JOINT BLIND CHANNEL ESTIMATION AND INTERFERENCE SUPPRESSION FOR OFDM SYSTEMS

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ABSTRACT

This paper presents a blind joint channel and interference suppression for orthogonal frequency-division multiplexing (OFDM) systems. Our approach uses a generalized multichannel minimum variance principle to design an equalizing filterbank that preserves the desired signal components and suppresses the overall interference. Channel estimate is then obtained by deriving an asymptotically tight lower bound of the filterbank output power, which reduces the problem to a quadratic minimization. While a channel estimate may be obtained by directly maximizing the filterbank output power through multidimensional nonlinear searches, such an approach is computationally prohibitive and suffers local convergence. Numerical examples show that the proposed scheme approaches the Cramér-Rao bound (CRB) as the SNR increases. It also exhibits low sensitivity to unknown narrowband interference and compares favorably with a subspace blind channel estimator.

1. INTRODUCTION

OFDM is a multicarrier digital modulation technique that allows high data rate transmission such as digital TV broadcasting and high-speed telephone line communications. In OFDM, the transmitted information is transformed by the inverse fast Fourier transform (IFFT) into parallel blocks. When the channel is dispersive, inter-block interference (IBI) between successive blocks occurs. To eliminate the IBI, a cyclic prefix (CP) is inserted at the beginning of each transmitted data block. By choosing the length of the CP to be greater than the channel impulse response, successive blocks will not interfere and can be reliably recovered at the receiver's end.

Numerous channel estimation schemes have been investigated recently. These schemes rely on either *explicit training* (e.g., [1]) or some *inherent structure* (e.g, subspace [2]) of the transmitted signal. Although the training-assisted schemes perform quite well, they reduce the spectral efficiency. Moreover, in order to track channel variations, training symbols have to be retransmitted periodically, leading to throughput reductions. Blind schemes, on the other hand, do not suffer from such drawbacks. A well-known blind channel estimation schemes for MC are the subspace-based methods proposed in (e.g., [2, 3]). However, when there is insufficient information about the interference so that prewhitening cannot be performed, subspace channel estimation is in general inaccurate.

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In this work, we design an equalizing filterbank to solve the problem of joint blind channel estimation and interference suppression for OFDM. Based on a generalized multichannel minimum variance principle, the desired signal components are preserved while suppressing the overall interference. Channel estimate is then obtained by deriving an asymptotically (in SNR) tight lower bound of the filterbank output power, which reduces the problem to a quadratic minimization. Even though multi-dimensional non-linear search methods can be applied to find channel estimates by directly maximizing the filterbank output power, such an approach is computationally prohibitive and suffers local convergence. Numerical examples show that the proposed scheme exhibits low sensitivity to unknown narrowband interference and compares favorably with a subspace blind channel estimator. Furthermore, we compare the proposed scheme with the CRB which, similar to the proposed blind channel estimator, does not assume the knowledge of the transmitted information symbols.

Notation: Vectors (matrices) are denoted by boldface lower (upper) case letters; all vectors are column vectors; superscripts $(\cdot)^T$, $(\cdot)^*$ and $(\cdot)^H$ denote the transpose, conjugate and conjugate transpose, respectively; $\mathbf{I}_N$ denotes the $N \times N$ identity matrix; $\mathbf{0}$ denotes an all-zero matrix or vector; $\mathrm{tr}\{\cdot\}$ denotes the trace; $\mathrm{vec}(\cdot)$ stacks the columns of its matrix argument on top of one another; $E\{\cdot\}$ denotes the statistical expectation and finally, $\otimes$ denotes the matrix Kronecker product.

