SISO AND MIMO CHANNEL ESTIMATION AND SYMBOL DETECTION USING DATA-DEPENDENT SUPERIMPOSED TRAINING*

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ABSTRACT

We address the problem of frequency-selective channel estimation and symbol detection using superimposed training. Both single and multiple antenna systems will be studied. The superimposed training consists of the sum of a known sequence and a data-dependent sequence, which is unknown to the receiver. The data-dependent sequence cancels the effects of the unknown data on channel estimation. The performance of the proposed approach is shown to significantly outperform existing methods based on superimposed training.

1. INTRODUCTION

Channel estimation is a major challenge for reliable wireless transmissions. In most practical systems, this task is accomplished by using pilot symbols that are known to the receiver. Often, these pilot symbols are time- or frequencydivision multiplexed (TDM or FDM) with the data, i.e., pilots and information-bearing symbols are transmitted on different time or frequency slots. Although accurate channel estimates can be obtained if the pilots are judiciously placed [1], this method wastes bandwidth. An alternative method is the superimposed training (ST) scheme where pilots are added to the data symbols [2, 3, 4, 5]. This scheme saves valuable bandwidth at the expense of a reduction in the information signal-to-noise ratio (SNR), since some of the transmitted energy is allocated to the hidden pilots. ST schemes offer tradeoffs between loss of rate (slots for training) and simplicity of the receiver, channel estimation vs. tracking, and possibly improved power efficiency.

The idea of using ST to estimate the channel has recently received renewed attention. Recent contributions are based on superimposing a known periodic sequence on the data [2, 3, 4, 5, 6]. In [6], the deterministic (or sample) mean of each data block was removed prior to transmission, and this approach was shown to reduce the effect of the unknown data on the performance of both channel estimation and equalization. Here, we propose a superimposed training scheme that fully cancels the effects of the unknown data on the performance of the channel estimator. Unlike the conventional ST scheme, the training sequence is the sum of a periodic sequence, which is known to the receiver, and a data-dependent sequence, which is unknown to the receiver. We show that by judiciously selecting the latter sequence, a very significant improvement in terms of estimation accuracy and symbol error rate can be obtained. We first focus on single antenna systems. Then, we extend the proposed technique to multiple antenna systems.

Notation: Superscripts *, and ^T denote Hermitian and transpose operators. The trace and statistical expectation are denoted by $\text{Tr} \{\cdot\}$ and $E\{\cdot\}$. The DFT of a $(N \times 1)$ vector \boldsymbol{x} is denoted by $\tilde{\boldsymbol{x}} = \mathbf{F}_N \boldsymbol{x}$, where \mathbf{F}_N has (m, n) entry $\frac{1}{\sqrt{N}}e^{-j2\pi mn/N}$. $\mathbf{F}_{P,L}$ will denote the leading $P \times L$ submatrix of \mathbf{F}_P . The *n*th element of a vector \boldsymbol{z} is denoted by z(n), and \otimes will denote the Kronecker product. The $(N \times N)$ identity matrix is denoted by \mathbf{I}_Q . Finally, diag $(a_1, ..., a_N)$ is the $(N \times N)$ diagonal matrix whose *n*th diagonal entry is a_n .

2. THE PROPOSED ST SCHEME

Consider a single-carrier block transmission system operating over a frequency-selective channel. Let N denote the block length. We assume the channel to be time-invariant over a single block, but it could vary across blocks. Assume that the discrete-time memory of the channel is upper bounded by L-1, which is known. Let $\boldsymbol{h} = [h_0, ..., h_{L-1}]^T$ denote the impulse response of the channel. In order to avoid interblock interference, a cyclic prefix (CP) of length $\geq L-1$ is inserted between the blocks. At the receiver, after removing the CP, the signal model for each block can be expressed as

$$\boldsymbol{x} = \mathcal{H}\boldsymbol{s} + \boldsymbol{v} \tag{1}$$

where \boldsymbol{s} is the $(N \times 1)$ transmitted block, \mathcal{H} is an $(N \times N)$ circulant matrix with first column, $[h_0, h_1, \dots, h_{L-1}, 0, \dots, 0]^T$, and \boldsymbol{v} is an additive white noise vector with covariance $\sigma_v^2 \mathbf{I}$. Further \boldsymbol{s} is assumed to be zero mean and independent of \boldsymbol{v} . Since we perform block-by-block processing, we do not need a block index in eq. (1).

