CAPACITY OF SPATIO-TEMPORALLY STRUCTURED MIMO CHANNELS WITH ESTIMATION ERRORS

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ABSTRACT

Practical MIMO channels often exhibit structure in both space and time, i.e. a spatio-temporal structure. The potential of exploiting this structure in training based schemes is studied using a common ray-based channel model that captures parts of the structure observed in measurements. A lower bound, the Cramér-Rao lower bound (CRB), on the channel estimation error and a lower bound on the capacity are used to study the potential gain in exploiting channel structure. It is found that the training based capacity may be substantially increased since a more parsimonious channel model with less parameters to estimate can be used. Numerical evaluations indicate that the capacity grows with the number of antennas similar to the case of a known channel if the structure is exploited. If it is not exploited, the training-based capacity reaches a maximum after which it decreases with the number of antennas. Furthermore, the temporal structure can be used to interpolate or predict the channel between training instants and it is found that prediction can improve performance for training based schemes.

1. INTRODUCTION

Using multiple antennas at both the transmitter and receiver or so called Multiple-Input Multiple-Output (MIMO) systems have been found to greatly increase the data rate compared to single antenna systems [1, 2]. Initially, most of the work was focused on the case where the channel is known at the receiver but the case with no Channel State Information (CSI) at the receiver (CSIR) has been addressed more recently. The most common approach is to use training symbols to obtain an estimate of the channel at the receiver and then detect the data symbols. Although not required by Shannon theory many practical wireless communication systems use training since it simplifies the receiver design.

Several recent publications have investigated the impact of training on the channel capacity [3]- [10] for a Gaussian distributed channels where each channel coefficient is considered an unknown parameter. However, most practical channels obtained in channel sounding experiments exhibit a structure in both time and space, i.e. a spatio-temporal structure. The potential benefit of exploiting this structure is investigated in this paper by using a common ray-based channel model [11] that captures part of the spatio-temporal structure observed in measurements. By analyzing the training-based capacity for this channel model, the fundamental performance gain of exploiting structure in the channel can be studied. For example, the training-based capacity indicates that the number of training symbols can be reduced when the spatio-temporal structure is exploited. Furthermore, the impact of physical parameters such as antenna separation and number of multipaths can also be studied using the ray-based channel model.

The training-based capacity for the structured channel model is investigated using a lower bound on capacity that has been derived in several recent publications [6, 7, 12]. This bound essentially depends on the channel estimation and training scheme through the channel estimation error covariance matrix. Since the aim of this paper is to study the fundamental limit of exploiting channel structure, the CRB on the estimation error is used which is valid of any unbiased estimator. It is found that the estimation error can be significantly reduced and the capacity increased by exploiting the structure of the ray-based channel model. This is mainly due to the fact that this model contains fewer unknown parameters than a direct parameterization of the channel coefficients. Furthermore, for time varying channels, this parsimonious model can be used to predict or interpolate the channel between training instants. Numerical evaluations where the CRB for channel prediction is inserted into the capacity lower bound indicate that the training-based capacity in moderately fast varying channels can be substantially increased.

2. SYSTEM MODEL

Consider a flat-fading MIMO system with M_t transmit and M_r receive antennas. The received signal at time t can be modeled as

$$\mathbf{y}(t) = \mathbf{H}(t)\mathbf{s}(t) + \mathbf{n}(t), \tag{1}$$

where the transmitted signal $\mathbf{s}(t)$ is an $M_t \times 1$ vector and the additive noise $\mathbf{n}(t)$ is an $M_r \times 1$ vector. The channel is modeled using a ray-based narrowband MIMO channel model which several recent measurement campaigns have used to describe and analyze measured data [13, 14]. Variations of this model have also been used in many other studies, see [11] for more details. The channel matrix at time t, $\mathbf{H}(t)$, is modeled as the sum of the contributions of L different plane waves (paths) as

$$\mathbf{H}(t) = \frac{1}{\sqrt{L}} \sum_{l=1}^{L} \alpha_l \mathbf{a}_{r,l} \mathbf{a}_{t,l}^T e^{j\omega_l t}, \tag{2}$$

where α_l is the scattering coefficient of path l, ω_l is the Doppler frequency of path l at time t, $\mathbf{a}_{r,l}$ is the steering vector at the receiving array associated with the l^{th} path, and $\mathbf{a}_{t,l}$ is the corresponding vector for the transmitter. Although the analysis in the

