ANALYSIS OF RICEAN MIMO CHANNELS BASED ON A VIRTUAL CHANNEL REPRESENTATION

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ABSTRACT

The ergodic capacity of Ricean MIMO (multiple-input multiple-output) fading channels is investigated in this paper, based on a virtual (Fourier) channel representation. Coherent reception is assumed at the receiver. The Ricean statistics are due to a line-of-sight (LOS) path joining the transmitter and receiver. In the virtual domain, this LOS path corresponds to a single entry in the virtual channel matrix. The optimal input distribution of the correlated Ricean MIMO channel in the virtual domain has a diagonal covariance matrix structure, the same as in Rayleigh fading MIMO case. Our method is used to get a tight lower bound and upper bound on the ergodic capacity. Further, the same analysis is used to obtain an approximate optimal input distribution analytically, in place of the usual Monte Carlo based numerical optimization methods.

1. INTRODUCTION

The use of multiple antennas can increase the capacity of wireless channels tremendously [1]. Most analyses of such multiple-input multiple-output (MIMO) channel capacities have been focussed on the Rayleigh fading MIMO case, and fewer results are available for Ricean MIMO channels. The Ricean statistics are due to a line-of-sight (LOS) path joining the transmitter and receiver. Since LOS paths frequently exist in practical channels, it is important to characterize their capacity as a function of the LOS path's relative strength and to investigate the optimal transmit covariance matrix.

The ergodic and outage capacity of Ricean MIMO channels can be investigated in two ways: (i) by simulations or measurements, and (ii) by analyses. Early investigations of the Ricean MIMO capacity were largely based on simulations and measurements (see, e.g., [2], [3]), where it was observed that the Ricean MIMO channel capacity can be reduced by nearly 50 percent when the Ricean factor K reached about 10 dB. Recently, results based on analyses have started to appear (see, e.g. [4], [5]). In [4] and [5], the optimal input covariance matrix was found for a Ricean MIMO channel, or a MIMO channel with mean feedback.

In this paper, we investigate the Ricean MIMO channel capacity using a virtual channel representation [6]. The LOS path component is transformed into a single entry in the virtual channel matrix. The capacity achieving input covariance matrix is found to have a diagonal structure in the virtual domain, the same as for the Rayleigh MIMO channel [7]. Although the optimal power distribution among the diagonal entries can be calculated by numerical methods [7], we use the diagonal structure to get upper and lower bounds analytically, and use the analysis to obtain an approximation to the optimal input power distribution.

2. CHANNEL MODEL AND VIRTUAL CHANNEL REPRESENTATION

We consider a flat-fading MIMO system with n_t transmit and n_r receive antennas. In complex base band, the received signals at the n_r receive antennas are organized into a vector $\underline{\boldsymbol{y}}$ as follows, which corresponds to one symbol interval:

$$\underline{\boldsymbol{y}} = \sqrt{\frac{\mathsf{SNR}}{n_t}}\boldsymbol{H}\underline{\boldsymbol{x}} + \underline{\boldsymbol{w}} \tag{1}$$

where $\underline{\boldsymbol{x}}$ is the n_t-dimensional transmit vector, $\boldsymbol{H}$ is the $n_r \times n_t$ channel matrix, and $\underline{\boldsymbol{w}} \sim \mathcal{CN}(\underline{0}, I)$ is the complex additive white Gaussian noise vector. We constrain the average input power via $\mathsf{E}\left[\underline{\boldsymbol{x}}^\dagger\underline{\boldsymbol{x}}\right] \le n_t$. The channel is normalized so that SNR represents the average signal-to-noise ratio at each receive antenna.

For the Ricean MIMO channel modelling, we constrain the total channel power to be fixed, so that the per receive antenna SNR is unchanged. This can be done through linear combination of a LOS channel matrix $\boldsymbol{H}_s$ and a diffuse channel matrix $\boldsymbol{H}_d$ as:

$$\boldsymbol{H} = \sqrt{r}\boldsymbol{H}_s + \sqrt{1-r}\boldsymbol{H}_d \tag{2}$$

where $r \in [0, 1]$ is the fraction of power in the LOS path.

