STABILITY OF WIRELESS RELAYS IN MOBILE AD HOC NETWORKS

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ABSTRACT

In mobile ad hoc networks, relaying is one of the most fundamental functions. In this paper, we analyze the stability of several relaying strategies. By using the notion of node lifetime instead of the commonly used link lifetime, our analysis offers a complementary and unique finding not available in the literature previously. Our results show that a route of parallel relays have a significantly longer lifetime than a route of serial relays although the latter of the two has previously predominated the networking research.

1. INTRODUCTION

Mobile ad hoc networks (MANET) are important for both military and civilian applications. In MANET, wireless relaying of data over many hops is essential. Relaying strategies can be divided into two groups: one is using serial relays and the other using parallel relays. The strategies using serial relays have so far predominated the networking research, e.g., see the routing schemes of AODV, DSR, ABR and SSR [1]. The reason for this is that each node in the network has often been assumed to be independent from all others at the physical layer. However, recent research of information theory and signal processing strongly suggests that a higher network capacity is achievable if the nodes are cooperative at the physical layer, e.g., see [2, 3]. A cooperation at the physical layer makes a route of parallel relays more efficient in throughput than that of serial relays [4].

In this paper, we present a stability analysis of routes using parallel relays and routes using serial relays. Stability is an important criterion to evaluate the performance of a routing strategy. For on-demand routing schemes that are highly desirable in applications [1], the time interval between two consecutive route discoveries is called *route lifetime* which is also a stability measure. A longer-lived route is more stable as it requires a less frequency of route rediscoveries and reduces the delay of data transmissions.

We will focus on four strategies: (1) single-path of serial relays; (2) completely disjoint multi-path of serial relays; (3) partially disjoint multi-path of serial relays; and (4) single path of parallel relays. The first strategy is the most conventional, which we use as a reference. The second and third strategies are popular in current networking research as they offer a longer lifetime than the first strategy [5,6,7]. However, a cost of the second and third strategies is a larger area of interference and higher energy consumption. The fourth strategy is relatively new. It hinges on the physical layer cooperation among cooperative nodes, and also raises new research issues across PHY, MAC and Network layers. Later in this paper, we demonstrate that the fourth strategy offers a longer lifetime than the first three strategies. This advantage is achievable without necessarily a larger area of interference or a higher energy consumption than the first strategy.

Our analysis differs from the most in the literature [5, 8, 9, 10, 11]. It is quite common that *link lifetime* is used as the base parameter and the link lifetimes of different links are assumed to be independent. This assumption is not always valid. For example, when two transmitting nodes are relatively close to each other with respect to a receiving node, their links to the receiving node are highly correlated [12]. Also, if a node experiences shadowing in all directions, battery drain or a failure of operation, then all the links associated with this node may lose connections simultaneously. As an alternative and complementary approach, *node lifetime* will be used as the base parameter in our analysis. The lifetime of a node is a random variable denoted by X. The cumulative distribution function (cdf) of X is denoted by $F_X(t)$. We

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will assume that all nodes in the network have independent and identically distributed (i.i.d.) lifetimes. We also assume $dF_X(t) = \lambda e^{-\lambda t} dt$. Given the exponential distribution, the expected lifetime of any route can be generally expressed by C/λ where C is a route-dependent constant and $1/\lambda$ is the expected node lifetime. For the four categories of routing as mentioned before, it is this constant C that we will be primarily interested in.

2. STABILITY ANALYSIS

2.1. Single-Path of Serial Relays

This scheme can be illustrated in Figure 1.

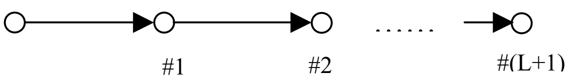

Figure 1: Single-Path of Serial Relays (SP-SR)

Assume that there are L+1 hops from the source to the destination. Then, the lifetime of this route is $T_{SP-SR} = \min\{X^{(1)},...,X^{(L+1)}\}$, and its expected value can be shown to be $E(T_{SP-SR}) = C_{SP-SR}(L)/\lambda$ where $C_{SP-SR}(L) = 1/(L+1)$.

2.2. Completely Disjoint Multi-Path of Serial Relays

Most of the recently proposed multipath routing schemes fall in this category [7]. The structure of such a routing scheme is illustrated in Figure 2.