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following sections is valid for any antenna arrangement, a Uniform Linear Array (ULA) will be used in the simulation section since the steering vector, in this case, exhibits a simple Vandermonde structure

$$\mathbf{a}_{t,l}^T = \begin{bmatrix} 1 & e^{j\Omega_{t,l}} & e^{j2\Omega_{t,l}} & \dots & e^{j(M_t-1)\Omega_{t,l}} \end{bmatrix}, \quad (3)$$

where the angular frequency of path l $\Omega_{t,l}$ is modeled as $\Omega_{t,l} = 2\pi d_t \sin\phi_{t,l}$ where d_t is the element separation distance in wavelengths and $\phi_{t,l}$ is the Direction Of Departure (DOD) at the transmitter.

In the following analysis it is assumed that the underlying parameters such as DODs and scattering coefficients are constant for N time instants and then change into new values. During a block of N symbols, N_t training symbols are used for channel estimation and $N_d = N - N_t$ data symbols are transmitted. Since the channel changes within each block but the parameters remain the same, this channel can be viewed as an extension of the typical block-fading channel [1, 6].

At each time instant t the transmitted symbol $\mathbf{s}(t)$ can be either a data symbol $\mathbf{s}_d(t)$ or a training symbol $\mathbf{s}_t(t)$. In the following, the training and data parts of the transmitted and received signals are stacked into matrices $\mathbf{Y}_t$, $\mathbf{S}_t$ and $\mathbf{Y}_d$, and $\mathbf{S}_d$. No CSI at the transmitter is assumed and a zero-mean complex normal distribution is assumed for the data part of the signal $\mathbf{S}_d$ of size $M_t \times N_T$ and $E[\mathbf{S}_d\mathbf{S}_d^H] = N_t\sigma_{S_d}^2/M_t\mathbf{I}_{S_d}$ where $\cdot^H$ denotes complex conjugate transpose. The training part of the signal is normalized in the same way. Furthermore, the stacked version of the additive noise $\mathbf{N}$ of size $M_R \times N$ is assumed to be complex normal distributed $\mathbf{N} \in \mathcal{CN}(\mathbf{O}, \sigma_N^2\mathbf{I})$.

3. CAPACITY LOWER BOUND AND CRB

Obtaining the capacity of training based systems is in general untractable since a complete statistical description of the channel estimate and transmitted and received signals is not available. However, lower bounds on the training based capacity has recently been modified to MIMO channels [6, 7, 12] where the true channel is divided into two parts; A known channel estimate $\hat{\mathbf{H}}(t)$ and an unknown random error term $\tilde{\mathbf{H}}(t)$. Using (1), this can be expressed as [6, 7, 12]

$$\mathbf{Y} = \mathbf{HS} + \mathbf{N} = \hat{\mathbf{H}}\mathbf{S} + \tilde{\mathbf{H}}\mathbf{S} + \mathbf{N} = \hat{\mathbf{H}}\mathbf{S} + \mathbf{N}_e, \quad (4)$$

where $\mathbf{N}_e = \mathbf{N} + \tilde{\mathbf{H}}\mathbf{S}$ denotes an effective noise term. Hence, signaling over a channel with partial channel knowledge can be interpreted as signaling on a channel equal to the channel estimate but with an effective noise term $\mathbf{N}_e$. Assuming no CSIT and that the power is distributed equally over antennas and time, the capacity can be lower bounded

