MMSE TRANSMIT OPTIMIZATION FOR MULTI-USER MULTI-ANTENNA SYSTEMS

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ABSTRACT

We address the problem of linear mean square error (MSE) transmitter design for point-to-multipoint multiuser systems, where the transmitter is equipped with multiple antennas and each of the independent receivers has a single antenna. This downlink scenario is more difficult to handle than its uplink counterpart since all users are coupled by transmit filters and powers. Our main result is to show that downlink and uplink share the same normalized MSE achievable region under a sum power constraint. Thus, the problem of downlink transmitter design can be solved by focusing on an equivalent uplink problem, which has a more suitable structure and allows for efficient algorithmic solutions. As application examples, we solve the problem of minimizing the maximal normalized MSE of all users (fairness), and the problem of minimizing the sum of all normalized MSE (overall efficiency).

1. INTRODUCTION

The mean square error (MSE) between the transmitted and received data symbols is an important performance measure for multiuser communication systems dominated by mutual interference. While MSE optimization strategies are known for the uplink receiver design, see e.g., [1,2], finding the optimal point-to-multi-point downlink transmitter is a complex problem, which involves the joint optimization of all transmit powers, pre-equalizers, and receivers.

1.1. Problem Statement

In this paper we propose a framework for MSE transmitter design based on uplink/downlink duality. We consider a system with M transmit antennas and K decentralized single antenna receivers, as depicted in Fig. 1. Note, that the same vector-valued model holds

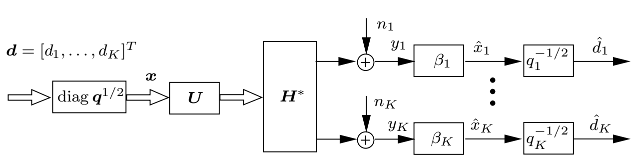

Fig. 1. Downlink system model

for synchronous DS-CDMA with fixed receivers, as well as for digital subscriber line (DSL) services, where many twisted pairs of telephone lines are bundled together in one cable leading to interference between users.

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Assume unity-energy random transmit symbols $\boldsymbol{d} = [d_1, ..., d_K]^{\mathrm{T}}$ i.e., $\mathrm{E}\{\boldsymbol{d}\boldsymbol{d}^*\} = \boldsymbol{I}$ and zero-mean white Gaussian noise $\boldsymbol{n} = [n_1, ..., n_K]^{\mathrm{T}}$ $\sim \mathcal{N}(0, \sigma_n^2 \boldsymbol{I})$. The vector $\boldsymbol{q} = [q_1, ..., q_K]$ contains the transmission powers. We define $\boldsymbol{Q} = \mathrm{diag}\{\boldsymbol{q}\}$. The normalized beamforming matrix $\boldsymbol{U} = [\boldsymbol{u}_1, \ldots, \boldsymbol{u}_K]$, with $\|\boldsymbol{u}_i\|_2 = 1$, is used to map the signal vector $\boldsymbol{d}$ onto the M transmit antennas. The signal received by the ith user is scaled by $\beta_i / \sqrt{q_i}$, where β_i adds additional degrees of freedom which can be used for MSE optimization. The estimated symbol is

$$\hat{d}_i = \frac{\beta_i}{\sqrt{q_i}} (\boldsymbol{h}_i^* \boldsymbol{U} \sqrt{\boldsymbol{Q}} \boldsymbol{d} + n_i) \qquad \forall i \in \{1, 2, \ldots, K\} \,, \quad (1)$$

where $\boldsymbol{h}_i \in \mathbb{C}^{M \times 1}$ models the channel between the ith user and the base station array and $(\cdot)^*$ denotes the conjugate transpose. Thus, the individual normalized MSE is given as

$$\varepsilon_i^{\mathrm{DL}} = \mathrm{E}\{|\hat{d}_i - d_i|^2\} = \mathrm{E}\{|\hat{x}_i - x_i|^2\} / p_i \qquad \forall i \,. \quad (2)$$

These quantities are coupled by the transmit filters $\boldsymbol{U}$ and transmit power allocation $\boldsymbol{q}$. Thus, choosing a joint transmit design means to finding an "optimal" trade-off between the individual MSE's. Since we are concerned with independent receivers, the notion of optimality clearly depends on the overall system design. That is, the desired MSE trade-off may be determined by higher layer requirements, like bit error rates, queuing priorities, latency constraints, etc. In the following we will focus on the following two design goals:

