SEPARATION OF CONVOLUTIVE MIXTURES OF LINEAR MODULATED SIGNALS USING CONSTANT MODULUS ALGORITHM

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ABSTRACT

The issue of separating linear mixtures of independent linearly modulated signals steming from unknown digital communication systems is addressed. The baud-rates of the various transmitted signals are in particular unknown and possibly different. Therefore, sampled versions of the received signal are cyclostationary sequences. Despite the non-stationary environment of the data, Godard's algorithm is shown to achieve separation in rather general contexts.

1. INTRODUCTION

The context of this work is the one of multi-users passive listening. Specifically, K unknown transmitters are supposed to share a common band of frequencies, and the goal of the receiver is to extract as much information as possible on the K transmitted signals (e.g. modulation types, baud-rates, carrier frequencies, symbol constellations, etc.). The receiver is assumed to have$M$-antennas ($M \geq K$): an appropriate linear combination of the data can be looked for in order to separate the various transmitted signals. This crucial step reduces the initial problems to K (much simpler) problems. In this paper, we assume that the K transmitted signals are linearly modulated by modulus 1 circular independent identically distributed sequences (with possibly different baud-rates and carrier frequencies), and address their blind separation.

In the context of blind source separation (BSS), the observed data $\mathbf{y}(n)$ is filtered by a multi-input filter $\mathbf{g}(z)$ (let $r(n)$ be the output), this latter been chosen so as to optimize a certain function depending on statistics of r. Such functions have been set forth, which ensure separation at their optima. These functions are called *contrasts*. Global approaches propose to extract the K components at a time (the filter $\mathbf{g}(z)$ is multi-output): see the interesting paper of Castella et al. [1] and the references therein. Iterative approaches aim at extracting a contribution of a single source ($\mathbf{g}(z)$ is mono-output); thanks to a deflating step, the other contributions are extracted one after the other (see e.g. [2], [3]). If the source signals are independent identically distributed (i.i.d.) sequences with negative kurtosis, then it has been shown that the kurtosis of r ([4]) defined by $\frac{c_4(r(n))}{(\mathbb{E}(|r(n)|^2))^2}$ ($c_4(x)$ stands for the fourth-order cumulant of x) and the Godard constant modulus cost function $\mathbb{E}(|r(n)|^2-1)^2)$ ([5]) allow to extract one of the source signal. These results have been generalized to the context of stationary source signals in [6].

In this paper, we focus on iterative approaches. The results of [4], [5] and [6] cannot be used in our context, because for a general configuration of the baud-rates, the received sampled signal is not stationary, but cyclostationary (the cyclic frequencies depend on the baud rates of the K source signals). The statistical properties of output signal $r(n)$ are thus time-dependent, and it has been shown recently in [7] that the minimization of $\frac{<c_4(r(n))>}{(<\mathbb{E}(|r(n)|^2)>)^2}$ allows to extract a filtered version of one of the source signal. Here, $<\ .\ >$ stands for the time average operator defined by $< u_n >= \lim_{N\to\infty} \frac{1}{N}\sum_{n=0}^{N-1} u_n$. However, the estimation $< c_4(r(n)) >$ requires the prior estimation of the cyclic frequencies of the received signal (see section 2).

In this paper, we rather study the behavior of the Godard cost function $J(r)$ defined by

$$J(r(n)) =< \mathbb{E}\left(|r(n)|^2-1\right)^2 > \tag{1}$$

which, in contrast with the above cost function, can be obviously consistently estimated by $\frac{1}{N}\sum_{n=1}^{N}(|r(n)|^2-1)^2$ where N represents the sample size. We show that the minimization of J over $\mathbf{g}$ allows us to achieve separation in rather general contexts.