In a TDM scheme, some of the entries of s are known pilots. In the conventional ST scheme, a known training sequence, c, is added to the data vector, w, i.e., s = w + c. The data symbols are assumed to be zero-mean, independent and identically distributed random variables drawn from a finite alphabet, e.g., PSK or QAM; let σ_w^2 denote the data symbol power. The channel coefficients can be consistently estimated using the first-order statistics of the received signal [2, 3, 4]. In order to simplify channel estimation, c is often chosen to be periodic; let P denote its period and assume that Q = N/P is an integer. A disadvantage of this method is that the performance of the channel estimator is affected by the embedded unknown data, which acts like input noise. In order to better explain this effect and also to motivate the proposed ST scheme, we use the following frequency domain interpretation. Since \mathcal{H} is circulant, the discrete-Fourier transform (DFT) of x can be written as

$$\tilde{\boldsymbol{x}} = \mathbf{H}\tilde{\boldsymbol{c}} + \mathbf{H}\tilde{\boldsymbol{w}} + \tilde{\boldsymbol{v}} \tag{2}$$

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where $\mathbf{H} = \text{diag}(H(0), ..., H(N-1))$ with H(k) being the frequency response of the channel at normalized frequency $2\pi k/N$,

$$H(k) = \sum_{\ell=0}^{L-1} h_{\ell} e^{-j2\pi k\ell/N}, \quad k = 0, ..., N-1 .$$

Since c is periodic with period P, its energy is concentrated only at the P equispaced frequency bins, $k = \ell Q$, $\ell = 0, ..., P - 1$, which we will refer to as pilot frequencies. In contrast, the energy of the data symbols is spread over all frequency bins. The channel coefficients are then estimated using the pilot frequencies, treating $\mathbf{H}\tilde{w}$ and \tilde{v} as additive noise sequences. Here, we propose to develop a channel estimator that is completely impervious to the unknown data. We propose to *distort* the data vector so that its DFT at the pilot frequencies, $k = \ell Q$, $\ell = 0, ..., P - 1$, is identically zero. If we let the distorted data vector be denoted by w-e, then it is easy to verify that the corresponding DFT at the pilot frequencies is zero provided that

$$\sum_{m=0}^{Q-1} w(i+mP) = \sum_{m=0}^{Q-1} e(i+mP), \quad i = 0, ..., P-1.$$
(3)

A trivial choice is e(n) = w(n) which leads to TDM. For superimposed training, the energy of the distortion sequence eshould be minimized for a fixed energy in the data sequence w. The solution in this case is found to be

$$e(i+mP) = e(i) = \frac{1}{Q} \sum_{m=0}^{Q-1} w(i+mP), \quad i = 0, ..., P-1 ,$$

which is the cyclic mean of the data. In this case, data distortion corresponds to simply removing the cyclic mean of the data. The distortion component can be written as $\mathbf{e} = \mathbf{J} \boldsymbol{w}$, where $\mathbf{J} = \frac{1}{Q} \mathbf{1}_Q \otimes \mathbf{I}_P$. The transmitted block is then given by

$$\boldsymbol{s} = (\mathbf{I} - \mathbf{J})\boldsymbol{w} + \boldsymbol{c} \; . \tag{4}$$

Now, the DFT of s at the pilot frequencies is identical to that of c. In other words, channel estimation will only be affected by the additive noise, v, and not by the unknown data vector, \boldsymbol{w} . The proposed ST technique can also be seen as a data-dependent superimposed training (DDST) scheme where the training sequence is the sum of a known sequence, c, and an unknown data-dependent sequence, $e := -\mathbf{J}w$.

The proposed ST scheme may seem similar to orthogonally multiplexing pilots and data tones in multicarrier (MC) systems [7]. However, in the proposed method, the symbol energy is spread over the entire bandwidth. Setting P DFT coefficients of the data to zero affects the symbols equally. Therefore, unlike MC systems, the proposed method for single carrier systems does not entail waste of bandwidth. Indeed, for MČ systems, only N - P information bearing symbols would be transmitted in each block. Both schemes involve a length L CP.

