ON IMPLEMENTING THE BLIND ML RECEIVER FOR ORTHOGONAL SPACE-TIME BLOCK CODES

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ABSTRACT

We consider the problem of blind maximum-likelihood (ML) detection for the orthogonal space-time block code (OSTBC) scheme. Our previous work has shown that for OSTBCs the blind ML detection problem can be simplified to a Boolean quadratic program (BQP). This sequel focuses on effective optimization methods for that BQP, which, from an optimization viewpoint, is still a computationally hard problem. First, we consider semidefinite relaxation (SDR), a high-precision BQP approximation algorithm with a computational cost that is polynomial in the problem size. We also propose a simple method that can significantly reduce the average complexity of the SDR technique. Second, we consider sphere decoding, an exact BQP solver that can be computationally expensive in the worst case, but generally incurs a reasonable average complexity particularly at high SNRs. Simulation results indicate that these two blind ML algorithms provide very similar bit error rate performance. Moreover, numerical studies show that SDR provides better complexity performance than sphere decoding in the worst-case sense, while sphere decoding provides better complexity performance in the average sense.

1. INTRODUCTION

Recently there has been much interest in developing blind receivers for space-time coding schemes; e.g., [1,2]. The orthogonal space-time block code (OSTBC) scheme [3] has been found to be particularly attractive in that the OSTBC matrix structures can be exploited to make the blind receivers less complex and more effective; e.g., blind subspace receivers [4], and blind maximum-likelihood (ML) receivers [5, 6]. The focus of this paper is on the blind ML method. It has been illustrated [5, 6] that the blind ML OSTBC detector with BPSK or QPSK constellations can be simplified to a Boolean quadratic program (BQP), the complexity of which is comparable to that of solving a coherent ML MIMO detection problem. The BQP is well known to be computationally hard to solve. There are simple suboptimal algorithms for the blind ML BQP, such as the methods [4–6], but to efficiently obtain a near-optimal or exactly optimal BQP solution is considerably more challenging. This paper proposes two alternatives for blind ML OSTBC detector implementations, both of which have been recently found to be powerful and computationally efficient BQP techniques. First, we consider *semidefinite relaxation* (SDR), a suboptimal BQP solver that has been shown [5, 7–11] to provide high-precision approximation accuracy with an affordable worst-case computational cost. We will propose a modified SDR algorithm that can provide considerable computation savings compared to the original SDR. Second, we consider a particularly effective exact BQP solver, namely *sphere decoding* [12–14]. The worst-case complexity of sphere decoding can be very expensive, but much evidence in the coherent MIMO detection application (e.g., [14]) has suggested that the average sphere decoding complexity is acceptable at high SNRs. Simulation results in Section 4 will show that these two blind ML implementation alternatives provide very similar bit error performance. It is then interesting to compare the complexity performance of the two methods. This aspect will be numerically studied in Section 4.

2. BACKGROUND

This section reviews the OSTBC scheme and the respective blind ML detection problem.

2.1. Orthogonal Space-Time Block Coding Scheme

We consider a standard space-time block code (STBC) transmission scenario in which the MIMO channel is flat in frequency. Let M_t and M_r denote the numbers of transmitter and receiver antennas, respectively. By letting T be the code block length, the received code block of a single STBC can be modeled as:

$$\mathbf{Y} = \mathbf{H}\mathbf{C}(\mathbf{s}) + \mathbf{V}, \tag{1}$$

where $\mathbf{s} \in \{\pm 1\}^K$ are transmitted bits, $\mathbf{C}(\mathbf{s}) \in \mathbb{C}^{M_t \times T}$ is an STBC function that maps information bits to a code matrix, $\mathbf{H} \in \mathbb{C}^{M_r \times M_t}$ is the MIMO channel matrix, and $\mathbf{V} \in \mathbb{C}^{M_r \times T}$ is an additive white Gaussian noise (AWGN) matrix with zero mean and variance $\mathcal{N}_o$. Orthogonal STBCs (OSTBCs) are a class of codes constructed based on the theory of orthogonal designs [3]. In the QPSK constellation case, an OSTBC function can be expressed as