General notations. If $(x(n))_{n\in\mathbb{Z}}$ is a discrete-time cyclostationary sequence, we denote by $R_x^{(\alpha)}(\tau)$ the cyclic correlation coefficient at lag τ of signal x at cyclic frequency α, and by $S_x^{(\alpha)}(e^{2i\pi\nu})$ the corresponding cyclic spectrum

2. PROBLEM STATEMENT.

We assume that for any k, $k=1,\dots,K$, the k-th transmitted signal is obtained by linearly modulating a centered and normalized i.i.d. sequence of circular symbols having modulus 1. The corresponding symbol period is denoted by T_k, and it is assumed that the band of frequencies of the k-th transmitted signal is $[-\frac{1+\gamma_k}{2T_k}, \frac{1+\gamma_k}{2T_k}]$ where the so-called excess bandwith factor γ_k belongs to $[0,1)$. The propagation channels between each transmitter and the receiver are assumed to be frequency selective. The continuous-time received signal is sampled at a period T_e which is supposed to satisfy the Shannon condition. Under these assumptions, the M-dimensional discrete-time received signal $\mathbf{y}(n)$ can be written as

$$\mathbf{y}(n) = \sum_k \mathbf{H}_k \mathbf{s}(n-k) = [\mathbf{H}(z)]\mathbf{s}(n) \tag{2}$$

where the components $s_1(n),\dots,s_K(n)$ of vector $\mathbf{s}(n)$ represent the sampled versions of the K transmitted signals, and where $\mathbf{H}(z)=\sum_{k\in\mathbb{Z}}\mathbf{H}_k z^{-k}$ is the transfer function of the K-inputs / M outputs discrete time equivalent channel. Each signal s_k is cyclostationary, and its second order cyclic frequencies are $0, \alpha_k, -\alpha_k$

where $\alpha_k = \frac{T_e}{T_k}$. In the following, we assume without restriction that source signals $(s_k)_{k=1,...,K}$ are normalized in such a way that

$$\forall k \;\; R^{(0)}_{s_k}(0) =< \mathbb{E}|s_k(n)|^2 >= 1. \qquad (3)$$

The observed data $\mathbf{y}(n)$ is filtered by a MISO filter $\mathbf{g}(z)$, and we denote by $r(n)$ the corresponding output given $r(n) = [\mathbf{g}(z)]\mathbf{y}(n)$. Signal $r(n)$ is of course cyclostationary, and its set of second order cyclic frequencies is included in the set I given by

$$I = \{0, \pm\alpha_1, ..., \pm\alpha_K\} \text{ with } \alpha_k = T_e/T_k.$$

In the following, we also denote I^*_+ the set of all strictly positive cyclic frequencies. In particular, the sequence $(\mathbb{E}(|r(n)|^2))_{n\in\mathbb{Z}}$ can be expanded as $\mathbb{E}(|r(n)|^2) = \sum_{\alpha\in I} R^{(\alpha)}_r(0)e^{2i\pi n\alpha}$. Prior to studying the Godard cost function, we explain why the contrast function $\frac{<c_4(r(n))>}{(<\mathbb{E}(|r(n)|^2)>)^2}$ is difficult to use in practice. Indeed, the fourth-order cumulant $c_4(r(n))$ of $r(n)$ can be written as $c_4(r(n)) = \mathbb{E}(|r(n)|^4) - 2(\mathbb{E}(|r(n)|^2))^2$ ($r(n)$ is circular because each transmitted symbol sequence is circular)). Hence, $< c_4(r(n)) >$ coincides with $< \mathbb{E}(|r(n)|^4 > -2 < (\mathbb{E}(|r(n)|^2))^2 >$. Using the Parseval identity, and the fact that $|R^{(-\alpha)}_r(0)| = |R^{(\alpha)}_r(0)|$, it yields

$$< (\mathbb{E}(|r(n)|^2))^2 >= \sum_{\alpha\in I} |R^{(\alpha)}_r|^2 = |R^{(0)}_r|^2 + 2\sum_{\alpha\in I^*_+} |R^{(\alpha)}_r|^2.$$

Hence

$$< c_4(r(n)) >=< \mathbb{E}(|r(n)|^4) > -2(|R^{(0)}_r|^2 + 2\sum_{\alpha\in I^*_+} |R^{(\alpha)}_r|^2) \qquad (4)$$

The first term of the right hand side of (4) and $R^{(0)}_r =< \mathbb{E}(|r(n)|^2) >$ can be consistently estimated by means of $\frac{1}{N}\sum_{n=1}^N |r(n)|^4$ and $\frac{1}{N}\sum_{n=1}^N |r(n)|^2$ respectively, but the estimation of the third term needs the prior estimation of the strictly positive cyclic frequencies. Due to the well-known difficulty of estimating these frequencies, we prefer to focus on an other approach, namely we proceed to the analysis of the cost function $J(r)$.