$$C \geq C_b = \frac{1}{N}\sum_{n=1}^{N_d} E_{\hat{H}}\left[\log\left|\mathbf{I} + \mathbf{\Upsilon}^{-1}\hat{\mathbf{H}}(t_n)\hat{\mathbf{H}}^H(t_n)\right|\right], \quad (5)$$

where $t_n \in [1, N]$, $\mathbf{\Upsilon} = \mathbf{R}_{\tilde{H}(t_n)} + M_t/\rho\mathbf{I}$, and $\rho = \sigma_{sd}^2/\sigma_N^2$ denotes the SNR. In achieving the bound in (5), the receiver assumes that the effective noise is Gaussian with spatial covariance $\mathbf{\Upsilon}$ and independent of the transmitted data. If the effective noise is not Gaussian, equation (5) represents a lower bound since Gaussian noise represents a worst-case noise. Since the potential performance of exploiting channel structure is investigated and not the performance of a particular estimator, the CRB that provides a lower bound of the error covariance $\mathbf{R}_{\tilde{H}}$ for any unbiased estimator will be used.

The CRB is found in a manner similar to [15] for the data model in (2) with parameters $\boldsymbol{\theta} = [\mathrm{Re}(\boldsymbol{\alpha}), \mathrm{Im}(\boldsymbol{\alpha}), \boldsymbol{\omega}, \mathbf{\Omega}_t, \mathbf{\Omega}_r]$ where $\boldsymbol{\alpha}$ is defined as $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \dots, \alpha_L]^T$ and the other parameters are defined similarly. Note that the total number of real unknowns is $5L$ and that the noise variance is omitted since the estimation of the noise variance decouples from the other parameters. In the following it is convenient to use a vectorized version of the channel model $\tilde{\mathbf{h}}(t) = \mathrm{vec}\left[\tilde{\mathbf{H}}(\mathrm{t})\right]$. Using a vector formulation of the CRB for functions of parameters, the $M_tM_r \times M_tM_r$ covariance matrix of the estimation error can be lower bounded as

$$\mathbf{R}_{\tilde{h}(t_t)} = E\left[\tilde{\mathbf{h}}(t_t)\tilde{\mathbf{h}}^H(t_t)\right] = E\left[\mathbf{H}'(t_t)\mathbf{B}\mathbf{H}'^H(t_t)\right], \quad (6)$$

where $\mathbf{H}'$ is a $M_tM_r \times 5L$ Jacobian matrix and $\mathbf{B}$ is the CRB of the parameters $\boldsymbol{\theta}$ which is given in the Appendix. The expression for the covariance of the estimation error is now obtained as

$$\mathbf{R}_{\tilde{H}(t_t)} = E\left[\tilde{\mathbf{H}}(t_t)\tilde{\mathbf{H}}^H(t_t)\right] = \sum_{k=1}^{M_t}\mathbf{E}_k\mathbf{R}_{\tilde{h}(t_t)}\mathbf{E}_k^H, \quad (7)$$

where the selection matrix $\mathbf{E}_k$ is defined as a $M_r \times M_tM_r$ matrix of zeros except for the k[th] block containing a unit matrix, i.e. $[\mathbf{E}_k]_{1:M_r,(k-1)M_t+1:kM_t} = \mathbf{I}_{M_r}$ using Matlab notation. Finding an analytic expression for (6)-(7) is in general difficult. A numerical Monte-Carlo approach will therefore be used to obtain initial channel capacity results.

The expression in (7) captures the error when training symbols are used to estimate the model in (2) which then is evaluated for the data transmission times. On the other hand, if no attempt is done to interpolate or predict the channel between training symbols, the channel estimate will become outdated before the next training segment. In that case, a second error term $\mathbf{R}_{\text{out}} = E\left[(\mathbf{H}(t) - \mathbf{H}(t_t))(\mathbf{H}(t) - \mathbf{H}(t_t))^H\right]$ that accounts for the aging of the channel estimate appears in (7) .