P1: optimal overall efficiency

$$\min_{\boldsymbol{U}, \boldsymbol{\beta}, \boldsymbol{q}} \sum_{i=1}^{K} \varepsilon_i^{\mathrm{DL}} \quad \text{s.t.} \quad \|\boldsymbol{q}\|_1 \leq P_{\max} \,, \quad (3)$$

P2: min-max fairness

$$\min_{\boldsymbol{U}, \boldsymbol{\beta}, \boldsymbol{q}} \Big(\max_{1 \leq i \leq K} \varepsilon_i^{\mathrm{DL}} \Big) \quad \text{s.t.} \quad \|\boldsymbol{q}\|_1 \leq P_{\max} \,. \quad (4)$$

1.2. Related Work

In [3–6] problem P1 has been studied for the *uplink*, where precoders at mobile terminals and MMSE decoder at base station are jointly adjusted to minimize the MSE. It was studied in [3,4] for a multiple-input-multiple-output (MIMO) single user scenario, and in [5] for a MIMO multiuser scenario. A weighted MSE optimization problem was proposed in [6], where different weights can lead to different optimization problems, e.g., maximum information rate, QoS-based design, etc.

For the downlink, a similar transmit optimization strategy was introduced in [7] (TO-MIMO) and in [8] (Transmit Wiener Filter, TxWF), which minimizes the modified MSE $\mathrm{E}\{\|\hat{\boldsymbol{x}} - \alpha \boldsymbol{x}\|^2\}$ by introducing a factor α. However, scaling all received signals by the same factor is generally not an optimum strategy. Better results are expected by using a vector scaling. Our proposed strategy employs different scalars for different users. This leads to a performance improvement over the TxWF, as will be shown in the simulation results. Additionally, in our scheme, adaptive power control can be applied to minimize the normalized MSE over all possible power allocations.

Problem P2 was only studied in an *uplink* context [2]. The downlink problem will be addressed in this paper. In [9], a similar problem was studied in the context of SINR balancing for single-input-multiple-output (SIMO) links. We will make use of this strategy later in Section 3, where an algorithmic solution for P2 will be proposed.

2. UPLINK/DOWNLINK NORMALIZED MSE DUALITY

It is important to constrain the transmit power, otherwise the optimization yields a zeroforcing solution [10]. On the other hand, power-constrained optimization problems of the form (3) and (4) are difficult to handle directly, which shows the analysis in [7, 8, 10]. Thus, it would be desirable to establish a duality between MSE optimization in uplink and downlink, similar to the one that was recently observed in the context of SINR optimization [9, 11]. Although the relationship between the downlink SINR and the MSE is not as straightforward as for the uplink, it will be shown in the following, that uplink and downlink indeed share the same MSE achievable region. This will be used later in Section 3 to derive solutions for the optimization problems P1 and P2.

2.1. Equivalent Uplink Model

We start by considering the uplink model that is obtained by switching the role of transmitter and receiver (see Fig. 2). The symbol vector $\boldsymbol{d}$ is now transmitted from K independent antennas over the propagation channel $\boldsymbol{H} = [\boldsymbol{h}_1, ..., \boldsymbol{h}_K]$. The matrix $\boldsymbol{U}^*$ now acts as a multiuser receiver, which separates the data streams. For convenience, we introduce $\widetilde{\boldsymbol{U}} = \boldsymbol{U}\boldsymbol{\beta}$, where the norm of each column in $\boldsymbol{U}$ is equal to one, and $\boldsymbol{\beta} = \mathrm{diag}\{[\beta_1, ..., \beta_K]\}$ is a diagonal matrix containing the column norms of $\widetilde{\boldsymbol{U}}$. For reasons that will become clear later, we assume that the quantities $\boldsymbol{H}, \boldsymbol{U}, \boldsymbol{\beta}$, are the same as for the downlink model. The power allocation $\boldsymbol{p}$, however, may be different from the downlink allocation $\boldsymbol{q}$. It is assumed that both links have the same sum power constraint, i.e., $\|\boldsymbol{p}\|_1 \le P_{\max}$ and $\|\boldsymbol{q}\|_1 \le P_{\max}$. We define $\boldsymbol{P} = \mathrm{diag}\{\boldsymbol{p}\}$. With the received signal $\boldsymbol{y} = \boldsymbol{H}\sqrt{\boldsymbol{P}}\boldsymbol{d} + \boldsymbol{n}$, the ith estimated symbol becomes