In order to express cost function $J(r)$ in a convenient way, we set $\mathbf{f}(z) = \mathbf{g}(z)\mathbf{H}(z)$. $r(n)$ can thus be written as $r(n) = \sum_{k=1}^K [f_k(z)]s_k(n)$. In the following, we set

$$||f_k||^2 = \int_{-1/2}^{1/2} |f_k(e^{2i\pi\nu})|^2 S^{(0)}_{s_k}(e^{2i\pi\nu})\, d\nu \qquad (5)$$

Notice that $r(n)$ is a filtered version of a source signal $s_{k_1}(n)$ if and only $\|f_k\| = 0 \;\forall k \neq k_1$. We now expand $r(n)$ as

$$r(n) = \sum_{k=1}^K ||f_k||\tilde{s}_k(n) \qquad (6)$$

Signal $\tilde{s}_k(n)$ is defined by $\tilde{s}_k(n) = [\tilde{f}_k(z)]s_k(n)$, where $\tilde{f}_k(z)$ represents the unit norm filter given by $\tilde{f}_k(z) = \frac{f_k(z)}{||f_k||}$. As $\|\tilde{f}_k\| = 1$, we have: $< \mathbb{E}(|\tilde{s}_k(n)|^2) >= 1$ for any k. From the statistical independance of signals $(\tilde{s}_k)_{k=1,...,K}$ and the identity (4), we deduce that

$$\begin{aligned} J(r) &= \sum_{k=1}^K \|f_k\|^4\beta(\tilde{s}_k) - 2\sum_{k=1}^K \|f_k\|^2 + 1 \\ &+ \sum_{k_1<k_2} \|f_{k_1}\|^2\|f_{k_2}\|^2\ell(\tilde{s}_{k_1}, \tilde{s}_{k_2}) \end{aligned}$$

where

$$\ell(\tilde{s}_1, \tilde{s}_{k_2}) = 4\left(1 + 2\sum_{\alpha\in I^*_+} \mathrm{Re}(R^{(\alpha)}_{\tilde{s}_{k_1}}(0)\overline{R^{(\alpha)}_{\tilde{s}_{k_2}}(0)})\right). \qquad (7)$$

and

$$\beta(\tilde{s}_k) =< c_4(\tilde{s}_k(n)) > +2 + 4|R^{(\alpha_k)}_{\tilde{s}_k}(0)|^2. \qquad (8)$$

Equation (4) shows that $\beta(\tilde{s}_k) =< \mathbb{E}|\tilde{s}_k(n)|^4) >$. As $< E|\tilde{s}_k(n)|^2) >= 1$, the Jensen inequality implies that $\beta(\tilde{s}_k) \geq 1$.

Proving that the argument-minima of J "separate" one of the sources requires to investigate (7). This optimization problem depends both 1) on the norms $\|f_k\|$ and 2) on the normalized functions $\tilde{f}_k$. What makes the problem intricate is that the β and ℓ both depend on the $\tilde{f}_k$.

3. RESULTS

3.1. First case: the sources have pair-wise different baud-rates

For each strictly positive cyclic frequency $\alpha \in I^*_+$, the term $R^{\alpha}_{\tilde{s}_{k_1}}\overline{R^{\alpha}_{\tilde{s}_{k_2}}}$ vanishes if $k_1 \neq k_2$. Therefore, the expression (7) reduces to

$$\begin{aligned} J(r) &= \sum_{k=1}^K \|f_k\|^4\beta(\tilde{s}_k) + 4\sum_{k_1<k_2} \|f_{k_1}\|^2\|f_{k_2}\|^2 \qquad (9) \\ &- 2\sum_{k=1}^K \|f_k\|^2 + 1 \qquad (10) \end{aligned}$$

We now define b as

$$b = \inf_{k,\|\tilde{f}_k\|=1} \beta([\tilde{f}_k(z)]s_k(n)) = \inf_{k,\|\tilde{f}_k\|=1} < \mathbb{E}(|[\tilde{f}_k(z)]s_k(n)|^4) > \qquad (11)$$