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The virtual channel representation [6] is a powerful channel model that links the physical scattering environments to the statistical properties of channel. Both the transmitter and the receiver are assumed to be equipped with a uniform linear array (ULA) of antennas. The array steering and response vectors can be expressed as

$$\underline{a}_t(\theta_t) = \frac{1}{\sqrt{n_t}}\left[1,\ e^{-j2\pi(\theta_t-0.5)},\ \ldots,\ e^{-j2\pi(n_t-1)(\theta_t-0.5)}\right]^\top$$

$$\underline{a}_r(\theta_r) = \frac{1}{\sqrt{n_r}}\left[1,\ e^{-j2\pi(\theta_r-0.5)},\ \ldots,\ e^{-j2\pi(n_r-1)(\theta_r-0.5)}\right]^\top$$

The variable θ is the delay of the signal transmited/received at two adjacent array elements from physical propagation angle ϕ: $\theta = \left[\frac{d\sin(\phi)}{\lambda}\right]_{mod[0,1]}$, where d is the spacing between transmit or receive antenna array elements, λ is signal wavelength. We define θ as the *virtual angle*.

In the actual array such as a ULA with n elements and critical antenna spacing, its spatial resolution in the virtual angle θ is limited to $1/n$. Thus the channel matrix can be expressed as:

$$\boldsymbol{H} = A_r\ \tilde{\boldsymbol{H}} A_t^\dagger \tag{3}$$

where the matrices A_t ($n_t \times n_t$) and A_r ($n_r \times n_r$) are the steering and response matrices corresponding to fixed virtual transmit and receive angles $\theta_{t,\ell} = (\ell - 0.5)/n_t$ for $\ell = 1, \ldots, n_t$ and $\theta_{r,k} = (k - 0.5)/n_r$ for $k = 1, \ldots, n_r$:

$$A_t = [\underline{a}_t(\theta_{t,1}),\ \underline{a}_t(\theta_{t,2}), \ldots,\ \underline{a}_t(\theta_{t,n_t})] \tag{4}$$

$$A_r = [\underline{a}_r(\theta_{r,1}),\ \underline{a}_r(\theta_{r,2}), \ldots,\ \underline{a}_r(\theta_{r,n_r})] \tag{5}$$

$\tilde{\boldsymbol{H}}$ is referred to as the *virtual representation* of the actual channel matrix $\boldsymbol{H}$. It corresponds to transmitting and receiving at fixed spatial angles. Furthermore this virtual representation can be interpreted as a 2-D spatial Fourier transform of the physical channel, since both A_t and A_r are unitary discrete Fourier transform matrices.

For the diffuse channel component, it can be reasonably assumed that the physical scatters are uncorrelated. Under this assumption, the virtual channel coefficients $\tilde{\boldsymbol{H}}_{k,\ell}$ are approximately uncorrelated as well [7]:

$$\mathsf{E}[\tilde{\boldsymbol{H}}_{k,\ell}\tilde{\boldsymbol{H}}^*_{k',\ell'}] \approx V_{k,\ell}\ \delta_{k-k'}\ \delta_{\ell-\ell'} \tag{6}$$

where δ_n denotes the Kronecker delta function and V is an $n_r \times n_t$ matrix that contains the variances of the components of the virtual channel matrix $\tilde{\boldsymbol{H}}$, $V_{k,\ell} = \mathsf{E}[|\tilde{\boldsymbol{H}}_{k,\ell}|^2]$.

3. CAPACITY ACHIEVING INPUT DISTRIBUTION

The LOS channel matrix can be expressed as:

$$\boldsymbol{H}_s = \underline{a}_r(\theta_{r,LOS})\underline{a}_t^\dagger(\theta_{t,LOS})e^{j\varphi} \tag{7}$$

where $\theta_{r,LOS}$ and $\theta_{t,LOS}$ are the virtual transmit and receive angles of the LOS path, and $\varphi \in [0, 2\pi)$ is the random phase of the channel response.

The sum of LOS component plus diffuse components is usually modelled as a non zero-mean Gaussian random variable. This model is based on the condition that the phase of the LOS path changes slowly, so that its phase can be tracked at the receiver. The LOS component is then deterministic and the non-zero mean model follows.

If the Ricean path is not from line of sight, but from reflection from some dominant scatter, its phase may not be trackable. Since the phase φ is random, the sum of Ricean component plus diffuse components can be modelled as a zero-mean but non-Gaussian random variable.