CHANNEL ESTIMATION AND TRAINING 3. DESIGN

At the receiver, channel estimation can be carried out as in the conventional ST scheme. A time-domain estimator based on synchronized averaging of the received signal was developed in [2, 4, 5]. The same estimator can be obtained using the frequency domain [3], which will be used here since equalization will be carried out in this domain.

Since the DFT of the distorted data is identically zero at the pilot frequencies, we have $\tilde{x}(kQ) = H(kQ)\tilde{c}(kQ) + \tilde{v}(kQ)$, k = 0, ..., P - 1. To ensure consistent channel estimates based on this, at least L of the P pilot cycles of c

should be non-zero. We estimate the frequency response of the channel at the pilot frequencies where $\tilde{c}(kQ) \neq 0$, via

$$\hat{H}(kQ) = \tilde{x}(kQ)/\tilde{c}(kQ), \ k = 0, ..., P-1$$
.

The channel coefficient vector is estimated as

$$\hat{\boldsymbol{h}} = \frac{1}{\sqrt{P}} \mathbf{F}_{P,L}^* \hat{\boldsymbol{d}}$$
(5)

where $\hat{\boldsymbol{d}} = [\hat{H}(0), \hat{H}(Q), ..., \hat{H}((P-1)Q)]^T$.

The channel estimate in eq. (5) is unbiased and its mean square error (MSE) is given by

$$\mathsf{mse}\left(\hat{\boldsymbol{h}}\right) := E\left\{\sum_{l=0}^{L-1} |\hat{h}_l - h_l|^2\right\} = \frac{\sigma_v^2}{P} \mathrm{Tr}\left\{\mathbf{F}_{P,L}^* \mathbf{C}^{-1} \mathbf{F}_{P,L}\right\}$$

where $\mathbf{C} = \text{diag}(|\tilde{c}(0)|^2, |\tilde{c}(Q)|^2, ..., |\tilde{c}((P-1)Q)|^2)$. Under the constraint of fixed training power $\frac{1}{N}\sum_{n=0}^{N-1}|c(n)|^2 = \sigma_c^2$, the above MSE is minimized when $|\tilde{c}(kQ)|^2 = \sigma_c^2(N/P), \ k = 0, ..., P-1$. The MSE expression becomes

$$\mathsf{mse}\left(\hat{\boldsymbol{h}}\right) = \frac{L\sigma_v^2}{N\sigma_c^2} \tag{6}$$

Further, since the above MSE is independent of P and data distortion increases with P (for fixed N), P should be as small as possible, i.e., $P = \dot{L}$. Note that the MSE of the channel estimate is now independent of the unknown data, unlike that in existing ST-based methods [4, 5].

Since there are infinitely many periodic sequences for which the cycles are all equal in magnitude, sequences with minimum PAR are desirable. In fact, 'ideal' sequences, i.e., optimal and constant envelope (i.e., unit PAR) sequences, exist for all values of P. One of these sequences is the chirp sequence $c(n) = \sigma_c \exp(j2\pi n(n+i)/P)$ with i = 2 when P is even and i = 1 if P is odd [4].

4. SYMBOL DETECTION

After the channel has been estimated, we can remove the contribution of c from x by simply computing (see (1), (4))

$$\boldsymbol{z} = (\mathbf{I} - \mathbf{J})\boldsymbol{x} \ . \tag{7}$$

In the frequency domain, this is equivalent to setting the DFT of x at the pilot frequencies to zero. Since both \mathcal{H} and **J** are circulant, they are commutative, and $(\mathbf{I} - \mathbf{J})$ is idempotent, z can be expressed as

$$oldsymbol{z} = \mathcal{H}(\mathbf{I} - \mathbf{J})oldsymbol{w} + (\mathbf{I} - \mathbf{J})oldsymbol{v}$$
 .