We make the purely technical assumption that the minimum over $(k, \tilde{f}_k)$ is reached for a at least a certain pair $(k_0, \tilde{f}^*_{k_0})$ (as the set of all unit norm filter is not compact, the existence of $\tilde{f}^*_{k_0}$ is not guaranteed). As $< \mathbb{E}(|[\tilde{f}_k(z)]s_k(n)|^4) > \geq 1$ for any unit norm filter $\tilde{f}_k$, we get immediately that $b \geq 1$. In order to face the minimization of J, we notice that

$$J(r) \geq M_1(\|f_1\|, ..., \|f_K\|) \qquad (12)$$

where $M_1(\|f_1\|, ..., \|f_K\|)$ is defined by

$$\begin{aligned} M_1(\|f_1\|, ..., \|f_K\|) &= b\sum_{k=1}^K \|f_k\|^4 + 4\sum_{k_1<k_2} \|f_{k_1}\|^2\|f_{k_2}\|^2 \\ &\quad -2\sum_{k=1}^K \|f_k\|^2 + 1. \end{aligned}$$

Lemma 1 *The stationary points of $M_1$are given by*

$$\forall k, \|f_k\|^2 = \begin{cases} 0 \\ or \\ \frac{1}{b+2(P-1)} \end{cases}$$

where P is the number of non-zero components of the vector $(\|f_1\|, ..., \|f_K\|)$. The value reached at these points is $1 - \frac{P}{2((P-1)+b)^2}$, which is a growing function of P if $b < 2$. In this case, the minimum of M_1 is equal to $1 - \frac{1}{b}$, and is reached if and only if $\|f_k\|^2 = \frac{1}{b}\delta(k - k_1)$ for some index k_1.

Corollary 1 *If $b < 2$, the minimum of $J(r)$ is equal to $(1 - \frac{1}{b})$, and is reached if and only filter $\mathbf{f}(z)$ for which $r(n) = [\mathbf{f}(z)]\mathbf{s}(n)$ satisfies $\|f_k\|^2 = \frac{1}{b}\delta(k - k_0)$ and $b = \beta([\tilde{f}_{k_0}(z)]s_{k_0}(n))$, where $\tilde{f}_{k_0}(z) = \frac{f_{k_0}(z)}{\|f_{k_0}\|}$.*

We first establish that $\min_{\mathbf{f}(z)} J(r) = 1 - \frac{1}{b}$. For this, we remark that Lemma 1 and inequality (12) imply that $J(r) \geq 1 - \frac{1}{b}$. Moreover, let $(k_0, \tilde{f}_{k_0})$ be a pair for which $b = \beta([\tilde{f}_{k_0}(z)]s_{k_0}(n)$. We denote f_{k_0} the filter $f_{k_0}(z) = \sqrt{b^{-1}}\tilde{f}_{k_0}(z)$, and by $r_0(n)$ the signal $r_0(n) = [f_{k_0}(z)]s_{k_0}(n)$. Then, it is easily seen that $J(r_0) = 1 - \frac{1}{b}$, thus showing that $\min_{\mathbf{f}(z)} J(r) = 1 - \frac{1}{b}$.

In order to complete the proof, we note that $J(r)$ is minimum if and only if function M_1 is minimum, a condition which implies that $\|f_k\|^2 = \frac{1}{b}\delta(k - k_0)$ and $b = \beta([\tilde{f}_{k_0}(z)]\tilde{s}_{k_0}(n))$.

This result proves that if $b < 2$, then the minimization of the Godard cost function achieves the extraction of one of the users as soon as the sources have pair-wise different baud-rates.