Based on (3), we can rewrite the input-output relationship (1) equivalently in the virtual domain as

$$\underline{\tilde{\boldsymbol{y}}} = \sqrt{\frac{\mathsf{SNR}}{n_t}}\tilde{\boldsymbol{H}}\underline{\tilde{\boldsymbol{x}}} + \underline{\tilde{\boldsymbol{w}}} \tag{8}$$

where $\underline{\tilde{\boldsymbol{y}}} = A_r^\dagger \underline{\boldsymbol{y}}$, $\underline{\tilde{\boldsymbol{x}}} = A_t^\dagger \underline{\boldsymbol{x}}$, and $\underline{\tilde{\boldsymbol{w}}} = A_r^\dagger \underline{\boldsymbol{w}}$. Due to the unitarity of A_t, the input power constraint in the virtual domain is unchanged, i.e., $\mathsf{E}\left[\underline{\tilde{\boldsymbol{x}}}^\dagger \underline{\tilde{\boldsymbol{x}}}\right] \le n_t$. The virtual domain channel matrix $\tilde{\boldsymbol{H}}$ is obtained similarly as:

$$\tilde{\boldsymbol{H}} = \sqrt{r}\tilde{\boldsymbol{H}}_s + \sqrt{1-r}\tilde{\boldsymbol{H}}_d \tag{9}$$

Without loss of generality, $\theta_{r,LOS}$ and $\theta_{t,LOS}$, can be assumed to correspond to the first fixed virtual angles $\theta_{r,1}$ and $\theta_{t,1}$. Thus, $\tilde{\boldsymbol{H}}_s$ has only a single non-zero entry $n_t n_r e^{j\varphi}$ at the $(1,1)$-th position, and zeros elsewhere. The diffuse virtual channel matrix $\tilde{\boldsymbol{H}}_d$ has independent but not necessarily identically distributed entries.

We make the reasonable assumption that the channel changes in a stationary ergodic manner from symbol to symbol, and also the receiver has complete channel state information (CSI). According to Lemma 2 of [1], the channel capacity is achieved by zero-mean proper (circularly symmetric) complex Gaussian input vector $\underline{\boldsymbol{x}} \sim \mathcal{CN}(\underline{0}, Q)$:

$$C = \max_Q \mathsf{E}[\log\ \det(I + \tfrac{\mathsf{SNR}}{n_t}\boldsymbol{H}Q\boldsymbol{H}^\dagger)] \tag{10}$$

where Q is subjected to the power constraint: $\mathrm{Tr}(Q) \le n_t$. This capacity formula can be equivalently expressed in the virtual domain as

$$C = \max_{\tilde{Q}} \mathsf{E}[\log\ \det(I + \tfrac{\mathsf{SNR}}{n_t}\tilde{\boldsymbol{H}}\tilde{Q}\tilde{\boldsymbol{H}}^\dagger)] \tag{11}$$

where $\tilde{Q}$ is the transmit signal covariance matrix in the virtual domain, with the same power constraint $\mathrm{Tr}(\tilde{Q}) \le n_t$. $\tilde{Q}$ is related to the physical domain covariance matrix Q by $Q = A_t\tilde{Q}A_t^\dagger$.

It has been proven in [7] that the optimal $\tilde{Q}$ is diagonal in structure for both general correlated Rayleigh fading MIMO channel and correlated Ricean MIMO channel, when the channel statistics is available at the transmitter. We denote this optimal input covariance matrix as $\Lambda^\circ = diag\{\lambda_1, \lambda_2, \ldots, \lambda_{n_t}\}$. Actually λ_i represents the power allocated to the i-th virtual transmit angle.

In [4] and [5], it was shown that the principal eigenvector of the optimal input covariance matrix must be in the same direction as the LOS path, and all other eigenvectors can be chosen arbitrarily as long as they are orthogonal to each other. The optimal input covariance matrix is diagonal in our virtual domain, thus the principle eigenvector corresponds to the first virtual transmit angle, and the other eigenvectors along other virtual transmit angles are orthogonal by the property of 2-D Fourier transform. However, proof in [7] is based on the independent but not necessarily identical entries in the virtual channel matrix, while [4] [5] assumes i.i.d. entries for the diffuse components.

4. ERGODIC CAPACITY

Previous work on capacity has used Monte Carlo method to find the optimal input distribution first, and then numerically calculate the capacity (see, e.g. [7]). In this section, instead of the numerical method, we provide tight bounds on the capacity by analysis. The bounds are used in both optimal input distribution characterization and capacity evaluation.