$$\hat{d}_i = \frac{\beta_i}{\sqrt{p_i}} \boldsymbol{u}_i^* (\boldsymbol{H}\sqrt{\boldsymbol{P}}\boldsymbol{d} + \boldsymbol{n}). \tag{5}$$

and the normalized MSE is

$$\varepsilon_i^{\mathrm{UL}} = \mathrm{E}\{|\hat{d}_i - d_i|^2\} = \mathrm{E}\{|\hat{x}_i - x_i|^2\}/p_i \;. \tag{6}$$

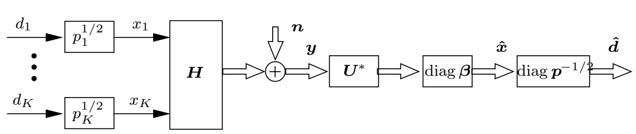

Fig. 2. Equivalent uplink system with K independent transmitters (single antenna) and joint multiuser reception.

2.2. Duality Relationship between Uplink and Downlink MSE

Next, we compare the achievable uplink MSE (6) with the downlink MSE (2) under a total power limit $P_{\max}$. To this end, we rewrite the downlink MSE $\varepsilon_i^{\mathrm{DL}}$ as

$$\begin{aligned}\varepsilon_i^{\mathrm{DL}} &= \beta_i^2 \boldsymbol{h}_i^* \boldsymbol{U}\boldsymbol{Q}\boldsymbol{U}^* \boldsymbol{h}_i / q_i \\ &\quad - \beta_i \boldsymbol{h}_i^* \boldsymbol{u}_i - \beta_i \boldsymbol{u}_i^* \boldsymbol{h}_i + 1 + \beta_i^2 \sigma_n^2 / q_i \;.\end{aligned} \tag{7}$$

The uplink MSE is

$$\varepsilon_i^{\mathrm{UL}} = \beta_i^2 \boldsymbol{u}_i^* \boldsymbol{Z} \boldsymbol{u}_i / p_i - \beta_i \boldsymbol{h}_i^* \boldsymbol{u}_i - \beta_i \boldsymbol{u}_i^* \boldsymbol{h}_i + 1 \;, \tag{8}$$

where $\boldsymbol{Z} = \boldsymbol{H}\boldsymbol{P}\boldsymbol{H}^* + \sigma_n^2 \boldsymbol{I}$.

It can be observed that $\varepsilon_1^{\mathrm{UL}}, \ldots, \varepsilon_K^{\mathrm{UL}}$ can be optimized independently. Collecting all optimizers in a matrix $\widetilde{\boldsymbol{U}}_{\mathrm{mmse}}$, we have

$$\widetilde{\boldsymbol{U}}_{\mathrm{mmse}} = (\boldsymbol{H}\boldsymbol{P}\boldsymbol{H}^* + \sigma_n^2 \boldsymbol{I})^{-1} \boldsymbol{H}\boldsymbol{P}. \tag{9}$$

Direct optimization of (7) is difficult, however. In the following, we derive a framework for an indirect optimization approach based on duality. Instead of solving problems (3) and (4) directly, a solution can be found by solving an equivalent uplink problem.

To this end, we need the following lemma, which characterizes the condition under which expressions (8) and (7) are equal.

Lemma 1. *Let* $\boldsymbol{U}$ *and* $\boldsymbol{\beta}$ *be fixed parameters, and suppose that SINR values* $\gamma_1, \ldots, \gamma_K$ *are achieved by uplink and downlink power allocations* $\boldsymbol{P} = \mathrm{diag}\{[p_1, \ldots, p_K]\}$ *and* $\boldsymbol{Q} = \mathrm{diag}\{[q_1, \ldots, q_K]\}$, *respectively, then* $\varepsilon_i^{\mathrm{DL}} = \varepsilon_i^{\mathrm{UL}}$, $\forall i \in \{1, 2, \ldots, K\}$.