2. PROBLEM FORMULATION

Consider an OFDM system where a serial of information symbols $\mathbf{s}(n) = [s(nK), \cdots, s(nK+K-1)]^T$ are blocked into $K \times 1$ vectors , which are linearly transformed into $\mathbf{u}(n) = \mathbf{F}\mathbf{s}(n)$ by $J \times K$ matrix $\mathbf{F} \triangleq [\bar{\mathbf{F}}_1^T, \bar{\mathbf{F}}^T]^T$, where $\bar{\mathbf{F}}$ denotes the $K \times K$ IDFT unitary matrix, and $\bar{\mathbf{F}}_1 \in \mathbb{C}^{\mu \times K}$ is formed from the last $\mu \triangleq J - K$ rows of $\bar{\mathbf{F}}$, where μ is the length of the cyclic-prefix. To avoid multipath-induced inter-block interference (IBI), transmission redundancy is introduced by choosing $J \geq K + L$, where L is the channel order.

In what follows, we will process a block of $N \geq 1$ OFDM symbols simultaneously. Let

$$\mathbf{s}_N(n) = \begin{bmatrix} \mathbf{s}(nN) \\ \vdots \\ \mathbf{s}(nN+N-1) \end{bmatrix}_{KN \times 1},$$
$$\mathbf{u}_N(n) = \begin{bmatrix} \mathbf{u}(nN) \\ \vdots \\ \mathbf{u}(nN+N-1) \end{bmatrix}_{JN \times 1} = (\mathbf{I}_N \otimes \mathbf{F})\mathbf{s}_N(n). \tag{1}$$

The discrete-time baseband equivalent channel, which includes the transmitter/receiver filter and the physical channel, is modeled as an FIR filter $\mathbf{h} \triangleq [h(0), h(1), \cdots, h(L)]^T$. Hence, the overall received samples resulting from the transmission of $\mathbf{u}_N(n)$ is $JN + L$. We discard the first and last L samples and form a $(JN - L) \times 1$ vector $\mathbf{y}_N(n)$ which can be expressed as [2]:

$$\begin{aligned} \mathbf{y}_N(n) =& \mathbf{H}(\mathbf{I}_N \otimes \mathbf{F})\mathbf{s}_N(n) + \mathbf{w}_N(n) + \mathbf{e}_N(n), \\ n =& 1, 2, \cdots, P \end{aligned} \tag{2}$$

where $\mathbf{w}_N(n)$ and $\mathbf{e}_N(n)$ denote interference and channel noise vectors respectively, and $\mathbf{H}$ is an $(JN - L) \times JN$ Toeplitz matrix defined as:

$$\mathbf{H} = \begin{bmatrix} h_L & \cdots & h_0 & 0 & \cdots & \cdots & 0 \\ 0 & h_L & \cdots & h_0 & 0 & \cdots & 0 \\ \vdots & & & & & & \vdots \\ 0 & \cdots & \cdots & 0 & h_L & \cdots & h_0 \end{bmatrix}. \tag{3}$$

It is seen that $\mathbf{H}(\mathbf{I}_N \otimes \mathbf{F})$ is a tall matrix with full column rank if $JN - L \geq KN$ or equivalently, $N \geq \frac{L}{J-K}$. If the length of the cyclic prefix is chosen as $J - K = L$, then the minimum value of N that is needed is equal to one.

The problem of interest is to estimate the channel coefficients $\{h(n)\}_{n=0}^{L}$ from the observed data without any knowledge of the transmitted symbols.

3. PROPOSED SCHEME

Equation (2) represents a multiple-input multiple-output system with KN inputs and $JN - L$ outputs. The mixing matrix $\mathbf{H}(\mathbf{I}_N \otimes \mathbf{F})$ is partially known since $\mathbf{H}$ has a known Toeplitz structure and $\mathbf{F}$ is also known to the receiver. We can exploit this knowledge to design a bank of KN FIR filters $\mathbf{G} \in \mathbb{C}^{(JN-L)\times KN)}$, each passing one symbol with unit-gain, completely annihilating the other $KN - 1$ interfering symbols, meanwhile suppressing interference $\mathbf{w}_N(n)$ as much as possible. In particular, we design an equalizing filterbank according to the following minimum variance criterion:

$$\begin{aligned} \mathbf{G} =& \arg \min_{\mathbf{G} \in \mathbb{C}^{(JN-L)\times KN}} \operatorname{tr}\left\{\mathbf{G}^H \mathbf{R} \mathbf{G}\right\}, \\ & \text{subject to } \mathbf{G}^H \mathbf{H}(\mathbf{I}_N \otimes \mathbf{F}) = \mathbf{I}_{KN}, \end{aligned} \tag{4}$$

where $\mathbf{R} \triangleq E\{\mathbf{y}_N(n) \times \mathbf{y}_N^H(n)\}$ denotes the sample covariance matrix, and the constraint $\mathbf{G}^H\mathbf{H}(\mathbf{I}_N \otimes \mathbf{F}) = \mathbf{I}_{KN}$ ensures that each filter (i.e.,one column of $\mathbf{G}$) will pass only one signal component $\big[$corresponding to one column of $\mathbf{H}(\mathbf{I}_N \otimes \mathbf{F})\big]$ undistorted with unit-gain, while completely eliminating inter-symbol interference (ISI) caused by the other columns of $\mathbf{H}(\mathbf{I}_N \otimes \mathbf{F})$. Using the Lagrange multiplier, the solution to the above constrained quadratic minimization problem is given by:

$$\mathbf{G} = \mathbf{R}^{-1}\mathbf{H}(\mathbf{I}_N \otimes \mathbf{F})\big[(\mathbf{I}_N \otimes \mathbf{F})^H \mathbf{H}^H \mathbf{R}^{-1} \mathbf{H}(\mathbf{I}_N \otimes \mathbf{F})\big]^{-1}. \tag{5}$$

Substituting (5) into (4), the minimized average power of the filterbank output is given by

$$\begin{aligned} V_1(\mathbf{h}) &= \operatorname{tr}\left\{\big[(\mathbf{I}_N \otimes \mathbf{F})^H \mathbf{H}^H \mathbf{R}^{-1} \mathbf{H}(\mathbf{I}_N \otimes \mathbf{F})\big]^{-1}\right\} \\ &= \operatorname{tr}\left\{\big[\mathbf{H}^H \mathbf{R}^{-1} \mathbf{H}(\mathbf{I}_N \otimes \mathbf{F}\mathbf{F}^H)\big]^{-1}\right\}, \end{aligned} \tag{6}$$

where we used the fact that $\operatorname{tr}(\mathbf{AB}) = \operatorname{tr}(\mathbf{BA})$ for any $\mathbf{A}$ and $\mathbf{B}$ with compatible size. We note here that channel estimate may be obtained by directly maximizing the output power $V_1(\mathbf{h})$ through multidimensional nonlinear searches, however, such approach is computationally prohibitive and suffers local convergence. To overcome this difficulty, we derive an asymptotically (in SNR) tight lower bound for the output power and then use it for channel estimation. Using the Schwartz inequality (see [4]), maximizing $V_1(\mathbf{h})$ w.r.t (with respect to) $\mathbf{h}$ is equivalent to minimizing the following asymptotic lower bound:

$$\begin{aligned} V_2(\mathbf{h}) &= \operatorname{tr}\left\{\mathbf{H}^H \mathbf{R}^{-1} \mathbf{H}\big[\mathbf{I}_N \otimes (\mathbf{F}\mathbf{F}^H)\big]\right\} \\ &= \operatorname{vec}^T(\mathbf{H}^*)\left\{\big[\mathbf{I}_N \otimes (\mathbf{F}\mathbf{F}^H)\big] \otimes \mathbf{R}^{-1}\right\}\operatorname{vec}(\mathbf{H}), \end{aligned} \tag{7}$$

which becomes a quadratic minimization problem. Next, we express $\operatorname{vec}(\mathbf{H})$ explicitly as a linear function in $\mathbf{h}$. In particular, we can write

$$\operatorname{vec}(\mathbf{H}) = \mathbf{S}\mathbf{h}, \tag{8}$$

where $\mathbf{S}$ is a $(JN - L)JN \times (L + 1)$ matrix formed by elements 0 and 1 only. It is seen that $\operatorname{vec}(\mathbf{H})$ is full column rank since the above mapping is one-to-one. For example, if $L = 2$ i.e., 3-tap FIR channel, then $\mathbf{S}$ can be expressed as in (9), shown on top of the next page.