The additive noise, $\breve{v} = (\mathbf{I} - \mathbf{J})v$, is now slightly colored. However, this color will fade away when Q is large, and will therefore be ignored in what follows. From the definition of **J**, it follows that the power of $\check{\boldsymbol{v}}$ is $\check{\sigma}_v^2 = \sigma_v^2(1-1/Q)$. Since \mathcal{H} is circulant, equalization can be carried out in

the frequency domain, i.e., the equalized signal is given by

$$\boldsymbol{u} = \mathbf{F}_N^* \mathbf{G} \tilde{\boldsymbol{z}} \tag{8}$$

where \tilde{z} , the DFT of z, is obtained by setting the DFT of \boldsymbol{x} at the pilot frequencies to zero, and \mathbf{G} is an $(N \times N)$ diagonal matrix whose kth entry G(k) is G(k) =1/H(k) in the case of zero-forcing equalization and G(k) = $\hat{H}^*(k)/(|\hat{H}(k)|^2 + \check{\sigma}_v^2)$ in the case of MMSE equalization.

Due to data distortion in the transmission, $u \neq w$ even in the absence of channel estimation error and noise. Indeed, in this ideal scenario, $\boldsymbol{u} = (\mathbf{I} - \mathbf{J})\boldsymbol{w}$. Since $(\mathbf{I} - \mathbf{J})$ is singular, w cannot be recovered *linearly*. However, using the fact that the data symbols are drawn from a finite alphabet and that $\mathbf{J}\boldsymbol{w}$ is small compared to \boldsymbol{w} , symbol detection can be accomplished by finding the vector of constellation points \boldsymbol{w} that minimizes the Euclidian distance between \boldsymbol{u} and $(\mathbf{I} - \mathbf{J})\boldsymbol{w}$. However, this sequence detection scheme is computationally cumbersome and will therefore not be considered here. Instead, we propose the following iterative symbol-by-symbol detection scheme.

The symbol-by-symbol detection algorithm is initialized by treating $\mathbf{J}\boldsymbol{w}$ as an additional noise term, and considering \boldsymbol{u} in eq. (8) as a soft detector of \boldsymbol{w} ; the initial hard detector of \boldsymbol{w} is given by

$$ar{m{w}}^{(0)} = ig\lfloorm{u}ig
vert$$

where $\lfloor u \rfloor$ denotes the vector of constellation points that are the closest to the vector u. The detected symbols are used to estimate $\mathbf{J}w$ to be used in the next iteration. The detected symbols at the *i*th iteration are given by

$$ar{oldsymbol{w}}^{(i)} = |oldsymbol{u} + \mathbf{J}ar{oldsymbol{w}}^{(i-1)}|$$

As we will see in section 6., most of the gain in symbol detection performance over existing ST-based methods is obtained in the very first iteration.

5. EXTENSION TO MIMO SYSTEMS

In this section, we extend the proposed approach to multiple antenna systems. Again, we assume that a CP is inserted between the data blocks. We first address the case of single-input multiple-output (SIMO) systems. Then, we study the more general case of multiple-input multipleoutput (MIMO) systems.

5.1. SIMO systems

The above ST scheme is also valid for SIMO systems since the estimation of the channels at different receive diversity branches are decoupled when based on first-order statistics. Symbol detection is based on maximum ratio combining (see eq. 10). Some details are given next.

Let \hat{R} denote the number of receive antenna and let x_r denote the signal vector at the rth receive antenna obtained after removing the CP. We first compute

$$\boldsymbol{z}_r = (\mathbf{I} - \mathbf{J})\boldsymbol{x}_r, \quad r = 1, \dots, R \tag{9}$$

and its DFT \tilde{z}_r . Then, we estimate the channels as in section 3., and compute their frequency responses, $\hat{H}_r(k)$, k = 0, ..., N - 1, r = 1, ..., R. Symbol detection is based on the following vector

$$\boldsymbol{u} = \frac{1}{R} \mathbf{F}_N^* \sum_{r=1}^R \mathbf{G}_r \tilde{\boldsymbol{z}}_r$$
(10)

where \mathbf{G}_r is an $(N \times N)$ diagonal matrix whose kth entry $G_r(k)$ is $G_r(k) = 1/\hat{H_r(k)}$ in the case of zero-forcing equalization and $G_r(k) = \hat{H}_r^*(k)/(|\hat{H}_r(k)|^2 + \check{\sigma}_v^2)$ in the case of MMSE equalization. The next steps of symbol detection are exactly the same as in Section (4.).