Proof. The uplink SINR is given by

$$\mathrm{SINR}_i^{\mathrm{UL}} = \frac{p_i \boldsymbol{u}_i^* \boldsymbol{h}_i \boldsymbol{h}_i^* \boldsymbol{u}_i}{\boldsymbol{u}_i^* \boldsymbol{Z}_i \boldsymbol{u}_i} \;, \tag{10}$$

where $\boldsymbol{Z}_i = \boldsymbol{Z} - p_i \boldsymbol{h}_i \boldsymbol{h}_i^*$. The downlink SINR is

$$\begin{aligned}\mathrm{SINR}_i^{\mathrm{DL}} &= \frac{q_i \boldsymbol{h}_i^* \boldsymbol{u}_i \boldsymbol{u}_i^* \boldsymbol{h}_i}{\sum_{\substack{j=1 \\ j \ne i}}^K q_j \boldsymbol{h}_i^* \boldsymbol{u}_j \boldsymbol{u}_j^* \boldsymbol{h}_i + \sigma_n^2} \\ &= \frac{q_i \boldsymbol{u}_i^* \boldsymbol{h}_i \boldsymbol{h}_i^* \boldsymbol{u}_i}{\sum_{j=1}^K q_j \boldsymbol{h}_i^* \boldsymbol{u}_j \boldsymbol{u}_j^* \boldsymbol{h}_i - q_i \boldsymbol{u}_i^* \boldsymbol{h}_i \boldsymbol{h}_i^* \boldsymbol{u}_i + \sigma_n^2} \\ &= \frac{\boldsymbol{u}_i^* \boldsymbol{h}_i \boldsymbol{h}_i^* \boldsymbol{u}_i}{\boldsymbol{h}_i^* \boldsymbol{U}\boldsymbol{Q}\boldsymbol{U}^* \boldsymbol{h}_i / q_i - \boldsymbol{u}_i^* \boldsymbol{h}_i \boldsymbol{h}_i^* \boldsymbol{u}_i + \sigma_n^2 / q_i} \;.\end{aligned} \tag{11}$$

With (10), (11) and the given condition $\mathrm{SINR}_i^{\mathrm{DL}} = \mathrm{SINR}_i^{\mathrm{UL}}$, we get

$$\boldsymbol{h}_i^* \boldsymbol{U}\boldsymbol{Q}\boldsymbol{U}^* \boldsymbol{h}_i / q_i - \boldsymbol{u}_i^* \boldsymbol{h}_i \boldsymbol{h}_i^* \boldsymbol{u}_i + \sigma_n^2 / q_i = \boldsymbol{u}_i^* \boldsymbol{Z}_i \boldsymbol{u}_i / p_i \;. \tag{12}$$

Substituting $\boldsymbol{Z}_i = \boldsymbol{Z} - p_i \boldsymbol{h}_i \boldsymbol{h}_i^*$ into (12) and multiplying with β_i^2 on both sides, we obtain

$$\beta_i^2 \boldsymbol{u}_i^* \boldsymbol{Z} \boldsymbol{u}_i / p_i = \beta_i^2 \boldsymbol{h}_i^* \boldsymbol{U}\boldsymbol{Q}\boldsymbol{U}^* \boldsymbol{h}_i / q_i + \beta_i^2 \sigma_n^2 / q_i \;. \tag{13}$$

Combining (13), (7) and (8), we can conclude that

$$\begin{aligned}\varepsilon_i^{\mathrm{DL}} &= \beta_i^2 \boldsymbol{u}_i^* \boldsymbol{Z} \boldsymbol{u}_i / p_i - \beta_i \boldsymbol{h}_i^* \boldsymbol{u}_i - \beta_i \boldsymbol{u}_i^* \boldsymbol{h}_i + 1 \\ &= \varepsilon_i^{\mathrm{UL}} \;.\end{aligned} \tag{14}$$

∎

It can be observed that the scaling matrix $\boldsymbol{\beta}$ does not change the SINRs, but affects the individual normalized MSE.

Combining Lemma 1 with the uplink/downlink duality [9,11], we can show that uplink and downlink have the same MSE achievable region.