4. NUMERICAL RESULTS

In this section, the capacity lower bound (5) will be evaluated by calculating the CRB lower bound and the channel estimate $\hat{\mathbf{H}}$. In evaluating the CRB, the scattering parameters $\boldsymbol{\alpha}$ are assumed to be complex Gaussian distributed $\boldsymbol{\alpha} \in \mathcal{CN}(0, \mathbf{I}_L)$. The Doppler frequencies ω_l are defined as $\omega_l = 2\pi f_d \sin\phi_l$ where f_d is the Doppler frequency normalized by the symbol rate. A uniform distribution for the angle between the propagation path and the direction of travel $\phi_l \in U[0, 2\pi)$ will be used although any angular distribution could be used in the analysis. The DOD and DOA are defined as $\Omega_{t,l} = 2\pi d_t \sin\phi_{t,l}$ and $\Omega_{r,l} = 2\pi d_r \sin\phi_{r,l}$ where the angles are assumed to be distributed according to a uniform $\phi_{t,l}, \phi_{r,l} \in U[0, 2\pi)$ distribution. Of course, other angular densities can also be explored but only these distributions will be studied here to reduce the number of scenario parameters. To avoid potentially unidentifiable scenarios, the parameter realizations with DOAs, DODs, or Doppler frequencies that essentially overlap are removed. This corresponds to a practical situation where two closely located clusters of scatterers can be combined into one [15]. For the above parameter distributions, the outdated error term becomes $\mathbf{R}_{\text{out}} = 2M_t\left(1 - J_0(2\pi f_d(t - t_t))\right)\mathbf{M}(d_r)$ where $\mathbf{M}(d_r)$ is defined as $[\mathbf{M}(d_r)]_{i,j} = J_0(2\pi d_r(i - j))$ and $J_0(x)$ denotes the zeroth order Bessel function of the first kind. In

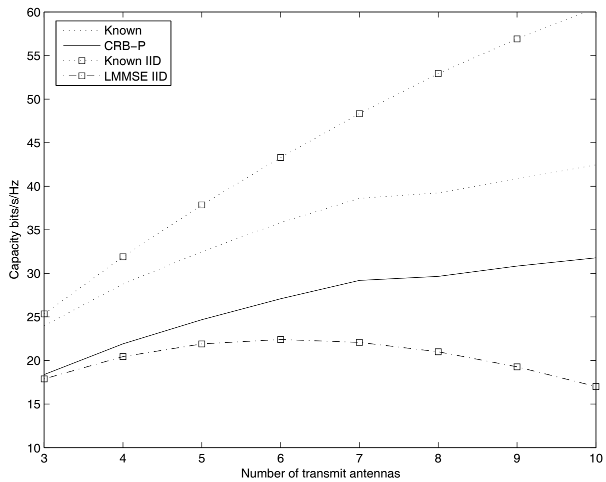

Fig. 1. Capacity lower bound as a function of the number of transmit antennas M_t with $\rho = 20$dB, $M_r = 12$, $f_d = 0.001$, and $L = 10$.

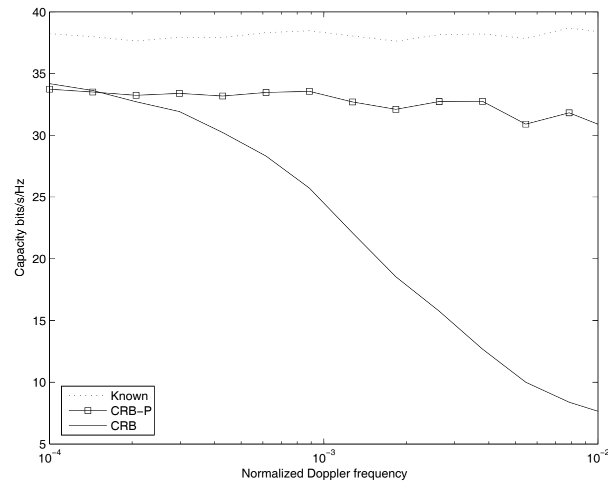

Fig. 2. Capacity lower bound as a function of the normalized Doppler frequency f_d with $\rho = 20$dB, $M_t = 6$, $M_r = 12$, and $L = 13$.

the following simulations $d_t = d_r = 5\lambda$, and an SNR of 20dB is assumed.

