BLIND CHANNEL ESTIMATION FOR PRECODED OFDM SYSTEM

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ABSTRACT

An orthogonal frequency division multiplex (OFDM) system with block-precoding is studied. It is observed that the correlation matrix of the post-DFT data is equal to the direct-product of the channel vector multiplied element-by-element with the product of the precoding matrix and its Hermitian. Therefore, an element-by-element division of the post-DFT correlation matrix by the product of the precoding matrix and its Hermitian gives a rank-one matrix whose only singular vector coincides the channel vector up to a phase ambiguity. The design criteria of the precoding matrix is discussed.

1. INTRODUCTION

Orthogonal frequency division multiplex (OFDM) has been identified as an effective technique to achieve high-speed data transmission over frequency-selective fading channels. It sends multiple data streams through a number of subbands of the total spectrum, each subband being narrow enough such that the channel fading is frequency flat. Orthogonality among the subcarriers is maintained while subbands are allowed to overlap.

An efficient implementation of OFDM may contain the following steps (c.f. Fig. 1). First, the information-bearing symbol, modulated via any type of constellation (e.g., BPSK, QAM) is segmented into blocks. IDFT is then performed on each block, and a preamble, consisting of the last several IDFT coefficients, called cyclic prefix (CP), is appended in front of the IDFT block. The augmented blocks are sent one after the other through the communication channel. As long as the effective time-spread of the channel is smaller than the length of the CP, inter-block interference only contaminates the CP part. After the CP being removed, the remaining of the received block, which is free and inter-block interference, can be shown to be equal to the cyclic convolution of the channel and the corresponding block of the information-bearing symbols. Thus, taking DFT on it leads to the original symbol block scaled symbol-by-symbol by the frequency samples of the channel at each individual subcarriers .

For coherent detection of the information symbols in each data block, reliable estimation of the channel at each frequency sample is crucial. Training/pilot symbols may be used for this purpose, e.g., [1]. However, blind channel estimation methods are well motivated for their spectral efficiency. Many blind methods estimate the time-domain channel using the pre-DFT channel output; the frequency-domain channel can be obtained by taking DFT of the estimates. Heath and Giannakis' method [2] uses the cyclostationarity induced by the CP. Also based on the cyclostationarity but in an implicit manner, the methods of Muquet's *et. al.* [3], Cai and Akansu's [4] and Zhuang's *et. al.* [5] uses the subspace structure of the pre-DFT data. There are also blind methods directly estimate the frequency-domain channel using the post-DFT signal. Zhou and Giannakis [6] have proposed one based on the finite-constellation property of the information symbols. Wang and Chen [7] have proposed to use the receiving diversity.

Frequency-domain precoding is an effective measure to create frequency diversity in OFDM so as to avoid the catastrophic effects of channel zeros at certain subcarriers [8]. The precoding also facilities blind channel estimation. The subspace-based methods in [9, 10] uses the redundancy introduced by the precoding. While, an non-redundant precoding scheme for blind channel estimation has also been proposed [11], which, by superimposing the information symbol of one subchannel. on all other ones, allows simple channel estimation through correlating the output of each subchannel to that of that superimposed one.

In this work, we show that blind OFDM channel estimation using non-redundant linear precoding is generally doable with almost any coding schemes. It is observed that the correlation matrix of the post-DFT data, if divided element-by-element with the product of the precoding matrix and its Hermitian, is equal to the direct-product of the channel vector. Therefore, an SVD can be revoked to recover the channel up to a phase ambiguity. Though some special precoder allows simple channel estimation algorithms such as that proposed in [11], others may simplify the encoding procedure. For example, a circular precoding matrix replaces the matrix multiplication operation in the coding procedure with a simple vector weighting. It also allows trade off the conflicting requirement for the precoder from the viewpoints of channel estimation and symbol error probability through a single parameter.

2. SIGNAL MODEL

Figure 1 shows the linearly precoded OFDM system (in its baseband, baud-rate, discrete-time form) that we are considering. The significant component is the linear precoder, which maps every, say the i-th, block of N information symbols $\{d_{i,n}, n = 0, \ldots, N-1\}$ to block of N coded symbols, $\{s_{i,n}, n = 0, \ldots, N-1\}$ by a linear transform. In matrix form, the precoding is described as

$$\boldsymbol{s}_i = \boldsymbol{A}\boldsymbol{d}_i \ , \tag{1}$$

where $\boldsymbol{d}_i = [d_{i,0} \ldots d_{i,N-1}]^T$, $\boldsymbol{s}_i = [s_{i,0} \ldots s_{i,N-1}]^T$, and $\boldsymbol{A}$ is an $N \times N$ coding matrix. Note that the precoding does not introduce any redundancy. For unique decodability, we require

a1) $\boldsymbol{A}$ be full rank.

