# Decode-Based Differential Modulation for Wireless Relay Networks

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***Abstract*— In this paper, we develop a differential binary phase shift keying (BPSK) modulation scheme for wireless relay networks composed of one source, one relay and one destination node. The proposed scheme, referred to as the *differential decode-and-forward* (DDF), utilizes the relay to assist data transmission from the source to the destination. We derive a maximum likelihood (ML) detector and a piece-wise linear (PL) detector for the proposed DDF scheme. A closed-form bit error rate (BER) expression is presented for the proposed PL detector. Both analytical and simulation results show that the proposed DDF scheme is capable of providing diversity gain at the destination node over Rayleigh fading channels.**

## I. Introduction

Diversity provides an efficient mechanism to combat multipath fading in wireless communication systems, and can be implemented in a space-time fashion. A basic premise behind all space-time coding schemes is the availability of multiple antennas at the transmitter. This may not be possible in some scenarios, e.g., a peer-to-peer ad-hoc mobile network, due to size, power and cost limitations on the mobile terminal. In such cases, multiple spatially distributed mobile nodes, referred to as *wireless relays* herein, can be exploited to assist data transmission.

Laneman et al. [1] investigated the *cooperative diversity*, and analyzed the outage capacity for the case of known channel state information (CSI). Sendonaris et al. addressed the achievable rates and implementation issues of user cooperation diversity for the code-division multiple-access (CDMA) systems in [2]. Coherent maximum likelihood (ML) detection was developed in [3]. The bit error rate (BER) was derived in [4] for the relay link. In [5], the symbol error rate was provided for the scenario of multi-branch multi-hop configurations. However, most of such analyses were based on the *amplify-and-forward* (AF) scheme [1], where the relay simply amplifies the received signals and retransmits to the destination, and the CSI is available to the receivers. In [6], the BER performance was studied for the non-coherent *decode-and-forward* (DF) applying binary frequency shift keying (BFSK) modulation.

Thus far, most of previous studies were focused on coherent detection. To obviate channel estimation, we develop in this paper a *differential decode-and-forward* (DDF) modulation scheme for cooperative wireless systems and analyze its BER performance.

## II. System Model

Consider a scenario depicted in Fig. 1, where a sequence of symbols are to be transmitted from the *source* node S to the *destination* node D. Suppose there is another *relay* node R that can hear S and transmit to R. To avoid interference, S and R use orthogonal channels for transmission, either by time-, frequency-, or code-division multiplexing. For ease of presentation, we assume time-division multiplexing for which the transmission is divided into *two* distinct phases. During *phase-I* transmission, S transmits, while R and D listen. During *phase-II* transmission, S is silent, while R transmits signals to D.

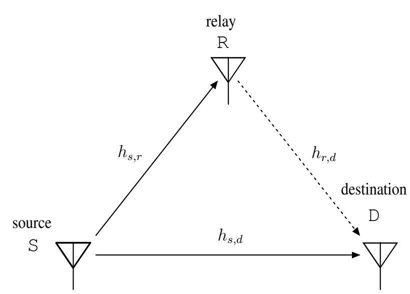


Fig. 1. A wireless relay system.

For phase-I transmission, the information bits $d(n) \in \{\pm 1\}$ at S are differentially encoded: $s(n) = s(n-1)d(n)$, $n = 1, 2, \cdots, N$, where $s(0) = 1$ and $N$ is the number of bits within the frame. The received baseband signals at R and D, respectively, are

$$x_r(n) = h_{s,r}s(n) + w_r(n), \quad n = 0, 1, \cdots, N, \qquad (1)$$

$$x_d(n) = h_{s,d}s(n) + w_d(n), \quad n = 0, 1, \cdots, N, \qquad (2)$$

where $h_{s,r}$ and $h_{s,d}$ are the channel coefficients, and $w_r(n)$ and $w_d(n)$ are complex additive white Gaussian noise (AWGN).