$$\mathbf{C}(\mathbf{s}) = \sum_{k=1}^{K/2} \mathbf{A}_k s_k + j \sum_{k=1}^{K/2} \mathbf{B}_k s_{k+K/2}, \tag{2}$$

where $\mathbf{A}_k, \mathbf{B}_k \in \mathbb{R}^{M_t \times T}$ are the constituent matrices of the code, $s_k \in \{\pm 1\}$ is the kth element of $\mathbf{s}$, and $j = \sqrt{-1}$. In the BPSK constellation case, the matrices $\mathbf{B}_k$ are absent from (2). Both the QPSK and BPSK OSTBCs can be represented by

$$\mathbf{C}(\mathbf{s}) = \sum_{k=1}^{K} \mathbf{X}_k s_k, \tag{3}$$

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where $\mathbf{X}_k \in \mathbb{C}^{M_t \times T}$. The OSTBC constituent matrices are specially designed such that for any $\mathbf{s} \in \{\pm 1\}^K$,

$$\mathbf{C}(\mathbf{s})\mathbf{C}^H(\mathbf{s}) = \|\mathbf{s}\|_2^2 \mathbf{I} = K\mathbf{I} \tag{4}$$

where $\|\cdot\|_2$ denotes the 2-norm. The semi-orthogonal code property in (4) has been shown to lead to the two important advantages of the maximum diversity gain and low detection complexity given channel state information (CSI) at the receiver [3]. The present paper considers blind detection of OSTBCs; i.e., detection in the absence of CSI at the receiver.

2.2. Formulation of Blind Maximum-Likelihood Detection

To formulate the problem of blind maximum-likelihood (ML) detection, we apply the two standard assumptions that lead to the so-called *deterministic blind ML* detector [2]. First, the MIMO channel is assumed to be slowly time varying such that $\mathbf{H}$ remains static over P consecutive code blocks. In this case it is appropriate to add a block index, p, to the OSTBC signal model in (1):

$$\mathbf{Y}_p = \mathbf{H}\mathbf{C}(\mathbf{s}_p) + \mathbf{V}_p, \qquad p = 1, \dots, P, \tag{5}$$

where $\mathbf{Y}_p$ is the pth received signal block, $\mathbf{s}_p \in \{\pm 1\}^K$ is the pth bit symbol block, and $\mathbf{V}_p$ contains the AWGN samples. Second, $\mathbf{H}$ is assumed to be a deterministic unknown. Define

$$\mathbf{s}_{1:P} = [\, \mathbf{s}_1^T, \dots, \mathbf{s}_P^T \,]^T \in \{\pm 1\}^{KP}. \tag{6}$$

With the two channel assumptions, the ML detector for the received signal frame $[\, \mathbf{Y}_1, \dots, \mathbf{Y}_P \,]$ is shown to be [2]

$$\{\hat{\mathbf{H}}, \hat{\mathbf{s}}_{1:P}\} = \arg \min_{\substack{\mathbf{H} \in \mathbb{C}^{M_r \times M_t} \\ \mathbf{s}_{1:P} \in \{\pm 1\}^{KP}}} \sum_{p=1}^{P} \|\mathbf{Y}_p - \mathbf{H}\mathbf{C}(\mathbf{s}_p)\|_F^2. \tag{7}$$

where $\|\cdot\|_F$ denotes the Frobenius norm. In [5, 6] it is further shown that by exploiting the OSTBC properties in (3) and (4), the ML detected symbols $\hat{\mathbf{s}}_{1:P}$ can be determined alone by solving