The point to inspect is: is the condition $b < 2$ true? We carry out the study for any k the minimum of $< \mathbb{E}(|[\tilde{f}(z)]s_k(n)|^4) >$ over the set of all unit norm filters $\tilde{f}(z)$. If T_e coincides with one of the $(T_k)_{k=1,\dots,K}$ (say T_1), then $(s_1(n))_{n\in\mathbb{Z}}$ is stationary: it is indeed a filtered version of the symbol $(a_1(n))_{n\in\mathbb{Z}}$ transmitted by user 1. The minimum of $\mathbb{E}(|[\tilde{f}(z)]s_1(n)|^4)$ over the set of unit norm filters is merely 1 and is reached for the filters for which $[\tilde{f}(z)]s_1(n)$ coincides with a delayed version of the symbol sequence. In this context, b is of course equal to 1. In general, however, $b > 1$ because the symbol period T_e is unlikely to be with one of the T_k's. In order to address the general case, we give an alternative expression of $< \mathbb{E}(|[\tilde{f}(z)]s_k(n)|^4) >$ by taking advantage of the structure of the sources. For each unit norm filter $\tilde{f}(z) = \sum_{l\in\mathbb{Z}} \tilde{f}_l z^{-l}$, it is possible to link $\tilde{s}_k(n) = [\tilde{f}(z)]s_k(n)$ to a continuous-time signal. For this, we denote by $s_{a,k}(t)$ the continuous-time signal transmitted by the k-th user. We recall that discrete-time signal $s_k(n)$ is defined by $s_k(n) = s_{a,k}(nT_e)$, and that $s_{a,k}(t)$ is given by $s_{a,k}(t) = \sum_m a_k(m)p_{a,k}(t - mT_k)$ where $(a_k(m))_{m\in\mathbb{Z}}$ represents the symbol sequence and $p_{a,k}$ is the shaping filter impulse response. It is straightforward to check that $\tilde{s}_k(n) = \tilde{s}_{a,k}(nT_e)$ where the continuous-time signal $\tilde{s}_{a,k}(t)$ is defined by $\tilde{s}_{a,k}(t) = \sum_m a_k(m)f_{a,k}(t - mT_k)$. The function $f_{a,k}(t)$ is given by $f_{a,k}(t) = \sum_l \tilde{f}_l p_{a,k}(t - lT_e)$. The support of the Fourier transform of function $f_{a,k}(t)$ is included in $[-\frac{1+\gamma_k}{2T_k}, \frac{1+\gamma_k}{2T_k}]$. As T_e is assumed to satisfy the Shannon condition, $\|\tilde{f}\| = 1$ implies that $\frac{1}{T_k}\int_{\mathbb{R}} |f_{a,k}(t)|^2 dt = 1$. It can be shown that $< c_4(\tilde{s}_n) >= c_4(a_k(n))\frac{1}{T_k}\int_{\mathbb{R}} |f_{a,k}(t)|^4 dt$. This equality holds true as soon as T_e **is none of the values** $T_k, T_k/2, T_k/3, T_k/4$ (these special cases have to be treated separately; this point is not developed here). We have assumed that the symbols have unit modulus, hence $c_4(a_k(n)) = -1$. In the same manner, we have

$$R^{(\alpha_k)}_{\tilde{s}_k}(0) = \frac{1}{T_k}\int_{\mathbb{R}} |f_{a,k}(t)|^2 e^{-i2\pi t/T_k} dt. \qquad (13)$$

Therefore, both $< c_4(\tilde{s}_n) >$ and $R^{(\alpha_k)}_{\tilde{s}_k}(0)$ do *not* depend on the value of T_e. More information on this fact can be found in [8].

Finally, $\beta(\tilde{s}_k)$ can be expressed as $\beta(\tilde{s}_k) = \varphi(f_{a,k}, T_k)$ where we have set:

$$\varphi(f, T) = \frac{-T\int_{\mathbb{R}} |f(t)|^4 dt + 4|\int_{\mathbb{R}} |f(t)|^2 e^{-i2\pi t/T} dt|^2}{(\int_{\mathbb{R}} |f(t)|^2 dt)^2} + 2. \qquad (14)$$

(notice that $\frac{1}{T_k}\int_{\mathbb{R}} |f_{a,k}(t)|^2 dt = 1$). This shows that the lower-bound of $\beta(\tilde{s}_k)$ is the lower-bound of $\varphi(f, T_k)$ over all the possible f, i.e. -summable functions whose Fourier Transform $\hat{f}$ has a support included in $(-\frac{1+\gamma_k}{2T_k}, \frac{1+\gamma_k}{2T_k})$. After a straight-forward change of variable, this lower-bound is readily seen not to depend on T_k, but only on the excess bandwith factor γ_k. We denote $\Phi(\gamma_k)$ this lower-bound, which, of course, is a decreasing function of γ_k. Letting γ_{max} be the maximum of all the excess bandwidth factors, it yields