A number of remarks on the structure of $\mathbf{S}$ are in order:

- There are JN blocks, each block is of size $(JN - L) \times (L + 1)$.
- Block number 1: 1st row is formed by last row of $\mathbf{I}_{L+1}$; zeros elsewhere.
- Block number 2: first 2 rows formed by last 2 rows of $\mathbf{I}_{L+1}$; zeros elsewhere.
- Block number $L + 1$: first $L + 1$ rows from $\mathbf{I}_{L+1}$; zeros elsewhere.

$$
\mathbf{S} = \left[\begin{array}{ccc}
0 & 0 & 1 \\
\multicolumn{3}{c}{\mathbf{0}_{(JN-L-1)\times(L+1)}} \\ \hline
0 & 1 & 0 \\
0 & 0 & 1 \\
\multicolumn{3}{c}{\mathbf{0}_{(JN-L-2)\times(L+1)}} \\ \hline
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\multicolumn{3}{c}{\mathbf{0}_{(JN-L-3)\times(L+1)}} \\ \hline
\multicolumn{3}{c}{\vdots} \\ \hline
\multicolumn{3}{c}{\mathbf{0}_{(JN-L-2)\times(L+1)}} \\
1 & 0 & 0 \\
0 & 1 & 0 \\ \hline
\multicolumn{3}{c}{\mathbf{0}_{(JN-L-1)\times(L+1)}} \\
1 & 0 & 0
\end{array}\right]
\begin{array}{l}
\} \text{ block } \#1 \\
\} \text{ block } \#2 \\
\} \text{ block } \#3 \\
\vdots \\
\} \text{ block } \#JN-1 \\
\} \text{ block } \#JN
\end{array}
. \quad (9)
$$

- Block number $JN - L$: last $L+1$ rows from $\mathbf{I}_{L+1}$; zeros elsewhere.
- Block number $JN - 1$: last 2 rows formed by first 2 rows of $\mathbf{I}_{L+1}$; zeros elsewhere.
- Block number JN: last row formed by first row of $\mathbf{I}_{L+1}$; zeros elsewhere.
- Explicit expression of $\mathbf{S}$ can be obtained by using elementary matrix $\mathbf{E}_{ij}$: a $(JN-L)\times(L+1)$ matrix with unit element at the ijth location and zeros elsewhere. For example, the first L blocks, $\mathbf{S}_l$, may be expressed as $\mathbf{S}_l = \sum_{j=1}^{l} \mathbf{E}_{l-j+1,L+2-j}, l = 1, 2, \cdots, L$. The rest of $\mathbf{S}_l$ for $l = L+1, \cdots, JN$, can be expressed in a similar fashion.

Using vec$(\mathbf{H}) = \mathbf{Sh}$ from (8) back in eq. (7), we have

$$V_2(\mathbf{h}) = \mathbf{h}^H\mathbf{S}^H\Big\{\big[\mathbf{I}_N \otimes (\mathbf{FF}^H)\big] \otimes \mathbf{R}^{-1}\Big\}\mathbf{Sh} \triangleq \mathbf{h}^H\boldsymbol{\Phi}\mathbf{h}, \quad (10)$$

where

$$\boldsymbol{\Phi}_{(L+1)\times(L+1)} \triangleq \mathbf{S}^H\Big\{\big[\mathbf{I}_N \otimes (\mathbf{FF}^H)\big] \otimes \mathbf{R}^{-1}\Big\}\mathbf{S}. \quad (11)$$