For phase-II transmission, relay R decodes the received signal $x_r(n)$, and generates a *unit-variance* signal $s_r(n)$ that is transmitted to destination D. The signal received at D is given by [1]

$$y_d(n) = h_{r,d}s_r(n) + u_d(n), \quad n = 0, 1, \cdots, N, \qquad (3)$$

[1]With some notation abuse, $n$ denotes the time index for both phase-I and phase-II transmissions.

where $h_{r,d}$ and $u_d(n)$ denotes the fading and channel noise, respectively.

For differential detection, the fading channels are assumed (approximately) static over two bit intervals. The dependence of the channels on time is dropped for brevity since the detection scheme to be discussed involves signals received over two adjacent bits. The channels are Rayleigh fading, i.e., $h_{i,j} \sim \mathcal{CN}(0, \sigma_{i,j}^2)$, $(i,j) \in \{(s,r),(s,d),(r,d)\}$, where $\mathcal{CN}(\mu, \sigma^2)$ denotes a complex Gaussian random variable with mean $\mu$ and variance $\sigma^2$. The channel noise $w_r(n)$, $w_d(n)$ and $u_d(n)$ are assumed independent $\mathcal{CN}(0, N_0)$ random variables. The *instantaneous SNR* between nodes $i$ and $j$, denoted by $\gamma_{i,j} = |h_{i,j}|^2/N_0$, is exponentially distributed with PDF

$$p_{\gamma_{i,j}}(\gamma_{i,j}) = \frac{1}{\bar{\gamma}_{i,j}} e^{-\gamma_{i,j}/\bar{\gamma}_{i,j}}, \tag{4}$$

where $\bar{\gamma}_{i,j} = \sigma_{i,j}^2/N_0$ denotes the *average SNR* between nodes $i$ and $j$. Finally, the channel coefficients are assumed independent of one another and also of the channel noise.

## III. Proposed Scheme

### A. Differential Demodulation and Encoding at the Relay

At the relay, the conventional differential demodulation is performed

$$\tilde{d}(n) = \begin{cases} 1, & \Re\{x_r^*(n-1)x_r(n)\} > 0 \\ -1, & \Re\{x_r^*(n-1)x_r(n)\} < 0 \end{cases} \tag{5}$$

for $n = 1, 2, \cdots, N$, where $\tilde{d}(n)$ denotes the estimate of $d(n)$, $\Re\{\cdot\}$ denotes the real part of the argument, and $(\cdot)^*$ denotes the complex conjugate of a complex number.

Then, the encoder at the relay performs differential encoding:

$$s_r(n) = s_r(n-1)\tilde{d}(n), \quad n = 1, 2, \cdots, N, \tag{6}$$

where $\tilde{s}(n) \in \{\pm 1\}$ denotes the actual transmitted signal from the relay to the destination, and $\tilde{s}(0) = 1$.

### B. Maximum Likelihood Detection at the Destination

Substituting (6) into (3), we have

$$y_d(n) = y_d(n-1)\tilde{d}(n) + u_d(n) - u_d(n-1)\tilde{d}(n). \tag{7}$$

Either a correct or wrong decision may occur at the relay. As a result, the conditional PDF of $y_d(n)$ takes the form of Gaussian mixture:

$$\begin{aligned} p_{y_d(n)}(y) =& (1-\epsilon)\Phi_c(y; y_d(n-1)d(n), 2N_0) \\ &+ \epsilon\Phi_c(y; -y_d(n-1)d(n), 2N_0), \end{aligned} \tag{8}$$

where $\Phi_c(y; \mu, \sigma^2)$ denotes the PDF of a complex Gaussian random variable with mean $\mu$ and variance $\sigma^2$, and $\epsilon = 1/(2 + 2\bar{\gamma}_{s,r})$ is the BER of the differential BPSK modulation over Rayleigh fading channels. Similarly, $x_d(n)$ can be rewritten as