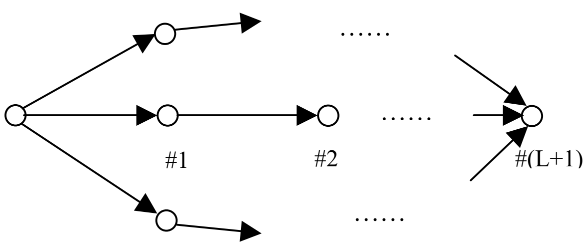

Figure 2: Completely Disjoint Multi-Path of Serial Relays (CDMP-SR)

Note that this scheme has a larger area of interference because the different paths must be sufficiently far from each other (or otherwise additional orthogonal radio channels are required). Assume that there are N-1 alternative paths, in addition to the primary path, from the source to the destination. The primary path is typically the shortest. Hence, we assume that the ith path has H_i+1 hops where $H_1 = L$ and $H_i \geq L$ for $2 \leq i \leq N$. The lifetime of the partial ith path (i.e., excluding the destination) is $Z_i = \min\{X_i^{(1)},...,X_i^{(H_i)}\}$ where $X_i^{(k)}$ is the node lifetime in tier k of path i. Then, the lifetime of the route is:

$$T_{CDMP-SR} = \max\{\min\{Z_1, X^{(L+1)}\},...,\min\{Z_N, X^{(L+1)}\}\}$$
$$= \min\{\max\{Z_1, Z_2,...,Z_N\}, X^{(L+1)}\}$$

We know that $\{Z_i\}_{i=1}^{N}$ are independent, and thus

$$P(\max\{Z_1, Z_2,...,Z_N\} \leq t) = \prod_{i=1}^{N}\left[1 - \prod_{j=1}^{H_i}(1 - F_{X_i^{(j)}}(t))\right].$$

Therefore, the cdf of the route lifetime is

$$F_{CDMP-SR}(t) = P(\min\{\max\{Z_1, Z_2,...,Z_N\}, X^{(L+1)}\} \leq t)$$
$$= 1 - \left\{1 - \prod_{i=1}^{N}\left[1 - \prod_{j=1}^{H_i}(1 - F_{X_i^{(j)}}(t))\right]\right\}[1 - F_{X^{(L+1)}}(t)] .$$

Under the i.i.d. condition of the node lifetime, we have $F_{CDMP-SR}(t) = 1 - \left[1 - \prod_{i=1}^{N}\left(1 - e^{-H_i \lambda t}\right)\right] e^{-\lambda t}$. Furthermore, if $H_i = L \ \forall i$, we can show that the expected route lifetime is $E(T_{CDMP-SR}) = C_{CDMP-SR}(N, L)/\lambda$ where

$$C_{CDMP-SR}(N, L) = \int_0^1 \left[1 - \left(1 - x^L\right)^N\right] dx = 1 - \frac{\Gamma(1 + 1/L) \cdot N!}{\Gamma(1 + N + 1/L)} \quad (1).$$

2.3. Partially Disjoint Multi-Path of Serial Relays

In [5], a partially disjoint multipath routing scheme as shown in Figure 3 is proposed.

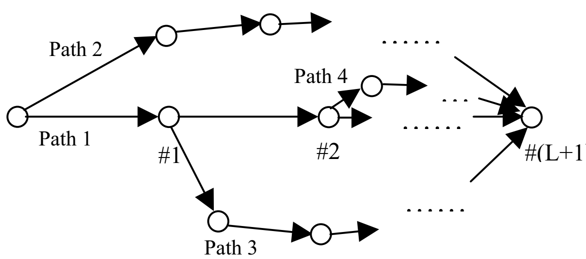

Figure 3: Partially Disjoint Multi-Path of Serial Relays (PDMP-SR)

This scheme ensures that any node on the primary path is connected to one or more alternative paths (But only one alternative path from each node is considered in [5] and here). Thus, when a node on the primary path fails, the node in the previous tier can detect the failure and readily forward packets along an alternative path to the destination. The authors of [5] also tried to show that the stability of this scheme would be better than CDMP-SR. However, their analysis assumed that $\{Z_i\}_{i=1}^{k}$ in Eqns. (14) and (15) of [5] were independent of each other. But they are actually not. Here, we use the notion of node lifetime to re-analyze this scheme. First, we introduce some definitions:

(a) The node lifetimes on the primary path are denoted by $\{X_1^{(1)}, X_1^{(2)},..., X_1^{(L)}, X_1^{(L+1)}\}$ where $X_1^{(L+1)}$ is the lifetime of the destination.

(b) The alternative path i with $H_i + 1$ hops is $\{X_1^{(1)}, X_1^{(2)}, ..., X_1^{(i-2)}, X_i^{(i-1)}, X_i^{(i)}, X_i^{(i+1)},..., X_i^{(H_i)}, X_i^{(H_i+1)}\}$ where $X_i^{(H_i+1)} = X_1^{(L+1)}$. Here, $X_1^{(1)}, X_1^{(2)}, ..., X_1^{(i-2)}$ are a portion of path i that coincides with the primary path, and $X_i^{(i-1)}, X_i^{(i)}, X_i^{(i+1)},..., X_i^{(H_i)}$ are a portion of path i that disjoints from the primary path.