5.2. MIMO systems

Let M and R respectively denote the number of transmit and receive antennas. We also assume that $R \ge M$. Let $\boldsymbol{h}_{m,r} = [h_{m,r}(0), ..., h_{m,r}(L-1)]^T$ denote the channel between the *m*th transmit antenna and *r*th receive antenna. The block received at the *r*th antenna is, after removing the CP, given by

$$\boldsymbol{x}_{r} = \sum_{m=0}^{M-1} \mathcal{H}_{m,r} \boldsymbol{s}_{m} + \boldsymbol{v}_{r}$$
(11)

where s_m , $\mathcal{H}_{m,r}$ and v_r are defined as in Section 2.. For the conventional ST, $s_m = w_m + c_m$. The DFT of x_r can be expressed as

$$\tilde{\boldsymbol{x}}_{r} = \sum_{m=0}^{M-1} \mathbf{H}_{m,r} \tilde{\boldsymbol{c}}_{m} + \sum_{m=0}^{M-1} \mathbf{H}_{m,r} \tilde{\boldsymbol{w}}_{m} + \tilde{\boldsymbol{v}}_{r} \qquad (12)$$

where $\mathbf{H}_{m,r}$ is defined like \mathbf{H} . In order to identify the channels, each \mathbf{c}_m is designed such that its DFT is nonzero at L frequencies, and these frequencies have to be distinct for different \mathbf{c}_m 's. It can be shown that for each antenna, choosing \mathbf{c}_m such that its DFT has only L nonzero elements minimizes channel estimation errors provided that these pilots are equispaced. Further, since data distortion for the DDST will increase with the total number of pilot frequencies, the latter should be as small as possible. The \mathbf{c}_m 's are therefore designed such that their DFTs satisfy the following:

$$|\tilde{c}_m(k)|^2 = \frac{N\sigma_c^2}{LM} \sum_{i=0}^{L-1} \delta(k - [m+iM]Q), \quad m = 0, ..., M-1$$
(10)

where Q = N/P, assumed to be integer and $P := \dot{M}L$. The above design implies that all pilot frequencies are equispaced; the spacing is equal to Q; and the spacing between pilots from the same antenna is N/L = MQ. In the above equation, we have assumed that the total power allocated to training is split equally between the transmit antennas. The above training sequences can be generated as follows. We design the training sequence for the 0th antenna using eq. (13), and then generate the sequences for the other antennas using

$$c_m(n) = e^{j2\pi mn/P} c_0(n), \quad m = 1, ..., M-1; \quad n = 0, ..., N-1$$
(14)

In order to cancel the effects of the unknown data on channel estimation performance, we set the DFT of the data vectors \boldsymbol{w} at all the pilot frequencies to zero. For the design in eq. (13), the distorted transmitted blocks can be implemented as in eq. (4), i.e. by removing the cyclic mean from the data vectors. The number of pilot frequencies, P = ML, is M times larger than that for the SISO case. Thus, data distortion increases not just with L but also with M, the number of transmit antennas. Channel estimation can be carried out as in Section 3. since the estimates of different channels are decoupled. Indeed, the pilot frequencies are distinct across the transmit antennas. The MSEs of the channel estimates are the same as in (6) after replacing σ_c^2 by σ_c^2/M .

For symbol detection, once we have estimated the channel coefficients, we first compute the frequency response of the channels at all frequency bins. Then, at each receive antenna, we remove the contribution of the c_m 's by computing z_r as in eq. (9) where **J** is defined with P = ML. Equalization and signal separation are carried out in the frequency domain. We compute the DFT of the z_r 's, the \tilde{z}_r 's. For each frequency bin, let $\tilde{z}(k) = [\tilde{z}_0(k), ..., \tilde{z}_R(k)]^T$, and let

$$\hat{\mathbf{H}}(k) = \begin{pmatrix} \hat{H}_{11}(k) & \hat{H}_{12}(k) & \cdots & \hat{H}_{1M}(k) \\ \hat{H}_{21}(k) & \hat{H}_{22}(k) & \cdots & \hat{H}_{2M}(k) \\ \vdots & & \ddots & \vdots \\ \hat{H}_{R1}(k) & \hat{H}_{R2}(k) & \cdots & \hat{H}_{RM}(k) \end{pmatrix}$$
(15)

The MMSE equalizer/separator output at the $k{\rm th}$ frequency bin is computed as

$$\boldsymbol{a}(k) = \mathbf{G}(k)\tilde{\boldsymbol{z}}(k) \tag{16}$$

 $\mathbf{G}(k) = (\hat{\mathbf{H}}(k)\hat{\mathbf{H}}^*(k) + \check{\sigma}_v^2 \mathbf{I})^{-1}\hat{\mathbf{H}}^*(k)$. Then, we form \boldsymbol{u}_m by grouping the *m*th entry of $\boldsymbol{a}(k), k = 0, ..., N-1$ in a vector. The vector \boldsymbol{u}_m is then used to detect \boldsymbol{w}_m , as in Section 4...