In order to evaluate (5), the value of the channel estimate $\hat{\mathbf{H}}$ is also needed but only a lower bound on the variance of the channel estimation error is available. For the case of interest where training is a suitable choice, the channel estimation error is substantially smaller than the channel itself and it is assumed that $\hat{\mathbf{H}} \approx \mathbf{H}$. Hence, additional effects on the effective SNR through the variance of the channel estimate are neglected [5]. Note that calculating the variance of the channel estimate requires that the statistical properties of the particular channel estimator are known. Although not reported here, simulations were performed for estimators with different statistical properties and the results showed similar qualitative behavior to those detailed below.

The benefit of exploiting channel structure is first studied in Figure 1 where the capacity lower bound (5) is plotted versus the number of transmit antennas M_t when the number of receive antennas is held fixed $M_r = 12$. A sub-block length of 15 symbols is used of which three symbols are used for training. Although shorter than a typical sub-block, it illustrates the general behavior of the system as a function of M_t. However, it is also assumed that the underlying parameters are the same for four sub-blocks so that previous training symbols can be used to improve performance. The results shown in Figure 1 represents the average for 100 blocks with a normalized Doppler frequency $f_d = 0.001$ and $L = 10$. For comparison, the performance of the LMMSE estimator in an i.i.d. Gaussian channel [6] and the capacity in that case with CSIR is also shown. Note that only the behavior is compared and not the actual values since the capacity lower bounds are obtained under different assumptions. For instance, the maximum rank of the structured channel matrix is $\min(L, M_t, M_r)$ so the capacity reaches a treshold at ten transmit antennas in this case. On the other hand, the i.i.d. channel, to which the LMMSE is applied, keeps growing with the number of transmit antennas.

For the structured channel, it is clear that the capacity grows with the number of transmit antennas in a manner similar to the case with CSIR. This is due to the fact that the number of unknown parameters does not grow with the number of transmit antennas unlike the case of an unstructured model where the number of parameters is $M_t M_r$. The number of training symbols used in the LMMSE simulation is M_t since that was found to be optimal (in LMMSE training-based capacity sense) in [6]. The number of transmit antennas that maximizes the capacity lower bound for the structured channel therefore differs from that obtained in the analysis of [6]. For a structured channel, higher capacities can be obtained by adding more transmit antennas if there is enough multipath and receive antennas. Since most practical channels do exhibit some structure, the analysis of the capacity lower bound indicates that training may still be a possible solution for systems with many antennas or rapidly varying channels if the channel structure can be exploited.

The impact of temporal structure is examined in Figure 2 where the capacity bound is evaluated versus normalized Doppler frequency. In this case, there are three sub-blocks each one hundred symbols long with three training symbols. It is clear that applying channel prediction can substantially improve the performance compared to traditional training. In principle, the second term in the error covariance that accounts for the aging of the channel estimate $\mathbf{R}_{\text{out}}$ dominates the no-prediction capacity for higher Doppler frequencies. It is important to note that the plot represents a fundamental upper limit of using channel prediction since the CRB is used to bound the variance and there is no modeling error. However, the results do indicate that significant gains can be achieved using channel prediction at moderately fast varying channels where training might be considered. Significant gains have already been observed for unstructured channel prediction and tracking schemes [16, 17]. Investigating MIMO channel prediction and tracking schemes that exploit channel structure is therefore interesting since training and channel state information is more important in MIMO than SISO systems.