$$x_d(n) = x_d(n-1)d(n) + v(n), \tag{9}$$

where $v(n) \triangleq w_d(n) - w_d(n-1)d(n)$. Clearly, we have $x_d(n) \sim \mathcal{CN}(x_d(n-1)d(n), 2N_0)$ conditioned on the information bit $d(n)$ and the previous received signal $x_d(n-1)$. The ML detection is shown to be:

$$f(t_1) + t_0 \underset{-1}{\overset{1}{\gtrless}} 0, \tag{10}$$

where

$$f(t_1) = \ln \frac{(1-\epsilon)e^{t_1} + \epsilon}{\epsilon e^{t_1} + 1 - \epsilon}, \tag{11}$$

$$t_1 = \frac{q_1}{N_0},\ q_1 = y_d^*(n-1)y_d(n) + y_d(n-1)y_d^*(n), \tag{12}$$

$$t_0 = \frac{q_0}{N_0},\ q_0 = x_d^*(n-1)x_d(n) + x_d(n-1)x_d^*(n). \tag{13}$$

It was shown that $f(t_1)$ can be approximated by a piecewise-linear (PL) function: [6]

$$f(t_1) \approx f_{PL}(t_1) \triangleq \begin{cases} -T_1, & t_1 \le -T_1 \\ t_1, & -T_1 \le t_1 \le T_1 \\ T_1, & t_1 \ge T_1 \end{cases} \tag{14}$$

where $T_1 = \ln[(1-\epsilon)/\epsilon]$ assuming $\epsilon < 0.5$. This leads to the following *PL detector*:

$$f_{PL}(t_1) + t_0 \underset{-1}{\overset{1}{\gtrless}} 0, \tag{15}$$

which is easier to implement than the ML detector (10). The PL detector achieves similar performance to that of the ML detector (see Section V) and admits tractable analysis.

## IV. Performance Analysis

### A. Average BER

Without lost of generality, we assume $d(n) = 1$ is transmitted. A close examination of the PL detector indicates that the error event can be represented using three mutually exclusively events. Specifically, the conditional BER is

$$\begin{aligned} P_b(\gamma_{s,d}, \gamma_{r,d}) =& \Pr\{t_0 - T_1 < 0 | t_1 < -T_1, d(n) = 1\} \\ &\times \Pr\{t_1 < -T_1 | d(n) = 1\} \\ &+ \Pr\{t_0 + T_1 < 0 | t_1 > T_1, d(n) = 1\} \\ &\times \Pr\{t_1 > T_1 | d(n) = 1\} \\ &+ \Pr\{t_0 + t_1 < 0, -T_1 \le t_1 \le T_1 | d(n) = 1\}. \end{aligned} \tag{16}$$

It is observed that $t_0$ and $t_1$ are mutually independent, and $t_0$ is of quadratic forms in complex Gaussian variables [7],we have

$$\begin{aligned} P_{b1}(\gamma_{s,d}) =& \Pr\{t_0 - T_1 < 0 | t_1 < -T_1, d(n) = 1\} \\ =& 1 - \frac{e^{-2\gamma_{s,d}}}{2} \sum_{k=0}^{\infty} \sum_{n=0}^{k} \frac{2^{k-n}\gamma_{s,d}^k \Gamma(k+1-n, T_1)}{k!(k-n)!}. \end{aligned}$$

where the upper incomplete Gamma function is defined as $\Gamma(a,x) = \int_x^\infty e^{-t}t^{a-1}dt$. Similarly,

$$\begin{aligned} P_{b4}(\gamma_{s,d}) =& \Pr\{t_0 + T_1 < 0 | t_1 > T_1, d(n) = 1\} \\ =& \frac{e^{-\gamma_{s,d} - T_1}}{2}. \end{aligned}$$