$$\begin{aligned} \hat{\mathbf{s}}_{1:P} &= \arg \max_{\mathbf{s}_{1:P} \in \{\pm 1\}^{KP}} \sum_{p=1}^{P} \sum_{q=1}^{P} \mathbf{s}_p^T \mathbf{G}_{\mathcal{Y},pq} \mathbf{s}_q \\ &= \arg \max_{\mathbf{s}_{1:P} \in \{\pm 1\}^{KP}} \mathbf{s}_{1:P}^T \begin{bmatrix} \mathbf{G}_{\mathcal{Y},11} & \dots & \mathbf{G}_{\mathcal{Y},1P} \\ \vdots & \ddots & \vdots \\ \mathbf{G}_{\mathcal{Y},P1} & \dots & \mathbf{G}_{\mathcal{Y},PP} \end{bmatrix} \mathbf{s}_{1:P} \end{aligned} \tag{8}$$

where $\mathbf{G}_{\mathcal{Y},pq} \in \mathbb{R}^{K \times K}$ has its (k, ℓ)th entry given by

$$[\mathbf{G}_{\mathcal{Y},pq}]_{k\ell} = \mathrm{Re}\{\mathrm{tr}\{\mathbf{Y}_p \mathbf{X}_k^H \mathbf{X}_\ell \mathbf{Y}_q^H\}\}. \tag{9}$$

The focus of this paper, which will be elaborated upon in the next section, is on practically realizable algorithms for finding the solution of (8).

3. EFFICIENT ALGORITHMS FOR BLIND ML OSTBC DETECTION

For notational simplicity, we rewrite the blind ML detection problem in (8) as

$$\max_{\mathbf{s} \in \{\pm 1\}^{KP}} \mathbf{s}^T \mathbf{G} \mathbf{s} \tag{10}$$

in which some subscripts in the original problem are dropped. Problem (10) is a Boolean quadratic program (BQP), whose optimal solution can be expensive to compute. (The BQP is NP-hard.) In Section 3.1, we describe the *semidefinite relaxation* (SDR) method for efficient approximation of the BQP. In this section, a method of reducing the complexity of SDR is also proposed. In Section 3.2, we describe an exact BQP solution using the *sphere decoding* method. The bit error performance and complexity performance of the two methods will be numerically compared in the subsequent section.

3.1. Semidefinite Relaxation

Semidefinite relaxation (SDR) has been shown, both theoretically and practically, to be a high-precision approximation algorithm for the BQP [7–10]. The SDR approximation advantages in the blind OSTBC ML application have also been examined in [5, 6]. SDR considers solving the following relaxed BQP problem:

$$\max_{\mathbf{S}} \ \mathrm{tr}\{\mathbf{S}\mathbf{G}\} \tag{11a}$$

$$\text{s.t. } S_{ii} = 1, \qquad i = 1, \dots, KP \tag{11b}$$

$$\mathbf{S} \succeq \mathbf{0} \tag{11c}$$

where $\mathbf{S} \succeq \mathbf{0}$ means that $\mathbf{S}$ is positive semidefinite (PSD). The SDR problem in (11) is a relaxation of (10) because any $\mathbf{S} = \mathbf{s}\mathbf{s}^T$, $\mathbf{s} \in \{\pm 1\}^{KP}$, is a feasible point of the SDR problem. An advantage of SDR is that Problem (11) is a convex semidefinite program, the globally optimal solution of which can be efficiently computed by readily available optimization algorithms [15] with an operational cost of $\mathcal{O}((KP)^{3.5})$. To use SDR to approximate the BQP solution, the following method can be used. Let $\hat{\mathbf{S}}$ denote the SDR solution in (11), and $\mathcal{P}\{\hat{\mathbf{S}}\}$ be the principal eigenvector of $\hat{\mathbf{S}}$. The BQP solution can be approximated by

$$\hat{\mathbf{s}}_{\text{SDR}} = \mathrm{sign}(\mathcal{P}\{\hat{\mathbf{S}}\}). \tag{12}$$

Another possible BQP solution approximation, which were numerically found to be very effective [9], is the Goemans-Williamson randomized algorithm [7]; please see [9] for the implementation details.