(c) $Z_i = \min\{X_i^{(i-1)}, X_i^{(i)}, X_i^{(i+1)},..., X_i^{(H_i)}\}, \quad 2 \le i \le L+2$ is the lifetime of the partial path i that disjoints from the primary one. $\{Z_i\}_{i=2}^{L+2}$ are mutually independent and also independent from $\{X_1^{(1)}, X_1^{(2)},..., X_1^{(L)}, X_1^{(L+1)}\}$.

(d) The lifetimes of all paths (excluding the destination) are:
$Y_1 = \min\{X_1^{(1)}, X_1^{(2)},..., X_1^{(L)}\}$ for the primary path;
$Y_2 = Z_2 = \min\{X_2^{(1)}, X_2^{(2)},..., X_2^{(H_2)}\}$ for path 2;
$Y_i = \min\{X_1^{(1)},..., X_1^{(i-2)}, Z_i\}, \quad 3 \le i \le L+2$ for path i.

Based on the above definitions, the route lifetime is $T_{PDMP-SR} = \min\{\max\{Y_1, Y_2,..., Y_{L+2}\}, X^{(L+1)}\}$. Then,

$$F_{PDMP-SR}(t) = 1 - P(\min\{\max\{Y_1, Y_2,..., Y_{L+2}\}, X^{(L+1)}\} > t) = 1 - [1 - P(Y_1 \le t, Y_2 \le t,..., Y_{L+2} \le t)]P(X^{(L+1)} > t).$$

We can show that

$$P(Y_1 \le t, Y_2 \le t,..., Y_{L+2} \le t) = P(Z_2 \le t) - \sum_{j=1}^{L-1}\left\{P(Z_{j+2} > t)\left[\prod_{i=1}^{j} P(X_1^{(i)} > t)P(Z_{i+1} \le t)\right]\right\} - \prod_{i=1}^{L} P(X_1^{(i)} > t)P(Z_{i+1} \le t)$$

Thus,

$$F_{PDMP-SR}(t) = 1 - \left\{P(Z_2 > t) + \sum_{j=1}^{L-1}\left\{P(Z_{j+2} > t)\left[\prod_{i=1}^{j} P(X_1^{(i)} > t)P(Z_{i+1} \le t)\right]\right\} + \prod_{i=1}^{L} P(X_1^{(i)} > t)P(Z_{i+1} \le t)\right\}P(X_1^{(L+1)} > t)$$

where $P(Z_i > t) = \prod_{k=i-1}^{H_i} P(X_i^{(k)} > t)$. Under the i.i.d. condition of the node lifetime, we have

$$F_{PDMP-SR}(t) = 1 - \left\{\begin{array}{l} e^{-H_2\lambda t} + \sum_{j=1}^{L-1} e^{-H_{j+2}\lambda t} \prod_{i=1}^{j}(1 - e^{-(H_{i+1}-i+1)\lambda t}) \\ + e^{-L\lambda t} \prod_{i=1}^{L}(1 - e^{-(H_{i+1}-i+1)\lambda t}) \end{array}\right\} e^{-\lambda t}.$$

Furthermore, if $H_i = L \ \forall i$, then the expected lifetime of the route is $E\{T_{PDMP-SR}\} = C_{PDMP-SR}(L)/\lambda$ where

$$C_{PDMP-SR}(L) = \int_0^1 x^L \left\{1 + \sum_{j=1}^{L}\prod_{i=1}^{j}\left(1 - x^{L-i+1}\right)\right\} dx \qquad (2).$$

2.4. Single-Path of Parallel Relays

The structure of a single-path route using parallel relays is shown in Figure 4 where the nodes in each tier transmit and receive data within a single channel. Symbol synchronization and space-time modulation can be achieved at the physical layer when a single or multiple narrowband symbol carrier(s) are used. As shown in [4], the diversity gain at each tier of this route is N^2 if N parallel relays are used in each tier, and a significant power saving is achievable in comparison to a single-path of serial relays. Furthermore, we have found that a route of parallel relays is uniformly more efficient than the duct routing scheme proposed in [13]. Also note that since the nodes at each tier do not have to be far apart, the area of interference here is much smaller than the schemes shown in Figures 2 or 3.