For the probability related to the random variable $t_1$, the error events can be grouped into two mutually exclusive error events: an error is made at the relay, or the relay detects the transmitted symbol correctly. Hence, we have

$$\Pr\{t_1 < -T_1 | d(n) = 1\} = (1-\epsilon)P_{b2}(\gamma_{r,d}) + \epsilon P_{b3}(\gamma_{r,d}),$$

where $P_{b2}(\gamma_{r,d}) = \Pr\{t_1 < -T_1 | d(n) = 1, \tilde{d}(n) = 1\}$, $P_{b3}(\gamma_{r,d}) = \Pr\{t_1 < -T_1 | d(n) = 1, \tilde{d}(n) = -1\}$. With the help of [7], we have

$$P_{b2}(\gamma_{r,d}) = \frac{e^{-\gamma_{r,d}-T_1}}{2},$$

$$P_{b3}(\gamma_{r,d}) = \frac{e^{-2\gamma_{r,d}}}{2} \sum_{k=0}^{\infty} \sum_{n=0}^{k} \frac{2^{k-n}\gamma_{r,d}^k \Gamma(k+1-n, T_1)}{k!(k-n)!}.$$

Similarly,

$$\Pr\{t_1 > T_1 | d(n) = 1\} = (1-\epsilon)P_{b5}(\gamma_{r,d}) + \epsilon P_{b6}(\gamma_{r,d}), \quad (17)$$

where $P_{b5}(\gamma_{r,d}) = P_{b3}(\gamma_{r,d})$, $P_{b6}(\gamma_{r,d}) = P_2(\gamma_{r,d})$ due to the symmetry of the PL function. The last term in (16) can be written as

$$\begin{aligned} &\Pr\{t_0 + t_1 < 0, -T_1 \le t_1 \le T_1 | d(n) = 1\} \\ &\quad = (1-\epsilon)P_{b7}(\gamma_{s,d}, \gamma_{r,d}) + \epsilon P_{b8}(\gamma_{s,d}, \gamma_{r,d}), \end{aligned} \quad (18)$$

where $P_{b7}(\gamma_{s,d}, \gamma_{r,d}) = \Pr\{t_0 + t_1 < 0, -T_1 \le t_1 \le T_1 | d(n) = 1, \tilde{d}(n) = 1\}$, $P_{b8}(\gamma_{s,d}, \gamma_{r,d}) = \Pr\{t_0 + t_1 < 0, -T_1 \le t_1 \le T_1 | d(n) = 1, \tilde{d}(n) = -1\}$. $P_{b7}$ and $P_{b8}$ can be found by integrating the joint PDF of $q_0$ and $q_1$ over the constrained domain:

$$\begin{aligned} P_{e7}(\gamma_{s,d}, \gamma_{r,d}) =& P_{e8}(\gamma_{s,d}, \gamma_{r,d}) \\ =& \int_{-N_0 T_1}^{N_0 T_1} p_{q_1}(y) dy \int_{-\infty}^{-y} p_{q_0}(x) dx. \end{aligned} \quad (19)$$

Due to space constraint, we skip the derivation. $P_{b7}(\gamma_{s,d}, \gamma_{r,d})$ and $P_{b8}(\gamma_{s,d}, \gamma_{r,d})$ are obtained as

$$\begin{aligned} P_{b7}(\gamma_{s,d}, \gamma_{r,d}) =& \frac{1}{2}e^{-\gamma_{r,d}} - \frac{1}{2}e^{-\gamma_{r,d}-T_1} - \frac{e^{-2\gamma_{s,d}-\gamma_{r,d}}}{4} \\ &\times \sum_{k=0}^{\infty} \sum_{n=0}^{k} \sum_{m=0}^{k-n} \frac{2^{k-n-m-1}\gamma_{s,d}^k}{k!m!} \gamma(m+1, 2T_1) \\ &+ \frac{e^{-\gamma_{s,d}-2\gamma_{r,d}}}{8} \sum_{k=0}^{\infty} \sum_{n=0}^{k} \frac{\gamma_{r,d}^k}{k!(k-n)!} \gamma(k-n+1, 2T_1), \end{aligned}$$