Theorem 1. *Given $\boldsymbol{U}$, $\boldsymbol{\beta}$, and a total power limit $P_{\max}$, normalized MSE values $\varepsilon_1, \dots, \varepsilon_K$ can be achieved in the uplink if and only if the same values can be achieved in the downlink. Thus, both links have the same achievable region under a sum power constraint.*

Proof. Suppose that uplink MSEs $\varepsilon_1, ..., \varepsilon_K$, are achieved with $\boldsymbol{U}$, $\boldsymbol{\beta}$ and power allocation $\boldsymbol{p}$, $\|\boldsymbol{p}\|_1 \leq P_{\max}$. This signaling strategy is associated with uplink SINRs $\gamma_1, ..., \gamma_K$. Thus, $\boldsymbol{p}$ is characterized by

$$\boldsymbol{p} = \sigma_n^2 (\boldsymbol{D}^{-1} - \boldsymbol{\Psi}^T)^{-1} \mathbf{1}_K \ ,$$

where $\mathbf{1}_K$ is the all-one vector and

$$\boldsymbol{D} = \mathrm{diag}\Big\{\Big[\frac{\gamma_1}{|\boldsymbol{u}_1^* \boldsymbol{h}_1|^2}, \dots, \frac{\gamma_K}{|\boldsymbol{u}_K^* \boldsymbol{h}_K|^2}\Big]\Big\},$$

and

$$\boldsymbol{\Psi}_{ik} = \begin{cases} |\boldsymbol{u}_k^* \boldsymbol{h}_i|^2 & k \neq i \\ 0 & k = i \ . \end{cases}$$

We have [9,12]

$$\begin{aligned} \|\boldsymbol{p}\|_1 &= \mathbf{1}_K^T \boldsymbol{p} = \sigma_n^2 \mathbf{1}_K^T \big((\boldsymbol{D}^{-1} - \boldsymbol{\Psi})^{-1}\big)^T \mathbf{1}_K \\ &= \sigma_n^2 \mathbf{1}_K^T (\boldsymbol{D}^{-1} - \boldsymbol{\Psi})^{-1} \mathbf{1}_K = \mathbf{1}_K^T \boldsymbol{q} = \|\boldsymbol{q}\|_1, \end{aligned} \tag{15}$$

where $\boldsymbol{q}$ is the downlink power allocation that achieves the same SINR values $\gamma_1, \dots, \gamma_K$ with the same filter $\boldsymbol{U}$ and with the same total power, i.e., $\|\boldsymbol{q}\|_1 \leq P_{\max}$. (Note that the scaling matrix $\boldsymbol{\beta}$ does not change the SINRs.) From Lemma 1 we know that $\boldsymbol{U}$, $\boldsymbol{\beta}$ and $\boldsymbol{q}$ achieve the same individual normalized MSEs $\varepsilon_1, ..., \varepsilon_K$ in the downlink. Therefore, if any individual normalized MSEs $\varepsilon_1, ..., \varepsilon_K$ are achievable in the uplink with total transmission power $P_{\max}$, then $\varepsilon_1, ..., \varepsilon_K$ are also achievable in the downlink under the same power constraint.

Conversely, it can be shown by the same reasoning, that each point from the downlink region is achievable in the uplink. ■

3. OPTIMIZATION STRATEGIES

An immediate consequence of Theorem 1 is that the optimization problems P1 (3) and P2 (4) can be solved by optimizing the mean square errors of the equivalent uplink system described in Section 2.1. The optimum equals the optimum of the original downlink problem. Moreover, the optimizer always has the familiar structure (9), which minimizes the uplink MSE for a given power allocation. Thus, a general framework for joint MSE transmitter design is as follows:

1. formulate the equivalent uplink problem and jointly optimize $\boldsymbol{U}$, $\boldsymbol{p}$, and $\boldsymbol{\beta}$ subject to a total power constraint.
2. compute associated SINR values $\gamma_i = \mathrm{SINR}_i(\boldsymbol{u_i}, \boldsymbol{p})$, $\forall i$.
3. compute the downlink power allocation which fulfills the targets obtained by step 2:

$$\boldsymbol{q} = \sigma_n^2 \big(\boldsymbol{D}^{-1}(\gamma_1, \dots, \gamma_K) - \boldsymbol{\Psi}(\boldsymbol{U})\big)^{-1} \mathbf{1}_K.$$

This power allocation together with $\boldsymbol{U}$ and $\boldsymbol{\beta}$ achieves the optimal downlink MSE according to the respective problem formulation. Having derived this duality framework, we are now able to propose algorithmic solutions for the problems P1 and P2.