When compared to some simple closed-form based blind detectors (such as those in [1, 4]), the SDR method is computationally more expensive due to the computational overhead for the SDR optimization algorithm. Here we propose a modified SDR algorithm [6] that finds the SDR solution without using the SDR optimization algorithm *sometimes*, by making use of a computationally cheap suboptimal blind detector. The following theorem provides the framework for the proposed method:

Theorem 1 *If there is a vector $\tilde{\mathbf{s}} \in \{\pm 1\}^{KP}$ that satisfies*

$$\mathrm{Diag}(\tilde{\mathbf{s}} \odot (\mathbf{G}\tilde{\mathbf{s}})) - \mathbf{G} \succeq \mathbf{0} \tag{13}$$

where $\odot$ is the Hadamard (elementwise) product and $\mathrm{Diag}(\mathbf{a})$ is a diagonal matrix with ith diagonal given by a_i, then

$$\mathbf{S} = \tilde{\mathbf{s}}\tilde{\mathbf{s}}^T \tag{14}$$

is an optimal SDR solution in (11). Such an $\tilde{\mathbf{s}}$ is also an optimal solution to the BQP in (10).

The proof of Theorem 1, which is available in [6], is not shown here due to lack of space. An alternative way to generate the principles behind that proof is to consider the rank-1 optimality result in [11].

Now, suppose that a suboptimal blind symbol decision, denoted by $\hat{\mathbf{s}}_{subopt} \in \{\pm 1\}^{KP}$, is easily available computationally. Sometimes $\hat{\mathbf{s}}_{subopt}$ will coincide with the blind SDR-ML decision. Using the optimality condition in (13), we can inspect whether $\hat{\mathbf{s}}_{subopt}$ is capable of forming the SDR solution. By doing so, the SDR optimization process is only necessary when $\hat{\mathbf{s}}_{subopt}$ fails to satisfy the optimality condition in (13). This idea leads to the following *modified SDR* algorithm:

Modified SDR Implementation

Given $\mathbf{G} \in \mathbb{R}^{KP \times KP}$, and a suboptimal blind symbol decision $\hat{\mathbf{s}}_{subopt} \in \{\pm 1\}^{KP}$.

if $\mathrm{Diag}(\hat{\mathbf{s}}_{subopt} \odot (\mathbf{G}\hat{\mathbf{s}}_{subopt})) - \mathbf{G} \succeq \mathbf{0}$
    output $\hat{\mathbf{s}}_{subopt}$ as the blind SDR-ML solution,
else
    run the original SDR algorithm to compute the blind SDR-ML solution.

3.2. Sphere Decoding

In sphere decoding, we are concerned with solving the following integer least squares (ILS) problem

$$\min_{\mathbf{s} \in \mathcal{A}^m} \|\mathbf{b} - \mathbf{R}\mathbf{s}\|_2^2 \tag{15}$$

where $\mathcal{A} \subseteq \mathbb{Z}$ is a set of integers. We will show that the blind ML BQP problem in (10) can be reformulated as an ILS. To describe the sphere decoding principle, define a subset

$$\mathcal{S}(d) = \{\mathbf{s} \in \mathcal{A}^m \mid \|\mathbf{b} - \mathbf{R}\mathbf{s}\|_2^2 \leq d\} \tag{16}$$

Now suppose that we are given a squared radius, denoted by d_0, such that the optimal ILS solution lies in $\mathcal{S}(d_0)$. In practice, such a d_0 can be determined by some heuristic means [12]; e.g., if a suboptimal ILS solution, denoted by $\hat{\mathbf{s}}_{subopt} \in \mathcal{A}^{KP}$, is easily available computationally, we can set $d_0 = \|\mathbf{b} - \mathbf{R}\hat{\mathbf{s}}_{subopt}\|_2^2$. Subsequently, solving the ILS problem is equivalent to solving the following sphere constrained ILS problem

$$\min_{\mathbf{s} \in \mathcal{S}(d_0)} \|\mathbf{b} - \mathbf{R}\mathbf{s}\|_2^2 \tag{17}$$