$$\begin{aligned} P_{b8}(\gamma_{s,d}, \gamma_{r,d}) =& \frac{e^{-2\gamma_{r,d}}}{2} \sum_{k=0}^{\infty} \sum_{n=0}^{k} \frac{2^{k-n}\gamma_{r,d}^k \gamma(k-n+1, T_1)}{k!(k-n)!} \\ &- \left( \sum_{k=0}^{\infty} \sum_{i=0}^{\infty} \sum_{n=0}^{k} \sum_{j=0}^{i} \sum_{m=0}^{i-j} \frac{\gamma_{r,d}^k \gamma_{s,d}^i \gamma(k-n+m+1, 2T_1)}{k!(k-n)!i!m!2^{j+m-i}} \right) \\ &\times \frac{e^{-2\gamma_{s,d}-2\gamma_{r,d}}}{8} + \frac{1}{8}e^{-\gamma_{s,d}-\gamma_{r,d}}(1-e^{-2T_1}). \end{aligned}$$

where the lower incomplete gamma function is defined as $\gamma(a, x) = \int_0^x e^{-t}t^{a-1}dt$.

Collecting all the terms in (16), we have

$$\begin{aligned} P_b =& (P_{b1}P_{b2} + P_{b3}P_{b4} + P_{b7})(1-\epsilon) \\ &+ (P_{b1}P_{b3} + P_{b2}P_{b4} + P_{b8})\epsilon. \end{aligned} \quad (20)$$

The average BER for DDF is obtained by averaging $P_b$ across the distribution of $\gamma_{r,d}$ and $\gamma_{s,d}$:

$$\bar{P}_b = \int_0^{\infty} \int_0^{\infty} P_b(\gamma_1, \gamma_2) p_{\gamma_{s,d}}(\gamma_1) p_{\gamma_{r,d}}(\gamma_2) d\gamma_1 d\gamma_2. \quad (21)$$

A close examination of (21) reveals that the 2-dimensional integration in (21) is separable. Using [8, Eqn. (3.351.3)], we arrive at the following closed-form expression of the average BER for DDF:

$$\begin{aligned} \bar{P}_b =& (\bar{P}_{b1}\bar{P}_{b2} + \bar{P}_{b3}\bar{P}_{b4} + \bar{P}_{b7})(1-\epsilon) \\ &+ (\bar{P}_{b1}\bar{P}_{b3} + \bar{P}_{b2}\bar{P}_{b4} + \bar{P}_{b8})\epsilon, \end{aligned} \quad (22)$$

where

$$\bar{P}_{b1} = 1 - \frac{1}{2(2\bar{\gamma}_{s,d}+1)} \sum_{k=0}^{\infty} \sum_{n=0}^{k} \frac{2^{k-n}\bar{\gamma}_{s,d}^k \Gamma(k+1-n, T_1)}{(k-n)!(2\bar{\gamma}_{s,d}+1)^k},$$

$$\bar{P}_{b2} = \frac{1}{2(\bar{\gamma}_{r,d}+1)} e^{-T_1},$$

$$\bar{P}_{b3} = \frac{1}{2(2\bar{\gamma}_{r,d}+1)} \sum_{k=0}^{\infty} \sum_{n=0}^{k} \frac{2^{k-n}\bar{\gamma}_{r,d}^k \Gamma(k+1-n, T_1)}{(k-n)!(2\bar{\gamma}_{r,d}+1)^k},$$