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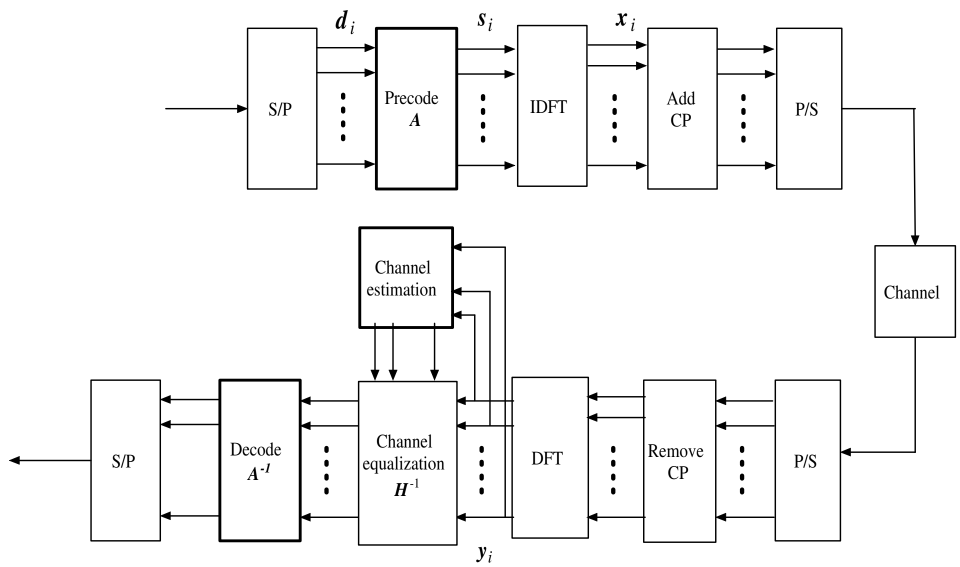

Fig. 1. OFDM system with linear precoder

The coded block $\boldsymbol{s}_i$ goes through the regular OFDM procedure: IDFT and CP-added before it is fed into the channel. The ith received signal block after removal of the CP and DFT is

$$\boldsymbol{y}_i = \boldsymbol{H}\boldsymbol{s}_i + \boldsymbol{v}_i = \boldsymbol{H}\boldsymbol{A}\boldsymbol{d}_i + \boldsymbol{v}_i \ , \tag{2}$$

where $\boldsymbol{H} = \text{diag}\{\tilde{h}(0), \ldots, \tilde{h}(N-1)\}$ and $\{\tilde{h}(k), k = 0, \ldots, N-1\}$ are the DFT coefficients of the channel, and $\boldsymbol{v}_i = [v_{i,0}, \ldots, v_{i,N-1}]^T$ models the noise. We assume that $v_{i,k}$ is complex, circular-Gaussian, zero-mean, σ^2-variance, and white across subcarriers and across blocks.

3. BLIND CHANNEL ESTIMATION THROUGH SVD

Our proposed blind channel estimation method is based on an interesting structure of the correlation matrix of the post-DFT signal $\boldsymbol{y}_i$ due to the linear precoding. The correlation matrix of $\boldsymbol{y}_i$ can be written as

$$\boldsymbol{R} = \mathrm{E}[\boldsymbol{y}_i \boldsymbol{y}_i^H] = \boldsymbol{H}\boldsymbol{A}\boldsymbol{A}^H\boldsymbol{H}^H + \sigma^2\boldsymbol{I} \ , \tag{3}$$

assuming whiteness and unit-power of the information symbol sequence. A critical observation as shown in (4) can be made, where b_{ij} is the (i, j)-th entry of $\boldsymbol{A}\boldsymbol{A}^H$, $\tilde{\boldsymbol{h}} = [\tilde{h}(0) \ldots \tilde{h}(N-1)]^T$, and $\odot$ means element-by-element multiplication.