Sphere decoding algorithms are point search methods particularly designed to solve (17). An advantage of sphere decoding is that if a large number of points in $\mathcal{A}^m$ are excluded from $\mathcal{S}(d_0)$, then sphere decoding will be much more efficient than a complete point search for (15). However, in a worst-case situation, such as when d_0 is poorly initialized, sphere decoding can be as expensive as the complete point search.

There are many implementation variants for sphere decoding; see [12–14] for the details of those algorithms. One popular implementation is the Viterbo-Boutros (VB) [12]. Moreover, recent development has indicated that the so-called Schnorr-Euchner (SE) sphere decoding implementation [13,14] can offer significant complexity reduction compared to many other sphere decoding implementations. We will evaluate both the VB and SE implementations of the sphere decoder in our blind ML OSTBC detector. To apply sphere decoding to blind ML detection, we reformulate the blind ML BQP problem in (8) as

$$\min_{\mathbf{s} \in \{\pm 1\}^{KP}} \mathbf{s}^T(\rho\mathbf{I} - \mathbf{G})\mathbf{s} \tag{18}$$

for some real constant ρ, $|\rho| < \infty$. Let us choose ρ to be greater than the largest eigenvalue of $\mathbf{G}$, such that $\rho\mathbf{I} - \mathbf{G}$ is PSD. Then we can perform Cholesky factorization $\mathbf{R}^T\mathbf{R} = \rho\mathbf{I} - \mathbf{G}$, where $\mathbf{R} \in \mathbb{R}^{KP \times KP}$ is the upper triangular Cholesky factor of $\rho\mathbf{I} - \mathbf{G}$. Thus, Problem (18) is equivalent to the ILS (with $\mathbf{b} = \mathbf{0}$).

4. SIMULATIONS

In this simulation example, we evaluate the BER performance of the proposed blind ML detectors. The simulation scenario is similar to that in [5], and is briefly described as follows: We employ the BPSK full-rate OSTBC with $M_t = 3$, and $T = 4$ [3]. Moreover, we set $M_r = 4$ and $P = 8$. The blind detectors tested are the SDR-ML detector, the ML detector by sphere decoding, the norm relaxed ML detector [5] (see also [4]), the cyclic ML detector [2], and the subspace detector [1]. Fig. 1 plots the BER performance of the blind detectors versus the SNR. (Since the original and modified SDR algorithms provide exactly the same symbol decision, we only plotted the BER performance of the original SDR algorithm.) We see that the BERs of the SDR-ML and sphere decoding ML detectors are very close. Moreover, the BER performance of the SDR-ML and sphere decoding ML detectors is significantly better than that of the other suboptimal detectors.

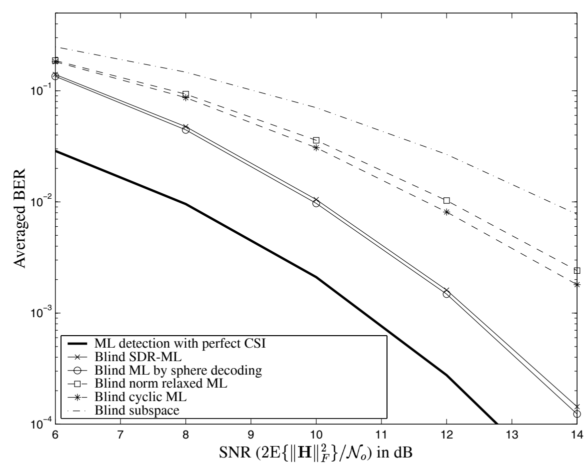

Fig. 1. BER performance in a 4 receiver antenna case.