$$\bar{P}_{b4} = \frac{1}{2(\bar{\gamma}_{s,d}+1)} e^{-T_1},$$

$$\begin{aligned} \bar{P}_{b7} =& \frac{1}{2(\bar{\gamma}_{r,d}+1)}(1-e^{-T_1}) - \frac{1}{8(2\bar{\gamma}_{s,d}+1)(\bar{\gamma}_{r,d}+1)} \\ &\times \sum_{k=0}^{\infty} \sum_{n=0}^{k} \sum_{m=0}^{k-n} \frac{2^{k-n-m}\bar{\gamma}_{s,d}^k \gamma(m+1, 2T_1)}{m!(2\bar{\gamma}_{s,d}+1)^k} \\ &+ \frac{1}{8(\bar{\gamma}_{s,d}+1)(2\bar{\gamma}_{r,d}+1)} \sum_{k=0}^{\infty} \sum_{n=0}^{k} \frac{\bar{\gamma}_{r,d}^k \gamma(k+1-n, 2T_1)}{(k-n)!(2\bar{\gamma}_{r,d}+1)^k}, \end{aligned}$$

$$\begin{aligned} \bar{P}_{b8} =& \frac{1}{2(2\bar{\gamma}_{r,d}+1)} \sum_{k=0}^{\infty} \sum_{n=0}^{k} \frac{2^{k-n}\bar{\gamma}_{r,d}^k \gamma(k+1-n, T_1)}{(k-n)!(2\bar{\gamma}_{r,d}+1)^k} \\ &- \frac{1}{8(2\bar{\gamma}_{s,d}+1)(2\bar{\gamma}_{r,d}+1)} \sum_{k=0}^{\infty} \sum_{i=0}^{\infty} \sum_{n=0}^{k} \sum_{j=0}^{i} \sum_{m=0}^{i-j} \\ &\frac{\bar{\gamma}_{r,d}^k (2\bar{\gamma}_{s,d})^i \gamma(k+m+1-n, 2T_1)}{(2\bar{\gamma}_{r,d}+1)^k (2\bar{\gamma}_{s,d}+1)^i (k-n)!m!2^{j+m}} \\ &+ \frac{1}{8(\bar{\gamma}_{s,d}+1)(\bar{\gamma}_{r,d}+1)}(1-e^{-2T_1}). \end{aligned}$$

BER given in (22) allows us to evaluate the performance of the DDF scheme for arbitrary average SNRs ($\bar{\gamma}_{s,d}, \bar{\gamma}_{s,r}, \bar{\gamma}_{r,d}$). As a result, one can exploit (22) to analyze problems like optimal geometric locations of nodes, and optimal power allocation strategy.

### *B. Outage Probability*

For DDF, we cannot employ a single instantaneous SNR at destination D as an indicator of outage due to decision errors at relay R. Alternatively, we can define that an outage of the cooperative system occurs when both the direct and relay links experience outage. Hence, the outage probability $P_{\text{out}}$ is given by

$$P_{\text{out}} = P_{\text{out}}^{s,d} P_{\text{out}}^{s,r,d}, \tag{23}$$

where $P_{\text{out}}^{s,d}$ and $P_{\text{out}}^{s,d}$ denote the outage probability of the direct and relay link, respectively. Specifically, $P_{\text{out}}^{s,d}$ is the probability with which $\gamma_{s,d}$ drops below threshold $\gamma_{\text{th}}$:

$$P_{\text{out}}^{s,d} \triangleq \int_0^{\gamma_{\text{th}}} p_{\gamma_{s,d}}(\gamma) d\gamma = 1 - e^{-\gamma_{\text{th}}/\bar{\gamma}_{s,d}}, \tag{24}$$

while $P_{\text{out}}^{s,r,d}$ is the probability with which either $\gamma_{s,r}$ or $\gamma_{r,d}$ drops below $\gamma_{\text{th}}$:

$$\begin{aligned} P_{\text{out}}^{s,r,d} &\triangleq P(\gamma_{s,r} \leq \gamma_{\text{th}} \cup \gamma_{r,d} \leq \gamma_{\text{th}}) \\ &= 1 - e^{-\gamma_{\text{th}}(1/\bar{\gamma}_{s,r} + 1/\bar{\gamma}_{r,d})}. \end{aligned} \tag{25}$$