If the matrix

a2) $\boldsymbol{A}\boldsymbol{A}^H$ has unit diagonal entries and no zero entries,

which can always be satisfied by properly choosing $\boldsymbol{A}$, we can perform an element-by-element division of $\boldsymbol{R}$ with $\boldsymbol{A}\boldsymbol{A}^H$. The quotient is

$$\boldsymbol{W} = \boldsymbol{R}./(\boldsymbol{A}\boldsymbol{A}^H) = \tilde{\boldsymbol{h}}\tilde{\boldsymbol{h}}^H + \sigma^2\boldsymbol{I} \ , \tag{5}$$

where ./ is a symbol of element-by-element division borrowed from Matlab™ notations. It can be seen that the singular value decomposition (SVD) of $\boldsymbol{W}$ directly gives the channel $\tilde{\boldsymbol{h}}$ with a phase ambiguity as the only singular vector. While, the phase ambiguity can be easily taken care of by a single training symbol or made irrelevant by using differential modulation. In addition, since only one singular vector which corresponding to the only nonzero singular value is needed, the SVD can be performed using simple algorithm.

3.1. Identifiability

The proposed method can identify any channels (up to a phase ambiguity), as long as a2) is satisfied. This fact can be easily seen from the uniqueness of the SVD. It deserves notice that the identifiability is guaranteed even when a channel has nulls at some subcarriers, a property that most channel estimation methods for OFDM do not possess.

3.2. Symbol Detection

After the channel has been obtained, symbol detection is straightforward. A simple zero-forcing solution is

$$\hat{\boldsymbol{d}}_i = (\hat{\boldsymbol{H}}\boldsymbol{A})^{-1}\boldsymbol{y}_i \ , \tag{6}$$

where $\hat{\boldsymbol{d}}$ and $\hat{\boldsymbol{H}}$ denoted the estimated versions of their counterparts. It may be noted that channel zeros at some subcarriers may cause the fail of the detection of those symbols on those subcarriers. A direct evidence is that $\boldsymbol{H}\boldsymbol{A}$ will lose rank in that case, causing the model in (2) under-determined. This is a common issue in OFDM systems. However, this problem can be solved with a redundant precoder. If the channel has a duration L, it has at most $L-1$ nulls. Suppose that the coding matrix $\boldsymbol{A}$ is an $N \times M$ matrix with $M \geq N + L - 1$. Then, $\boldsymbol{H}\boldsymbol{A}$ is guaranteed full column-rank and thus reliable symbol detection is achieved. Our propose channel estimation method applies to this redundant precoding case, though the derivation of the method have assumed non-redundant precoding.

$$
\begin{aligned}
\boldsymbol{H}\boldsymbol{A}\boldsymbol{A}^H\boldsymbol{H}^H &= \begin{bmatrix} b_{11}|\tilde{h}(0)|^2 & b_{12}\tilde{h}(0)\tilde{h}^*(1) & \dots & b_{1N}\tilde{h}(0)\tilde{h}^*(N-1) \\ b_{21}\tilde{h}(1)\tilde{h}^*(0) & b_{22}|\tilde{h}(1)|^2 & \dots & b_{2N}\tilde{h}(1)\tilde{h}^*(N-1) \\ \vdots & \vdots & \ddots & \vdots \\ b_{N1}\tilde{h}(N-1)\tilde{h}^*(0) & b_{N2}\tilde{h}(N-1)\tilde{h}^*(1) & \dots & b_{NN}|\tilde{h}(N-1)|^2 \end{bmatrix} \\
&= (\boldsymbol{A}\boldsymbol{A}^H) \odot (\tilde{\boldsymbol{h}}\tilde{\boldsymbol{h}}^H)
\end{aligned} \tag{4}
$$

4. DESIGN CRITERIA FOR THE PRECODER

Though the precoder $\boldsymbol{A}$ enables blind channel estimation, it causes inter-carrier interference. Consider the symbol detection procedure of (6). Since

$$\hat{\boldsymbol{d}}_i = \boldsymbol{d}_i + (\hat{\boldsymbol{H}}\boldsymbol{A})^{-1}\boldsymbol{v}_i \ , \tag{7}$$

the symbol error probability is determined by the signal-to-noise ratio (SNR):

$$
\begin{aligned}
\text{SNR} &= \frac{\text{E}[\|\boldsymbol{d}_i\|^2]}{\text{E}[\|(\hat{\boldsymbol{H}}\boldsymbol{A})^{-1}\boldsymbol{v}_i\|^2]} \\
&= \frac{N}{\text{E}[\text{tr}\{(\boldsymbol{A}\boldsymbol{A}^H)^{-1}\boldsymbol{H}^{-1}\boldsymbol{v}\boldsymbol{v}^H\boldsymbol{H}^{-H}\}]} \\
&= \frac{1}{\sigma^2}\frac{N}{\text{tr}[(\boldsymbol{A}\boldsymbol{A}^H)^{-1}]} \ .
\end{aligned} \tag{8}
$$