The solution $\hat{\mathbf{h}}$ that minimizes $V_2(\mathbf{h})$ is given by the eigenvector of $\boldsymbol{\Phi}$ associated with the smallest eigenvalue. We note that the calculation of $\boldsymbol{\Phi}$ has to be performed carefully because of the large dimensions of the matrices involved: $\mathbf{FF}^H$ is $J \times J$, $\mathbf{I}_N \otimes (\mathbf{FF}^H)$ is $JN \times JN$, $\mathbf{S}$ is $(JN-L)JN \times (L+1)$ and $\big[\mathbf{I}_N \otimes (\mathbf{FF}^H)\big] \otimes \mathbf{R}^{-1}$ is a $(JN-L)JN \times (JN-L)JN$ matrix (e.g. 14336×14336 for $N = 2, J = 64$ and $L = 16$). Hence, brute-force computation is impractical/inefficient except for small values. The spars structure of the matrices involved has to be exploited for efficient implementation. Because of the limited space, algorithm implementation was omitted. We also note that the matrix $\mathbf{R}$ has to be replaced by some covariance matrix estimate, e.g., the sample covariance matrix $\hat{\mathbf{R}} = P^{-1}\sum_{n=0}^{P-1}\mathbf{y}_N(n)\mathbf{y}_N^H(n)$ or some adaptive estimate of $\mathbf{R}$. It can be shown (e.g., [5]) that $\hat{\mathbf{h}}$ converges to the true channel $\mathbf{h}$ (up to a scalar factor) as the interference and noise vanish. For finite SNR and in the presence of interference, we evaluate the accuracy of $\hat{\mathbf{h}}$ via simulations in Section 4. Finally, like all other blind schemes, the channel estimate $\hat{\mathbf{h}}$ has a scalar ambiguity, which can be resolved either by differential coding or by transmitting a *few* pilot symbols.

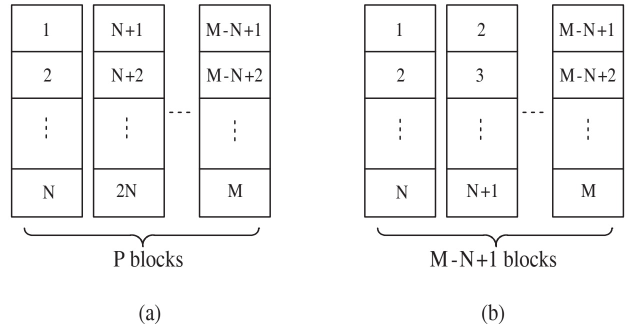


Fig. 1. (a) Non-overlapping OFDM symbols. (b) Overlapping OFDM symbols

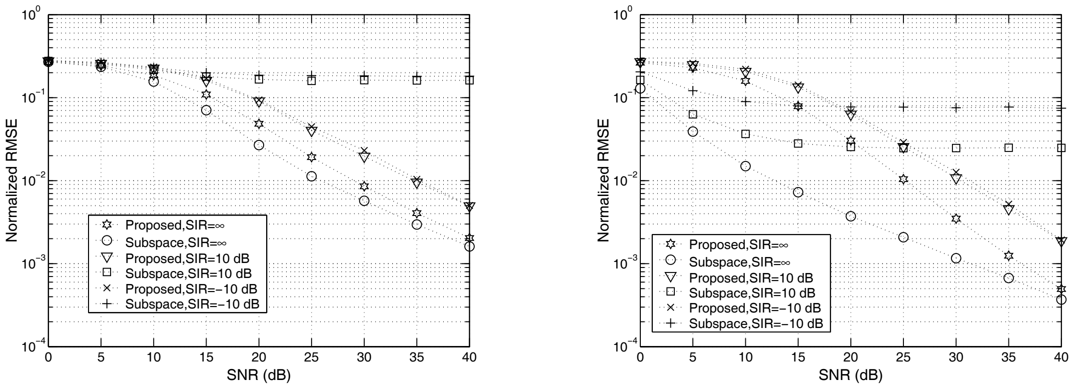
4. NUMERICAL RESULTS

In what follows, we present simulation results reflecting two different scenarios based on how the received signal is being processed. Precisely, we form blocks of N symbols each, and have the N-symbols arranged in overlapping and non-overlapping fashion. To see this, let the total number of OFDM symbols M equals to an integer multiples of N i.e. $M = NP$ for any integer P. As illustrated by Figure 1, this will result in P and $M - N + 1$ non-overlapping and overlapping blocks simultaneously.