3.1. P1: Best Overall Efficiency

Let us first consider the problem P1 in the uplink. We begin with a fixed power allocation $\boldsymbol{P}$. In this case, the minimum individual normalized MSEs and the maximum SINRs are achieved simultaneously by the MMSE beamformers (9). The minimum MSE is related to the maximum SINR (see e.g. [1]) as follows

$$\varepsilon_i^{\mathrm{UL,min}} = \frac{1}{1 + \mathrm{SINR}_i^{\mathrm{UL,max}}}, \ \ i = 1, ..., K. \tag{16}$$

The normalized MSE can be further minimized if we apply power control. Since the normalized MSE can be expressed as

$$\begin{aligned} \mathrm{NMSE}^{\mathrm{UL}} &= \mathrm{E}\{\|\hat{\boldsymbol{d}} - \boldsymbol{d}\|^2\} = \sum_{i=1}^{K} \varepsilon_i^{\mathrm{UL}} \\ &= K - M + \sigma_n^2 \mathrm{Tr}([\boldsymbol{HPH}^* + \sigma_n^2 \boldsymbol{I}]^{-1}) \ , \end{aligned} \tag{17}$$

the optimization problem P1 can be reformulated as

$$\begin{aligned} &\min_{p_1, \dots, p_K} \ \mathrm{Tr}([\boldsymbol{HPH}^* + \sigma_n^2 \boldsymbol{I}]^{-1}) \\ &\text{s.t.} \sum_{i=1}^{K} p_i = P_{\max}, \ \text{ and } \ p_i \geq 0, 1 \leq i \leq K \ . \end{aligned} \tag{18}$$

This problem is convex with respect to the power allocation. So it can be easily solved by interior point methods. Assume the optimal power allocation is $\boldsymbol{P}^{\mathrm{opt}}$, then the optimal MMSE filter is $\widetilde{\boldsymbol{U}}_{\mathrm{mmse}}^{\mathrm{opt}} = (\boldsymbol{HP}^{\mathrm{opt}}\boldsymbol{H}^* + \sigma_n^2 \boldsymbol{I})^{-1} \boldsymbol{HP}^{\mathrm{opt}} = \boldsymbol{U}_{\mathrm{mmse}}^{\mathrm{opt}} \boldsymbol{\beta}_{\mathrm{mmse}}^{\mathrm{opt}}$.

Note that the scaling matrix $\boldsymbol{\beta}_{\mathrm{mmse}}^{\mathrm{opt}}$ plays an important role in minimizing the individual normalized MSE.

3.2. P2: Fairness

Problem P2 was studied for the more general MIMO case in [2]. It was shown that the optimum is characterized by equal MSE and the power constraint is fulfilled with equality. For each power allocation, the filter $\widetilde{\boldsymbol{U}}$ has the optimal structure (9), in which case the MSE is given by (16) as a function of the transmit powers. Thus, balancing the MSE is equivalent to balancing the quantities $\mathrm{SINR}_i^{\mathrm{UL,max}}$, $\forall i$. An algorithmic solution of this problem was recently proposed in [9]. The algorithm converges monotonically to the global optimum. Each iteration consists of the following steps:

1. for given $\boldsymbol{p}$ update the filters with (9).
2. for given $\boldsymbol{U}$, compute the new powers $\boldsymbol{p}$ by eigendecomposition

$$\begin{bmatrix} \boldsymbol{D\Psi}^T(\boldsymbol{U}) & \boldsymbol{D}\mathbf{1}_K \sigma_n^2 \\ \frac{1}{P_{\max}} \mathbf{1}_K^T \boldsymbol{D\Psi}^T(\boldsymbol{U}) & \frac{1}{P_{\max}} \mathbf{1}_K^T \boldsymbol{D} \mathbf{1}_K \sigma_n^2 \end{bmatrix} \begin{bmatrix} \boldsymbol{p} \\ 1 \end{bmatrix} = \lambda_{\max} \begin{bmatrix} \boldsymbol{p} \\ 1 \end{bmatrix}$$

Having found the optimal uplink power allocation $\boldsymbol{p}^{\mathrm{bal}}$ and the associated filter $\widetilde{\boldsymbol{U}}_{\mathrm{mmse}}^{\mathrm{bal}}$, we can proceed with the steps 2 and 3 listed at the beginning of Section 3.