The previous simulation settings were used to evaluate the computational costs of the SDR algorithm, the modified SDR algorithm, the VB sphere decoder, and the SE sphere decoder. We use the norm relaxed ML detector to initialize the sphere decoding algorithms and to provide the preliminary decision for the modified SDR algorithm. The complexities of the above methods are measured by counting the total number of floating point operations (FLOPs) required to jointly detect KP symbols (note that $KP = 32$ in this example). Fig. 2(a) shows the average (expected) complexity performance of the various algorithms. We notice that for a wide range of SNRs, the SE sphere decoder generally provides better average complexity performance than the other

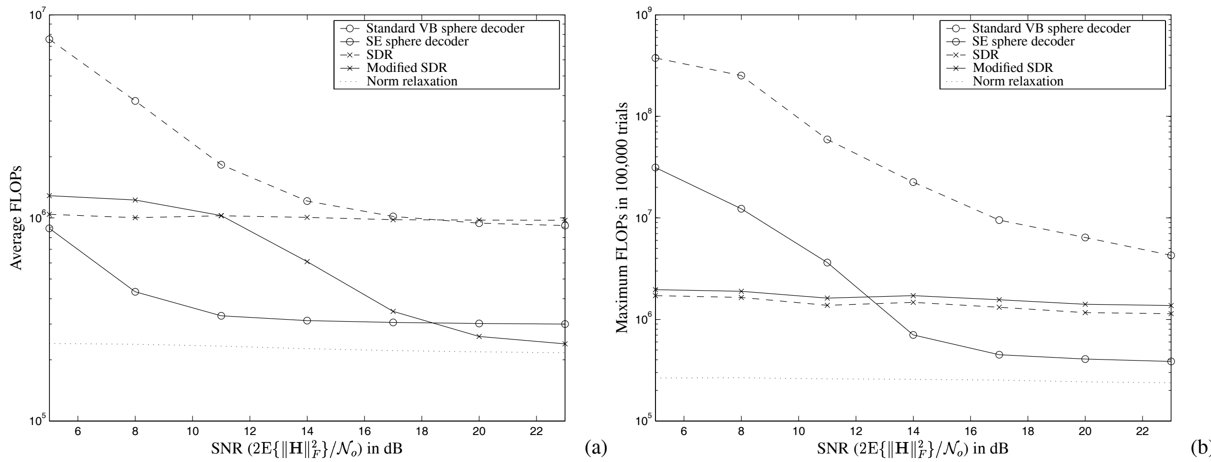

Fig. 2. Average and worst-case complexity performance of the SDR and sphere decoding algorithms.

methods. Unlike the sphere decoders, the SDR algorithm has an average complexity that is essentially invariant to the SNR. As for the modified SDR algorithm, it has an average complexity that decreases with the SNR. In particular, the modified SDR algorithm manages to provide a slightly lower average complexity than the SE sphere decoder when the SNR is greater than 20dB.

The worst-case complexity of the various algorithms is plotted in Fig. 2(b). The worst-case complexity is measured by picking the largest FLOP count in $100,000$ independent trials. The figure shows that at low SNRs, the two sphere decoders yield poor worst-case complexity performance compared to the SDR algorithms. It is worthwhile pointing out that for a fixed SNR, the worst-case complexity of the sphere decoders can become unacceptably large as the problem size increases [6].

5. CONCLUSION

This paper presents two highly effective methods for realizing the blind ML OSTBC detector, namely SDR and sphere decoding. SDR leads to an approximate blind ML receiver, but it exhibits good approximation accuracy and has an affordable worst-case computational complexity. Sphere decoding results in an exact blind ML receiver that yields poor complexity performance in the worst case, but it generally offers a reasonable average computational cost. Using simulations, we have illustrated that the two methods provide very similar bit error performance. Numerical complexity studies have indicated that while the average complexity performance of sphere decoding is generally better than that of SDR, sphere decoding can yield poor worst-case complexity performance compared with SDR.

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