In order to proceed the analysis, we need to make further assumption: a circular d-diagonal channel model as in [8] is adopted in this section, which is an extension of the d-diagonal model. In the d-diagonal structure, d denotes the number of non-zero entries $\sigma^2_{k,\ell}$ above and below diagonal.

$$V_d(k,\ell) = \begin{cases} \sigma^2_{k,\ell}, & \max(n_t, k-d) \le \ell \le \min(0, k-d) \\ 0, & \text{otherwise} \end{cases}$$

The circular model makes the number of nonvanishing elements in each row to be equal. We further assume that these nonvanishing elements have same distribution $\sigma^2_{k,\ell} = \sigma^2$ in the following analysis. When $d = 0$, the diffuse channel matrix is diagonal, which is refereed to as *diagonal scattering*. When $d = n_r/2$, the channel matrix has non-zero entries everywhere, which is refereed to as *maximally rich scattering*.

We use techniques similar to those used in [9] to get the analytical expression for the lower bound and the upper bound of the Ricean MIMO channel capacity. Without loss of generality, we assume that $n_r \ge n_t$. To characterize the capacity lower bound and upper bound, we first apply a QR decomposition on $\tilde{\boldsymbol{H}}$ as $\tilde{\boldsymbol{H}} = Q'R$, where Q' is an unitary matrix and R is an upper triangular matrix with independent entries.

The capacity can be lower bounded as:

$$\begin{aligned} C &= \mathsf{E}\left[\log\ \det\left(I + \frac{\mathsf{SNR}}{n_t} R\Lambda^\circ R^\dagger\right)\right] \\ &\ge \mathsf{E}\left[\sum_{i=1}^{n_t} \log\left(1 + \frac{\mathsf{SNR}}{n_t}\lambda_i |R_{i,i}|^2\right)\right] \end{aligned} \tag{12}$$

where $|R_{i,i}|^2$, are chi-square distributed with $2df(i)$ degrees of freedom $\chi^2_{2df(i)}$. The degree of freedom $df(i)$ is a function of n_t , n_r and d, taking values in the range $[1, \min(n_r, 2d+1)]$. $|R_{i,i}|^2$ is central (zero-mean) chi-square distributed for $2 \le i \le n_t$, and non-central (non zero-mean) chi-square distributed for $i = 1$.

The capacity can also be upper bounded as:

$$\begin{aligned} C \le \mathsf{E}\Big[\sum_{i=1}^{n_t} \log\Big(1 + \frac{\mathsf{SNR}}{n_t}\lambda_i \big(|R_{i,i}|^2 \\ + \sum_{m=i+1}^{\min(i+d,n_t)} |R_{i,m}|^2\big)\Big)\Big]. \end{aligned}$$

The techniques used in [10] can be applied to further characterize the bounds, by the convexity of $\log_2(1 + ae^x)$ and Jensen's Inequality:

$$\mathsf{E}\left[\log_2(1 + \frac{\mathsf{SNR}}{n_t}|R_{i,i}|^2)\right] \ge \log_2(1 + \frac{\mathsf{SNR}}{n_t}\exp(\mathsf{E}[\ln|R_{i,i}|^2]))$$

Since $|R_{i,i}|^2 \sim \chi^2_{2df(i)}$ central for $2 \le i \le n_t$, the expectation of its logarithm can be calculated as:

$$\mathsf{E}\left[\ln|R_{i,i}|^2\right] = \psi(2df(i)) \tag{13}$$

where $\psi(x)$ is the $digamma$ function. Similarly, the expectation of the logarithm of the $|R_{1,1}|^2$ term can be calculated as:

$$\begin{aligned} \mathsf{E}\left[\ln|R_{1,1}|^2\right] &= e^{-\frac{r}{1-r}n_t n_r} \sum_{k=0}^{\infty} \frac{\psi(k+df(1))\left(\frac{r}{1-r}n_t n_r\right)^k}{k!} \\ &\stackrel{\text{def}}{=} S\left(\frac{r}{1-r}n_t n_r\right) \end{aligned} \tag{14}$$

where $df(1) = \min(n_r, 2d+1)$.

The lower bound on the capacity is then obtained by substituting (13) and (14) into (12):

$$C_{LB} = \log\left(1 + \frac{\mathsf{SNR}}{n_t}(1-r)\lambda_1 \exp\left(S\left(\frac{r}{1-r}n_t n_r\right)\right)\right) + \sum_{i=2}^{n_t} \log\left(1 + \frac{\mathsf{SNR}}{n_t}\frac{n_r}{\min(n_r, 2d+1)}(1-r)\lambda_i \exp\left(\psi(2df(i))\right)\right)$$