$$b = \Phi(\gamma_{max}). \qquad (15)$$

We have not been able, so far, to characterize Φ *analytically*. However, the integrals in φ can be expressed as sums due to the band-limited character of the functions f. Hence the minimization over square summable functions coincides with the minimization of a criterion depending on a *series*. This makes the computation of Φ possible. In particular, it can be shown that if the Fourier transform of f vanishes outside the interval $[-\frac{1+\gamma}{2}, \frac{1+\gamma}{2}]$, then $\varphi(f, T)$ is

$$-(1+\gamma)\frac{\sum_{n\in\mathbb{Z}} |\overline{f}(n/2)|^4}{(\sum_{n\in\mathbb{Z}} |\overline{f}(n)|^2)^2} + 4\frac{|\sum_{n\in\mathbb{Z}} |\overline{f}(n/2)|^2 e^{-2i\pi n/(1+\gamma)}|^2}{(\sum_{n\in\mathbb{Z}} |\overline{f}(n)|^2)^4} + 2$$

where $(\overline{f}(n))_{n\in\mathbb{Z}}$ is the square summable sequence defined by $\overline{f}(n) = f(\frac{nT}{1+\gamma})$ and $\overline{f}(n/2)$ is the "interpolated" sequence defined by $\overline{f}(n/2) = \int_{-1/2}^{1/2}(\sum_{n\in\mathbb{Z}} e^{-2i\pi n\nu} f(n))e^{2i\pi\nu n/2}\, d\nu$. The left part of the figure shows the graph of Φ as a function of the excess bandwidth factor. It is clear that $\Phi(0) < 2$, so that for any $\gamma > 0$, then, $b < 2$.

Remark 1 *It follows from Eq.* (15) *and Corollary 1 that the extracted source is the one having the biggest excess bandwidth factor.*

3.2. Second case: all the sources share a common baudrate

We let T be the common symbol period. Contrary to the previous section, the terms ℓ do not reduce to a constant:

$$\ell(\tilde{s}_{k_1}, \tilde{s}_{k_2}) = 4\left(1 + 2\Re e\left(R^{(\alpha)}_{\tilde{s}_{k_1}}(0)\overline{R^{(\alpha)}_{\tilde{s}_{k_2}}(0)}\right)\right) \qquad (16)$$

where we set $\alpha = T_e/T$. Cancealing the angles of the $R^{(\alpha)}_{\tilde{s}_k}(0)$s yields the inequality $J(r) \geq J_1(r)$ with

$$\begin{aligned} J_1(r) \quad &= \quad \sum_{k=1}^{K} \beta(\tilde{s}_k)\|f_k\|^4 - 2\sum_{k=1}^{K} \|f_k\|^2 + 1 \\ &\quad +4\sum_{k_1<k_2} (1 - \lambda(\tilde{s}_{k_1})\lambda(\tilde{s}_{k_2})\, \|f_{k_1}\|^2\|f_{k_2}\|^2 \end{aligned}$$

where we have set $\lambda(\tilde{s}_k) = |R^{(\alpha)}_{\tilde{s}_k}(0)|$.

The main difficulty the minimization of J_1 is based on is that the $\beta(\tilde{s}_k)$ and $\lambda(\tilde{s}_k)$ are related with one another - this may be seen in (14). At the first sight, an idea consists in uniformely lower (respectively upper) bounding the $\beta(\tilde{s}_k)$ (respectively $\lambda(\tilde{s}_k)$:

$$\beta(\tilde{s}_k) \geq b = \Phi(\gamma_{max}) \qquad (17)$$

(as is seen in the previous section); as far as $\lambda(\tilde{s}_k)$ is concerned, it can easily be proved (this is a straight-forward application of the Cauchy-Schwartz inequality) that

Lemma 2

$$|R^{(\alpha)}_{\tilde{s}_k}(0)| \leq \frac{1}{2}. \quad \textit{That is} \quad \lambda(\tilde{s}_k) \leq \frac{1}{2}. \tag{18}$$