5. CONCLUSIONS

Several recent publications have investigated the impact of training on the capacity using a lower bound on the capacity where each channel coefficient is an unknown parameter. In practice the MIMO channel will exhibit a spatio-temporal structure and the gain of exploiting this structure was studied in this paper. A common ray-based channel model that captures parts of this structure and a lower bound (CRB) on the channel estimation error was used to study the potential gain of exploiting channel structure. The ray-based channel model also allows for studies of the impact of physical parameters such as antenna separation and the number of multipaths. It was found that significantly better performance can be obtained by exploiting channel structure since a more parsimonious channel model with less parameters to estimate can be used. Numerical evaluations indicated that the capacity grows with the number of antennas similar to the case of a known channel if the structure is exploited. This is different from earlier analysis of unstructured channels [6] where it was found that the training-based capacity reaches a maximum after which it decreases with the number of antennas.

Furthermore, the temporal structure can be used to interpolate or predict the channel between training instants. Numerical evaluations of the capacity lower bound indicated that prediction can improve performance for training based schemes for moderately fast varying channels. Future work includes analysis of the number and placement of the training symbols to maximize capacity and other ways of including channel information or constraints that might lower the CRB [18].

APPENDIX A.

For the data model given in Section 2, the received signal $\mathbf{y}(t)$ at training instants is a Gaussian distributed random variable with mean $\boldsymbol{\mu}_n = \boldsymbol{\mu}(t_n) = \left(\mathbf{s}_t^T(t_n) \otimes \mathbf{I}_{Mr}\right) \mathbf{h}(t_n)$ and covariance $\mathbf{C} = \mathbf{C}(t_n) = \sigma_n \mathbf{I}$. Assuming that the noise at different times is independent, the CRB for the parameters θ becomes $\mathbf{R}_\theta \geq \mathbf{B}_\theta = \mathbf{J}^{-1}$, where the Fisher information matrix is

$$[\mathbf{J}_d]_{p,q} = \sum_{n=1}^{N_t} 2\mathrm{Re}\left(\frac{\partial \boldsymbol{\mu}_n^H}{\partial \theta_p}\mathbf{C}^{-1}\frac{\partial \boldsymbol{\mu}_n}{\partial \theta_q}\right). \tag{A.1}$$

Since $\partial\boldsymbol{\mu}_n/\partial\theta_p = \left(\mathbf{s}_t^T(t_n) \otimes \mathbf{I}_{Mr}\right) \partial\mathbf{h}(t_n)/\partial\theta_p$, all that is needed to evaluate the CRB bound in (A.1) is to differentiate the channel. Straightforward calculations give

$$\begin{aligned} \partial\mathbf{h}(t)/\partial\mathrm{Im}[\alpha_k] &= j\partial\mathbf{h}(t)/\partial\mathrm{Re}[\alpha_k] = je^{j\omega_k t}\mathbf{a}_{t,k} \otimes \mathbf{a}_{r,k}/\sqrt{L} \\ \partial\mathbf{h}(t)/\partial\omega_k &= jt\alpha_k e^{j\omega_k t}\mathbf{a}_{t,k} \otimes \mathbf{a}_{r,k}/\sqrt{L} \\ \partial\mathbf{h}(t)/\partial\Omega_{t,k} &= \alpha_k e^{j\omega_k t}\mathbf{d}_{t,k} \otimes \mathbf{a}_{r,k}/\sqrt{L} \\ \partial\mathbf{h}(t)/\partial\Omega_{r,k} &= \alpha_k e^{j\omega_k t}\mathbf{a}_{t,k} \otimes \mathbf{d}_{r,k}/\sqrt{L}, \end{aligned} \tag{A.2}$$

where $\mathbf{d}_{t,k} = \partial\mathbf{a}_{t,k}/\partial\Omega_{t,k}$ and $\mathbf{d}_{r,k}$ is defined similarly. Although the derivation is valid for any antenna arrangement, only ULAs will be simulated in order to reduce the number of scenario parameters.

6. REFERENCES

[1] G.J. Foschini and M.J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Communications*, vol. 6, pp. 311–335, March 1998.