If the MIMO channel is flat-fading, the following observations/simplifications apply

- No cyclic prefix is required
- The equalizing matrices **G**(k) are frequency independent. Therefore, equalization can be simplified.

6. SIMULATION RESULTS

We compare the proposed DDST scheme with both the conventional ST and TDM schemes in terms of channel estimation performance and symbol error rate (SER). The length of the data block is set to N = 256. The channel is randomly generated at each Monte-Carlo run and is assumed to be Rayleigh with length L = 8; the coefficients are uncorrelated and their powers are given by the exponential delay profile $E\{|h_\ell|^2\} = \exp(-0.2\ell)$. The periodic sequence is chosen to be optimal according to Section 3. and its power is set to be 10% of the total transmitted power, σ_s^2 ; its period is set to P = L. The data symbols are drawn from QPSK constellations. Due to lack of space, we limit our simulation study to the SISO case.

The channel estimation performance of the proposed DDST scheme is the same as that of the TDM scheme if the percentage of power used in the DDST scheme is the same as that of pilot symbols used in the TDM scheme, as is verified in Figure 1 where the number of training symbols in the TDM scheme is $N_t = 26$. Figure 2 displays the SER for the three schemes as well as that when the channel is exactly known at the receiver and full power is assigned to the data symbols. It is seen that the SER for the DDST scheme is close to that for the TDM scheme. The latter however consumes 10% of the bandwidth. Full investigation of the effects of N, constellation size, and σ_c^2 on performance will be carried out in a longer version of this paper.

7. CONCLUSIONS

We have presented a new pilot assisted transmission scheme to estimate frequency-selective channels. The scheme consists of setting a few points of the DFT of the data to known values. This operation can be easily implemented in the time domain when these DFT points are equispaced. The channel is estimated using the DFT of the received signal at these pilot frequencies. Detection of the distorted symbols is carried out using an iterative scheme. The proposed method was shown to outperform existing methods based on superimposed training, and compares well with the time-division multiplexing training scheme, in terms of both channel estimation performance and symbol error rate. Unlike the TDM scheme, the proposed pilot transmission scheme does not entail waste of bandwidth.

REFERENCES

- S. Adireddy, L. Tong and H. Viswanathan, "Optimal placement of training for frequency-slective blockfading channels," *IEEE Trans. Info. Theory*, 48(8), 2338-2352, Aug 2002.
- [2] G.T. Zhou, M. Viberg, and T. McKelvey, "A first-order statistical method for channel estimation," *IEEE Sig. Proc. Lett.*, 10(3), 57-60, Mar 2003.
- [3] J. Tugnait and W. Luo, "On channel estimation using superimposed training and first-order statistics," *IEEE Commun. Lett.*, 7(9), 413-5, Sep 2003.

- [4] A. G. Orozco-Lugo, M. M. Lara and D. C. McLernon, "Channel estimation using implicit training," *IEEE Trans. Sig. Proc.*, 52(1), 240-254, Jan 2004.
- [5] M. Ghogho and A. Swami, "Improved channel estimation using superimposed training," *Proc. SPAWC'04*, Lisbon, July 2004.
- [6] D. McLernon, E. Alameda-Hernandez and A. G. Orozco-Lugo, "Implicitly-Trained Channel Estimation and Equalisation with Zero Mean Input Data Packets," to appear in the Proc. of *IEEE ISSPIT'2004*, Rome, Italy, Dec. 2004.
- [7] S. Ohno, and G. B. Giannakis, "Optimal training and redundant precoding for block transmissions with application to wirelesss OFDM", *IEEE Trans. Commun.*, 50(12), 2113-2123, Dec 2002.



Figure 1. Empirical Mean square error of channel estimates



Figure 2. Symbol error rate for different PAT schemes