3.3. Simulation Results

We illustrate the solutions of P1 and P2 and compare our strategies with the TxWF [8]. Consider the following channel realization

$$\boldsymbol{H} = \begin{bmatrix} -0.0420 & 0.0994 & 0.3254 & -1.7313 \\ 0.3240 & -0.1164 & -0.0952 & 0.4788 \\ 0.5065 & 0.6892 & 0.0312 & -0.4478 \\ -1.0286 & 1.8833 & -0.6138 & 0.3868 \end{bmatrix} . \tag{19}$$

In the simulation, independent unity-energy data streams and same total transmission powers are assumed for different schemes. Fig. 3 shows the normalized MSE versus the total transmission power over noise variance. The line with circles denotes the total normalized MSE for P1, which is the minimum that can be achieved in both uplink and downlink for the given channel realization (19) and total transmission powers. It can be seen that there is a transition between 12dB and 14dB. Before the transition, user 3 is switched off due to the bad channel quality, and after that, the total transmission power is high enough to support all users. We can see that the normalized MSEs for P2 and TxWF are larger. For fairness of comparison with TxWF, user 3 is also switched off for low powers, which is denoted by TxWF2.

However, the optimal power allocation which minimizes the normalized MSE in most cases is unfair for the users who experience bad channels. This can be observed in Fig. 4. At $P_{\max}/\sigma_n^2 =$ 10dB, user 3 is switched off, and at $P_{\max}/\sigma_n^2 =$ 20dB, user 3 suffers a larger normalized MSE compared with other users, especially with user 2. In the MSE balancing case, all users have the same individual normalized MSE. For TxWF, the individual normalized MSEs are neither balanced nor is the sum minimized.

4. CONCLUSIONS

We have studied the problem of MSE transmit optimization for a multi-antenna transmitter and several independent single-antenna receivers under a sum power constraint. This model also holds for multi-user MIMO with fixed receivers. One main result of the paper is to show that the achievable region in terms of normalized linear MSE is the same as the region of an equivalent uplink problem, which is obtained by switching the role of transmitter and receivers. Thereby, the optimal transmit strategy can be found indirectly, by solving an equivalent problem. Many MSE optimization strategies, which are known for joint reception, can be transferred to the joint transmit problem. This is a big advantage since the downlink is difficult to handle directly. Examples are the min-max-MSE and sum-MSE optimization problems, for which we have provided algorithmic solutions in Section 3.

This "duality" between the normalized MSE regions extends previous results [9, 12, 13], where a similar relationship was found in the context of SINR optimization. The close relationship between the maximum SINR and the minimum MSE is well known for the uplink. An interesting consequence of the results in this paper is that a similar link exists between the downlink quantities. This is an immediate consequence of both duality relationships.

Other forms of duality have already been observed, e.g. in the context of information theoretic capacity [11]. An interesting topic for future work will be to prove whether or not a similar duality holds for other scenarios, e.g. the capacity of MIMO multiuser systems with linear processing.

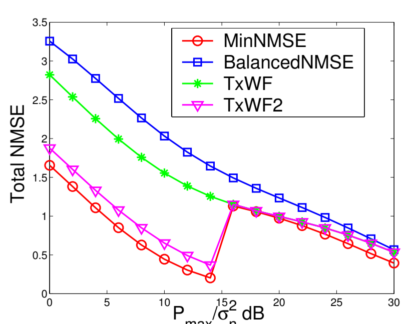

Fig. 3. Normalized MSE for different $P_{\max}/\sigma_n^2$

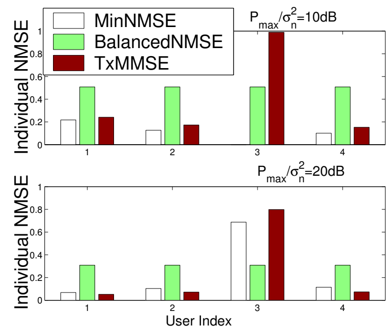

Fig. 4. Individual normalized MSEs $P_{\max}/\sigma_n^2 =$ 10dB, 20dB

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