Here, we have assumed normalized channel. Because tr$[\boldsymbol{A}\boldsymbol{A}^H]$ is the transmission power (c.f. (1)) and is thus fixed to N for normalization, it can be easily derived that tr$[(\boldsymbol{A}\boldsymbol{A}^H)^{-1}] \leq N$ and the equality holds only when $\boldsymbol{A}$ is a unitary matrix, i.e., $\boldsymbol{A}\boldsymbol{A}^H = \boldsymbol{I}$. However, unitary matrices will not satisfy a2), hence is not applicable. Therefore, the precoder will always introduce an SNR loss. To reduce this loss, $\boldsymbol{A}\boldsymbol{A}^H$ should be as close to the identity matrix as possible, i.e., having small off-diagonal entry.

However, from (5), we see that small values of the entry of $\boldsymbol{A}\boldsymbol{A}^H$ may cause higher errors due to its amplification of any errors in estimating the correlation matrix $\boldsymbol{R}$. Therefore, the requirements for $\boldsymbol{A}$ for channel estimation and for error probability need to trade off.

4.1. Circular Precoder

We conjecture that a "good" precoder $\boldsymbol{A}$ could in the form of

$$
\boldsymbol{A} = \begin{bmatrix} \sqrt{\rho} & \sqrt{\frac{1-\rho}{N-1}} & \cdots & \sqrt{\frac{1-\rho}{N-1}} \\ \sqrt{\frac{1-\rho}{N-1}} & \sqrt{\rho} & \cdots & \sqrt{\frac{1-\rho}{N-1}} \\ \vdots & \vdots & \ddots & \vdots \\ \sqrt{\frac{1-\rho}{N-1}} & \cdots & \sqrt{\frac{1-\rho}{N-1}} & \sqrt{\rho} \end{bmatrix} \tag{9}
$$

for some $0 < \rho < 1$. We note that this $\boldsymbol{A}$ satisfies a1) and a2) and has unit norm for all rows. The circular symmetry of it is motivated by the symmetry of the subcarriers and the fact that circular matrices have a DFT eigen-structure. The latter property may simplify the implementation of the precoder.

Since $\boldsymbol{A}$ as given in (9) is circular, its eigen-decomposition is

$$\boldsymbol{A} = \frac{1}{N}\boldsymbol{F}\boldsymbol{\Lambda}\boldsymbol{F}^H \ , \tag{10}$$

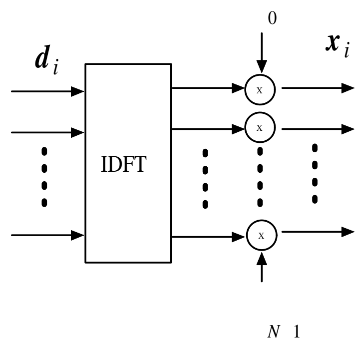

Fig. 2. Alternative implementation of circular precoder

where $\boldsymbol{F} = [e^{-j2\pi/Nnk}]_{0\leq n\leq N-1}^{0\leq k\leq N-1}$ is the DFT matrix, $\boldsymbol{\Lambda} = \text{diag}\{\lambda_0, \dots, \lambda_{N-1}\}$ and λ_n is the eigen-value of $\boldsymbol{A}$ and given by the DFT of the first row of $\boldsymbol{A}$. If we substitute (10) in to (1) and note that the transmitted signal $\boldsymbol{x}_i$ is the IDFT of $\boldsymbol{s}_i$, then we have

$$\boldsymbol{x}_i = \boldsymbol{F}^H\boldsymbol{F}\boldsymbol{\Lambda}\boldsymbol{F}^H\boldsymbol{d}_i = \boldsymbol{\Lambda}\boldsymbol{F}^H\boldsymbol{d}_i \ . \tag{11}$$

(11) gives us an alternative implementation of the precoding, as shown in Figure 2, in which the precoding is achieved by weighing the time-domain data.

With $\boldsymbol{A}$ in the form of (9), $\boldsymbol{A}\boldsymbol{A}^H$ can be obtained as

$$
\boldsymbol{A}\boldsymbol{A}^H = \begin{bmatrix} 1 & \alpha & \dots & \alpha \\ \alpha & 1 & \dots & \alpha \\ \vdots & \vdots & \ddots & \vdots \\ \alpha & \dots & \alpha & 1 \end{bmatrix} \tag{12}
$$

where $\alpha = 2\sqrt{\rho(1-\rho)/(N-1)} + (1-\rho)(N-2)/(N-1)$. We can control the value α of the off-diagonal entries of $\boldsymbol{A}\boldsymbol{A}^H$ by adjusting ρ.