The rough lower-bound of J_1 hence obtained can be shown not to provide any positive result concerning the minimization of $J(r)$. The point making the uniform bounds (17) and (18) inefficient is that b and $\frac{1}{2}$ are not reached for the same filters. Taking into account that $\beta(\tilde{s}_k)$ and $\lambda(\tilde{s}_k)$ depend on one another may tighten the minoration. In order to specify this point, we show that there exist couples (β_*, λ_*) verifying $\lambda_* < 1/2$ and

$$\text{If } \beta(\tilde{s}_k) \leq \beta_* \text{ then } \lambda \leq \lambda_*.$$

Up to now, we have failed to provide any results when the number of sources is bigger than two. However, for the case of two sources, the following holds:

Result 1 *$P = 2$ (i.e. 2 sources). If $\gamma_{max} \in [0.05; 0.95]$, the minimum of $J(r)$ is equal to $1 - \frac{1}{b} = 1 - \frac{1}{\Phi(\gamma_{max})}$. Moreover, this lower-bound is reached if and only if filter $\mathbf{f}(z)$ for which $r(n) = [\mathbf{f}(z)]\mathbf{s}(n)$ satisfies $\|f_k\|^2 = \frac{1}{b}\delta(k-k_0)$ and $b = \beta([\tilde{f}_{k_0}(z)]s_{k_0}(n)) = \Phi(\gamma_{max})$, where $\tilde{f}_{k_0}(z) = \frac{f_{k_0}(z)}{\|f_{k_0}\|}$.*

The proof is rather technical and is being written in a complete paper to be soon submitted.

3.3. Third case: groups of different baud-rates

This section provides a more general result. We call *group of sources* a set of sources having a common baud-rate. As the previous developments suggest, we are not able up to now to say much when a group of sources has more than two sources. On the contrary, the following holds

Result 2 *If all the groups of sources have at most two sources, then $J(r) \geq 1 - \frac{1}{b}$ with $b = \Phi(\gamma_{max})$. Moreover, the equality holds if and only if filter $\mathbf{f}(z)$ for which $r(n) = [\mathbf{f}(z)]\mathbf{s}(n)$ satisfies $\|f_k\|^2 = \frac{1}{b}\delta(k-k_0)$ and $b = \beta([\tilde{f}_{k_0}(z)]s_{k_0}(n)) = \Phi(\gamma_{max})$, where $\tilde{f}_{k_0}(z) = \frac{f_{k_0}(z)}{\|f_{k_0}\|}$.*

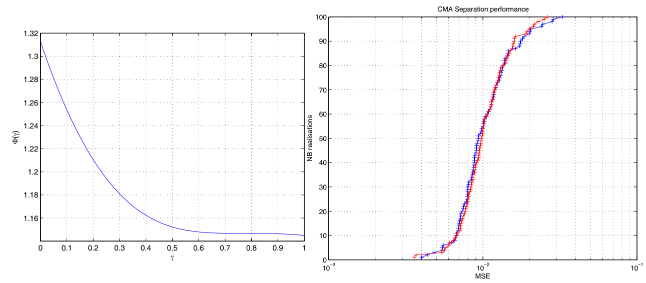
This says that the arguments minima of J are filters capable of extracting one of the sources having the biggest excess band-width factor.

4. SIMULATIONS

We denote by $c_{k,l}(n)$ the contribution of the k-th source on the l-th sensor at time n, and $\tilde{c}_{k,l}(n)$ its estimated version provided by the source separation algorithm. The performance criterion for the source k is defined as:

$$C_k = \sum_{l=1}^{N} \frac{< |\tilde{c}_{k,l}(n) - c_{k,l}(n)|^2 >}{\sum_{k=1}^{K} < |c_{k,l}(n)|^2 >} \tag{19}$$

On figure(b), the criterion for the first extracted source is plotted for 100 trials. The number of sources is $K = 2$ and the number



(a) Proof Illustration

(b) Contrast performance

of sensors $M = 3$. The baud-rates are the same. At each trial, the characteristics of the sources (QPSK or 8-PSK, excess-bandwidth in $\{0.5\ ,\ 0.7\ ,\ 0.3\}$), the propagation channels (Rayleigh with 3 paths) are randomly chosen. The estimation of $J(r)$ relies on 3000 time-units. The results obtained confirm that the minimization of Godard's function achieves separation.

5. REFERENCES

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