### *C. Asymptotic Analysis and Diversity Order*

We may find the diversity gain from the average BER in (22) by letting the SNR approach infinity, this turns out a tedious process. Instead, we will use the outage probability to determine its diversity gain. Let $\bar{\gamma}_{s,r} = \bar{\gamma}_{s,d} = \bar{\gamma}_{r,d} = \rho$ and $\rho \to \infty$. A first-order Taylor expansion of (24) and (25), followed by a substitution back into (23), yields:

$$P_{\text{out}} \propto C_{\text{DDF}} \gamma_{\text{th}}^2 \rho^{-2}, \quad \text{for large } \rho, \tag{26}$$

where $C_{\text{DDF}}$ is a constant. Hence, the DDF has a diversity order of 2.

## V. Numerical Results

Fig. 2 plots the average BER curves for DDF applying the PL detector (both simulation and analytical result), along with the average BER of the non-cooperative differential BPSK. For fair comparison, we set $\bar{\gamma}_{s,r} = \bar{\gamma}_{r,d} = \bar{\gamma}_{s,d} = 0.5E_b/N_0$, so that the sum of the transmitted energy from both S and R for the cooperative system is identical to that of the conventional system. The simulation BER of DDF with ML detection is also shown in Fig. 2. It is seen that the PL detector performs slightly worse than the ML detector. Overall, the analytical BER performance for the PL detector agrees the simulation result, and our cooperative scheme achieves cooperative diversity and outperforms the non-cooperative DBPSK.

Fig. 3 illustrates the outage probabilities for the proposed and the non-cooperative schemes, where the threshold $\gamma_{\text{th}} =$ 10 dB. The horizontal axis in Fig. 3 is the average SNR normalized by $\gamma_{\text{th}}$. DDF is again seen to yield diversity gain over the non-cooperative scheme.

## VI. Conclusions

We have presented a differential BPSK modulation scheme for wireless relay networks. Both analytical and simulation results show that the proposed DDF scheme can provide diversity gain to the destination when CSI is not available at the receiver in Rayleigh fading channels.

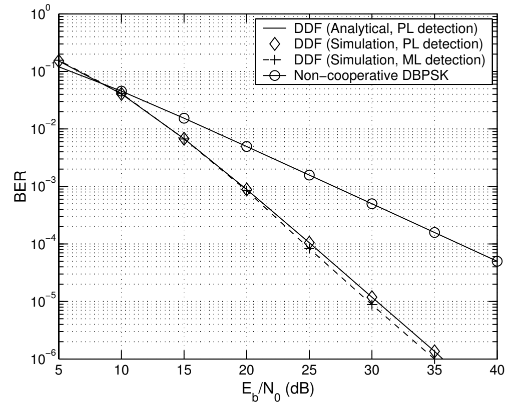


Fig. 2. BER performance in Rayleigh fading channels.

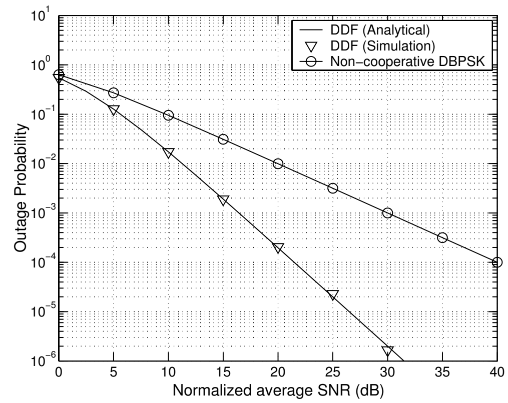


Fig. 3. Outage probability in Rayleigh fading channels.

## VII. Acknowledgment

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