The mathematical expression for the received ofdm non-overlapping symbols is given by equation (2). By forming an overlapping symbols, the received data will have similar expression as in equation (2) with the index n running from 1 to $M + N - 1$.

We compare here the proposed method with the subspace blind channel estimators [2]. The system under study utilizes the IDFT transform and a BPSK constellation with $K = 48$ and $N = 2$. Additionally, both estimators use a total of $M = 204$ OFDM symbols for channel estimation. The channel is a four-tap ($L = 3$) FIR channel. Two narrowband interfering signals are added with various values of signal-to-interference ratio (SIR). As a performance measure, we consider here the normalized root mean-squared error (RMSE) defined as $\frac{1}{\|\mathbf{h}\|}\sqrt{\frac{1}{D(L+1)}\sum_i^D \|\hat{\mathbf{h}}_i - \mathbf{h}\|^2}$ that is averaged over $D = 500$ Monte Carlo runs.

Figures 2(a) and 2(b) show the performance versus SNR and SIR for both non-overlapping and overlapping scenarios respectively. In the absence of interference (i.e., SIR $= \infty$), the subspace estimator outperforms the proposed scheme



(a) (b)

Fig. 2. Normalized RMSE of the proposed and subspace blind channel estimates versus SNR and SIR, for $K = 48$, $L = 3$ and $N = 2$. (a) No overlapping. (b) Overlapping.

slightly. However, even with fairly weak interference (i.e., SIR $= 10$dB), the subspace estimator degrades significantly and exhibit irreducible error. By employing an overlapping structure on the received data symbols, a significant performance improvement can be obtained for both estimators as seen in Figure 2(b).

Figure 3 depicts the RMSE of the proposed and subspace channel estimators along with the CRB for SIR = 10 dB. We see that the proposed scheme approaches the CRB as the SNR increases for both the overlapping and non-overlapping symbols. Meanwhile, the subspace scheme is suffering from the moderately increased interference level.

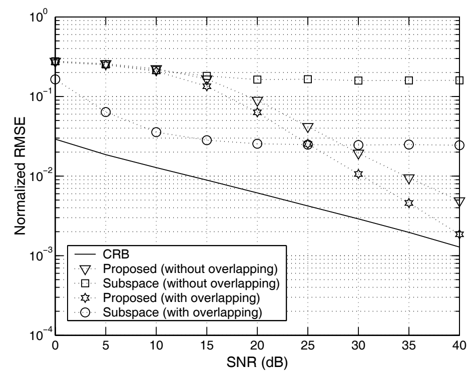


Fig. 3. Normalized RMSE of the CRB, proposed and subspace blind channel estimates versus SNR, for $K = 48$, $L = 3$, $N = 2$ and SIR $= 10$ dB .

5. CONCLUSIONS

We have presented a blind joint channel and interference suppression for orthogonal frequency-division multiplexing (OFDM) systems. A generalized multichannel minimum variance principle was invoked to design an equalizing filterbank that preserves desired signal components and suppresses the overall interference. To overcome computational difficulty and local convergence problems that accompany multidimensional search methods, we've derived an asymptotically (in SNR) tight lower bound of the filterbank output power and used it for channel estimation, which reduces the problem to a quadratic minimization. To assess the performance of the proposed scheme, numerical examples were presented. The proposed scheme compares favorably with a subspace blind channel estimator in the presence of unknown narrowband interference and approaches the CRB as the SNR level is increased. By imposing an overlapping structure on the received data symbols, the performance of both estimators was significantly improved.

6. REFERENCES

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