However, a closed-form expression for (14) cannot be obtained. When r gets close to 1.0, i.e., the Ricean factor approaches infinity, the first term in C_{LB} is not easy to calculate. Under this condition, the following approximation to the first term is found to be accurate:

$$\log\left(1 + \frac{\mathsf{SNR}}{n_t}\lambda_1 \exp\left(S(rn_t n_r)\right)\right) \tag{15}$$

In practice, the optimal input distribution matrix Λ° has to be calculated at the transmitter, based on knowledge of the channel statistics. The optimal allocation of power among the diagonal entries is a constrained convex optimization problem and can be solved numerically via gradient descent algorithms. However, this numerical calculation is very slow [7]. We propose an analytical method to find the optimal power allocation, which we denote as *power allocation by analysis*. The following terms from the above analysis are used as approximate eigenvalues of the transmit correlation matrix: $\mu_1 = \exp\left(S(rn_t n_r)\right)$, and $\mu_i = (1-r)\exp(\psi(2df(i)))$ for $2 \le i \le n_t$.

Based on the approximate eigenvalues μ_i, the power allocation is the well-known waterfilling strategy [1]. Since the transmitter can only determine the strongest eigenmode in the LOS direction but not the other eigenmodes, the transmitter allocates the desired power λ_1 in the LOS virtual angle direction, and distributes the remaining power evenly among the other virtual transmit angles, i.e., $\lambda_i = (n_t - \lambda_1)/(n_t - 1)$ for $i = 2, \ldots, n_t$.

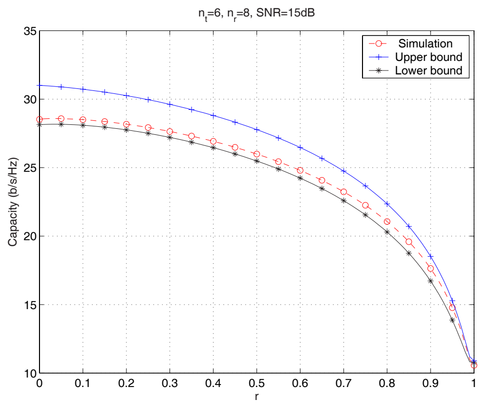

Fig. 1. Specular MIMO ergodic capacity bounds.

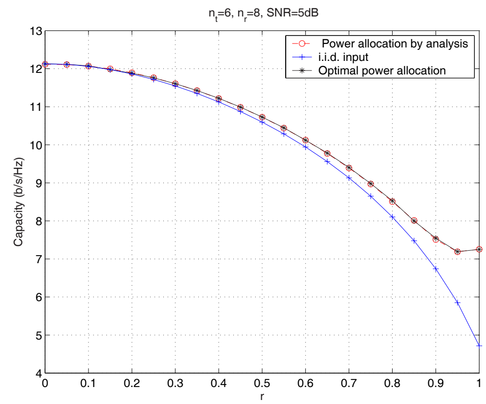

Fig. 2. Different power allocation strategies.

5. NUMERICAL RESULTS

We select the diffuse part of the Ricean MIMO channel to be maximally rich scattering i.i.d. Rayleigh fading, i.e., the d value in the d-diagonal model is set as $d = n_r/2$. In Fig. 1, we simulate the capacity of an $n_t = 6$ and $n_r = 8$ Ricean MIMO channel at SNR of 15 dB. We approximate the infinite summation with the sum of the first 500 terms. From the figure, the lower bound and the simulation results are very close for this high SNR case.

In Fig. 2, we compare the mutual information curves for three different power allocation strategies: optimal power allocation via numerical method, proposed power allocation by analysis method, and equal power allocation with i.i.d. input. From the figure, power allocation by analysis is almost optimal. It is also interesting that the capacity with optimal power allocation starts to increase when $r > 0.95$. This can be explained that $r = 0.95$ is the threshold for beamforming to be the optimal transmit power allocation strategy. When $r > 0.95$, beamforming is optimal. As r increases from 0.95 to 1.0, the channel energy wasted in the diffuse channel components (non-beamforming directions) decreases and thus the beamforming channel capacity increases.

6. CONCLUSIONS

We have exploited using virtual channel representation to analyze the ergodic capacity of a Ricean MIMO fading channel. We considered both deterministic and random LOS cases, where non zero-mean Gaussian modelling and zero-mean non-Gaussian modelling were applied respectively. Previous work on capacity used Monte Carlo technology to find the optimal input distribution first, and then numerically calculate the capacity. We provided tight bounds that are useful in both optimal input distribution characterization and capacity evaluation. The analysis was verified by numerical results.

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