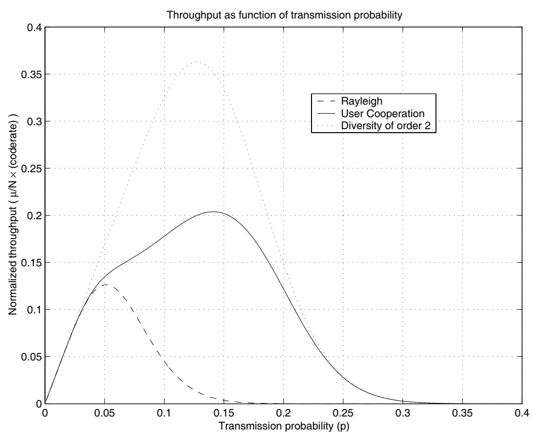

Fig. 5. Throughput of the CSSRA protocol lies in between the throughput of systems with and without diversity ($J = 128$, $N = 32$, $L = 1024$, $215/255$ BCH code capable of correcting $t = 5$ errors)

and the steady state distribution can be found by solving the eigen-equation

$$\mathbf{M}^T \begin{bmatrix} s_0 \\ (1 - s_0) \end{bmatrix} = \begin{bmatrix} s_0 \\ (1 - s_0) \end{bmatrix} . \quad (13)$$

Assuming that packets are long enough, we can ignore the probability of collision and obtain the throughput as

$$\eta = (J + 1)p(1 - \bar{P}_e) . \quad (14)$$

5. NUMERICAL RESULTS

The throughput of the proposed protocol for an example BCH code is shown in Fig. 5, in comparison with the throughput of SSRA over Rayleigh fading channels with and without diversity. As expected, the maximum stable throughput is in between the corresponding values of these systems; and even though the proposed protocol does not capture the maximum possible diversity, we witness a clear improvement relative to a system without diversity.

6. CONCLUSIONS

Aiming to migrate the well-established advantages of cooperative diversity from fixed multiplexing to the random access regime, we have introduced a collaborative random access protocol based on spread Aloha using random shifts of a long PN code. Analyzing this protocol under certain simplifying assumptions, we have confirmed that diversity provided through user cooperation is a viable technique, particularly suited for the unique features of long PN-spread Aloha random access networks. Our protocol takes advantage of nodes with idle queues by having them serve as cooperating relays to aid active nodes. We also exploited the fact that user cooperation can be seen as a form of multipath whose components can be effectively resolved through the use of long PN spreading codes.

By capturing even part of the diversity gain, we showed that our novel protocol can significantly increase throughput in wireless random access networks[1].

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