Since $\boldsymbol{A}\boldsymbol{A}^H$ is also circular, its eigen-values can be obtained as the DFT of its first row, which are $\eta_0 = 1 + (N-1)\alpha$ and $\eta_k = 1 - \alpha$, $k = 1, \dots, N-1$. We know that

$$\text{tr}[(\boldsymbol{A}\boldsymbol{A}^H)^{-1}] = \sum_{k=0}^{N-1}\frac{1}{\eta_k} = \frac{1}{1+(N-1)\alpha} + \frac{N-1}{1-\alpha} \ . \tag{13}$$

Therefore, the SNR loss due to the precoder can also be controlled by the parameter ρ.

5. SIMULATIONS

We have simulated an OFDM system with 16 subcarriers (also the DFT size). The precoder is chosen according to (9) with different ρ values. The channel model used is a 3-tap FIR filter with tap coefficients independently chosen from a white Gaussian process. The CP length is set to 5 to accommodate the channel spread. The noise is additive white Gaussian noise. The modulation scheme is 16-QAM

We use the normalized MSE as the figure of merit for the channel estimation, which is defined as

$$\overline{NMSE} = \frac{1}{M_1 M_2} \sum_{i=1}^{M_1} \sum_{j=1}^{M_2} \frac{\sum_{k=0}^{N-1} [\tilde{h}^i(k) - \hat{\tilde{h}}^{i,j}(k)]^2}{\sum_{k=0}^{N-1} [\tilde{h}^i(k)]^2}, \quad (14)$$

where M_1 is the number of channel realizations in the simulations, M_2 is the number of data and noise realizations for each channel realization, and $\tilde{h}^i(k)$, $\hat{\tilde{h}}^{i,j}(k)$ are the i-th realization of the channel and its estimate at the j-th Monte Carlo run, respectively. For one point of $\overline{NMSE}$ value, we do the simulation for 60 channel realizations, each for 100 data and noise realization. In addition, we use 50 OFDM blocks to compute the correlation matrix for each channel estimation.

Fig. 3 shows $\overline{NMSE}$ figure versus the signal-to-noise (SNR) ratio for several different ρ value, and Fig. 4 shows the bit-error-rate (BER) using the estimated channel for coherent detection.

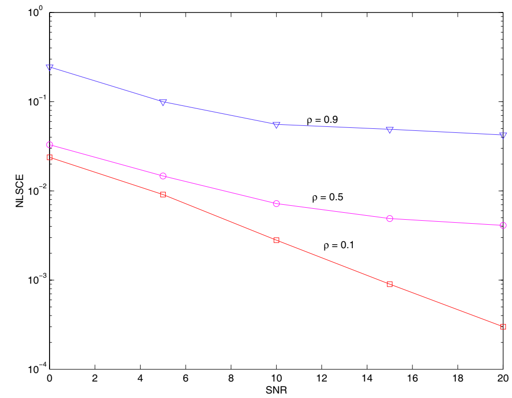

Fig. 3. NMSE performance of the proposed blind channel estimation method

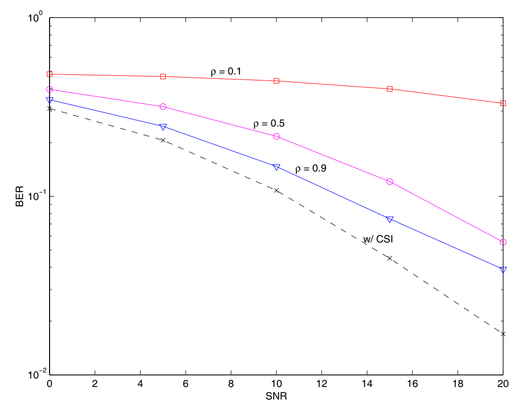

Fig. 4. BER performance of the proposed blind channel estimation method

6. CONCLUSION

We have proposed an SVD-based blind channel estimation method for linearly precoded OFDM systems. This method works with non-redundant precoders to identify even channels with zeros on the unit circle. We have also discussed some design criteria of the precoder and give heuristically a "good" precoder which can be optimized with only one parameter. However, the "optimal" precoder is still yet to be studied. Another interesting research may be how to explore traditional codes, such as a block code for error correcting in the place of the precoder, for the channel estimation purposes.

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