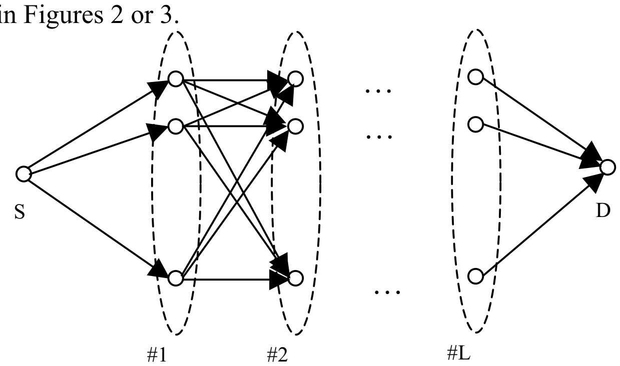

Figure 4: Single-Path of Parallel Relays (SP-PR)

Let N_k be the number of nodes in tier k $(1 \le k \le L+1)$ with $N_{L+1} = 1$, and $X_i^{(k)}$ the lifetime of the ith node in tier k. Then, the lifetime of tier k is $Z_k = \max\{X_1^{(k)}, X_2^{(k)},..., X_{N_k}^{(k)}\}$. The lifetime of the SP-PR route is $T_{SP-PR} = \min\{Z_1, Z_2,..., Z_{L+1}\}$. Assume that $N_k = N \quad (1 \le k \le L)$ and $N_{L+1} = 1$. Then, we can show that

$$F_{SP-PR}(t) = P(T_{SP-PR} \le t) = 1 - \left[1 - (1 - e^{-\lambda t})^N\right]^L e^{-\lambda t}$$

and the expected lifetime of the route is $E(T_{SP-PR}) = C_{SP-PR}(N, L)/\lambda$ where

$$C_{SP-PR}(N, L) = \int_0^1 \left(1 - (1-x)^N\right)^L dx = \frac{\Gamma(1 + 1/N) \cdot L!}{\Gamma(1 + L + 1/N)} \qquad (3).$$

One should notice a similarity and a difference between (1) and (3).

3. NUMERICAL RESULTS

The expected route lifetimes for all four strategies can be compared via the C constants. Assuming $N = 2$, Figure 5 compares the C constants for $L = 2 \sim 7$. As expected, as L increases, all the C constants decrease. However, the SP-PR scheme shows the longest route lifetime among all the routing strategies. In particular, SP-PR lives twice as long as SP-SR. In general, we have

$$C_{SP-SR} < C_{CDMP-SR} < C_{PDMP-SR} < C_{SP-PR} .$$

Table 1 compares our analysis of CDMP-SR with that from [5]. The results are surprisingly similar although their analysis is based on link lifetime and ours is based on node lifetime. From the table, we also notice that the route lifetime based on link lifetime is slightly longer than the route lifetime based on node lifetime. One explanation is as follows. When a node fails, all the links associated with this node also fail. But in [5], the lifetimes of all links are assumed to be independent.

4. CONCLUSIONS

We have analyzed and compared the lifetimes of four different routing schemes. Our results suggest that a route of parallel relays has a longer lifetime than a route of serial relays. Using parallel relays, we can increase the lifetime of a route without (or with little) increase of interference area. The multi-path routing schemes widely considered in the literature do not have this advantage. Further results along these lines have been reported in [14].

5. REFERENCES

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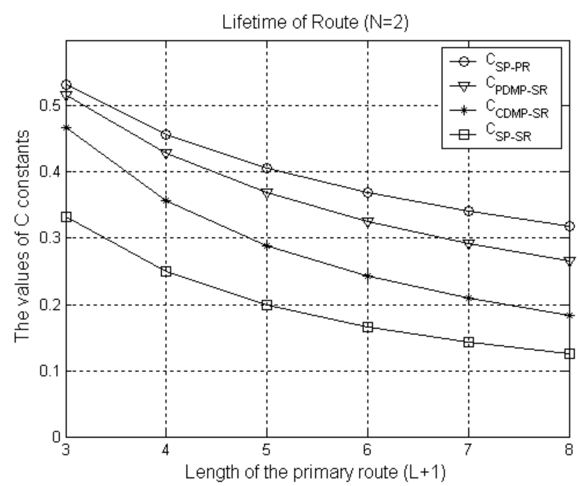

Figure 5: Comparison of expected route lifetimes of SP-PR, CDMP-SR, PDMP-SR and SP-SR.

CDMP-SR (N=2, 1/λ=5)		
L +1 (k in [5])	$E(T)$ in [5]	$E(T)$ in our analysis
3	2.5	2.334
4	1.875	1.786
5	1.50	1.445
6	1.25	1.212
7	1.07	1.044

Table 1: Comparison of our analysis with that from [5] for CDMP-SR