[2] E. Telatar, "Capacity of multi-antenna gaussian channels," Tech. Rep., AT&T Bell Labs, June 1995.

[3] J. Zheng and B.D. Rao, "Capacity analysis of MIMO systems with unknown channel state information," in *Proc. Information Theory Workshop 2004*, October 2004.

[4] S. Adireddy, L. Tong, and H. Viswanathan, "Optimal placement of training for frequency-selective block-fading channels," *IEEE Trans. on Information Theory*, vol. 48, no. 8, pp. 2338–2353, August 2002.

[5] J. Baltersee, G. Fock, and H. Meyr, "Achievable rate of MIMO channels with data-aided channel estimation and perfect interleaving," *IEEE Journal on Selected Areas in Comm.*, vol. 19, no. 12, pp. 2358–2368, December 2001.

[6] B. Hassibi and B.M. Hochwald, "How much training is needed in multiple-antenna wireless links?," *IEEE Trans. on Information Theory*, vol. 49, no. 4, pp. 951–963, April 2003.

[7] M. Medard, "The effect upon channel capacity in wireless communications of perfect and imperfect knowledge of the channel," *IEEE Trans. on Information Theory*, vol. 46, no. 3, pp. 933–946, May 2000.

[8] S. Ohno and G.B. Giannakis, "Capacity maximizing MMSE-optimal pilots for wireless OFDM over frequency-selective block Rayleigh-fading channels," *IEEE Trans. on Information Theory*, vol. 50, no. 9, pp. 2138–2145, September 2004.

[9] A. Vosoughi and A. Scaglione, "Channel estimation for precoded MIMO systems," in *Proc. 2003 IEEE Workshop on Stat. Sig. Proc.*, September 2003, pp. 442–445.

[10] T. Yoo and A. Goldsmith, "Capacity of fading MIMO channels with channel estimation error," in *Proc. IEEE ICC 2004*, Paris, France, June 2004.

[11] K. Yu and B. Ottersten, "Models for MIMO propagation channels, a review," *Wiley Journal on Wireless Comm. and Mobile Comp.*, vol. 2, no. 7, pp. 653–666, Nov. 2002,

[12] D. Samardzija and N. Mandayam, "Pilot-assisted estimation of MIMO fading channel response and achievable data rates," *IEEE Trans. on Signal Processing*, vol. 51, no. 11, pp. 2882–2890, November 2003.

[13] A.F. Molisch, M. Steinbauer, M. Toeltsch, E. Bonek, and R.S Thomä, "Capacity of MIMO systems based on measured wireless channels," *IEEE Journal on Selected Areas in Communications*, vol. 20, no. 3, pp. 561–569, April 2002.

[14] J.W. Wallace, M.A. Jensen, A.L. Swindlehurst, and B.D. Jeffs, "Experimental characterization of the MIMO wireless channel: Data acquisition and analysis," *IEEE Trans. on Wireless Comm.*, vol. 2, no. 2, pp. 335–343, March 2003.

[15] T. Svantesson and A. L. Swindlehurst, "A performance bound for prediction of a multipath MIMO channel," in *Proc. 37th Asilomar Conf. Sig., Syst., Comput.*, Nov 2003.

[16] Z. Liu, X. Ma, and G.B. Giannakis, "Space-time coding and Kalman filtering for time-selective fading channels," *IEEE Trans. on Comm.*, vol. 50, no. 2, pp. 183–186, Feb 2002.

[17] G.E. Øien, H. Holm, and K.J. Hole, "Impact of channel prediction on adaptive coded modulation performance in Rayleigh fading," *IEEE Trans. on Vehicular Technology*, vol. 53, no. 3, pp. 758–769, May 2004.

[18] A.K. Jagannatham and B.D. Rao, "Complex constrained CRB and its applications to semi-blind MIMO and OFDM channel estimation," in *Proc. of the third IEEE Sensor Array and Multichannel Sig. Proc. Workshop*, July 2004.