LIMITING THROUGHPUT OF MIMO AD HOC NETWORKS

Biao Chen *

Syracuse University
bichen@ecs.syr.edu

Michael J. Gans

AFRL/IFGC
gansm@rl.af.mil

ABSTRACT

We study the limiting throughput of an MIMO (multiple-input multiple output) *ad hoc* network with K simultaneous communicating transceiver pairs. Assume that each transmitter is equipped with t antennas and receiver with r antennas, we show that in the absence of channel state information (CSI) at the transmitters, the asymptotic network throughput is limited by $r\ nats/s/Hz$ as $K \to \infty$. With CSI corresponding to the desired receiver available at the transmitter, we demonstrate that an asymptotic throughput of $t+r+2\sqrt{tr}\ nats/s/Hz$ can be achieved using a simple beamforming approach. Further, we show that the asymptotically optimal transmission scheme with CSI amounts to a single-user water£lling for a properly scaled channel.

1. INTRODUCTION

Multiple-input multiple-output (MIMO) systems by using multiple antenna transceivers have shown great promise in providing spectral ef£ciencies that are several orders of magnitude higher than that of the traditional communication systems [1, 2]. Recently, there is an increasing need for mobile networks with distributed transmitters and receivers, typically referred to as mobile *ad hoc* networks (MANET). It is, therefore, of great interest to study the performance limit of MANET with MIMO transceivers, i.e., when all the nodes are equipped with multiple antennas.

In [3], Blum studied the capacity of a MIMO network with simultaneous pairwise transmissions. Without knowing the channel state information (CSI) at the transmitter, the author showed that, depending on the interference to noise power ratio, the transmitter should either put equal power into each antenna (optimal in the interference-free MIMO transmission) or operate in a singular mode (i.e., it puts all power on a single element). In this paper, we establish the limiting throughput as the number of transmitting pairs, denoted by K, increases. By assuming t transmit and r receive antennas for each transceiver pair, we show that as K increases, the total throughput of is fundamentally limited by the receive antenna size r and is independent of t and the transmit power. This results in a per user throughput of $O(\frac{1}{K})$ for £xed r which decreases to 0 as $K \to \infty$. To achieve non-zero per node throughput, one needs to scale up r in the absence of CSI at the transmitter.

When the CSI corresponding to the desired receiver is available at the transmitter, we show that a simple "beamforming" approach achieves a throughput of approximately $t+r+2\sqrt{tr}\ nats/s/Hz$ for large t and r as $K \to \infty$. For example, with $t = r$, i.e., each transceiver uses the same number of transmit and receive antennas, the total throughput is $4r\ nats/s/Hz$. Nonetheless, the asymptotic per node throughput still decreases to zero for £xed t and r as the number of pairs K increases. Thus, either t or r or both need to be scaled up in the presence of CSI at the transmitter for non diminishing per-user throughput. We further derive the asymptotically optimal transmission schemes which amounts to a water£lling solution for a composite channel incorporating the interference power. The asymptotically achievable throughput with CSI remains an open problem.

The rest of the paper is organized as follows. Section 2 describes the system model. We show in Section 3 that in the absence of CSI, the asymptotic network throughput with interference transmission is fundamentally limited by the receive antenna number r and independent of other system parameters. With CSI available at the transmitter, we establish in Section 4 that a simple beamforming approach can improve the throughput over the blind transmission. In particular, the throughput scales both in t and in r and is strictly larger than that of the CSI absent cases. Asymptotically optimal signaling scheme is also derived. Numerical examples are presented in Section 5. We conclude in Section 6 with remarks on future research topics for MIMO MANET. Natural logarithm is assumed throughout this paper hence the obtained spectral ef£ciency is in $nats/s/Hz$ and we use $|\mathbf{A}|$ to denote the determinant of matrix $\mathbf{A}$.

2. NETWORK MODEL AND ASSUMPTIONS

The system layout is essentially the same as that of [3] where all MIMO nodes communicate in the same channel. The following assumptions are used throughout this paper.

A1 All users have identical power constraint P.

A2 A rich scattering environment: each channel matrix consists of independent identically distributed ($i.i.d.$) entries with zero mean and unit variance.

*This work was supported by the AFRL/IF through the 2004 summer visiting faculty research program.

A3 The combined path loss/shadow fading, denoted by η_{kj}, between the jth transmitter and the kth receiver is $i.i.d.$ with mean $\bar{\eta}$. Further η_{kj} (large scale fading) is independent with the corresponding channel matrix $\mathbf{H}_{kj}$ which captures small scale fading.

A4 Circularly complex Gaussian noise with covariance matrix $\sigma^2\mathbf{I}_r$ is assumed at each receiver.

A5 Gaussian codes are assumed for each user. This does not lose any optimality in Gaussian noise.

Assume that the transmit vector for the jth transmitter has a covariance matrix $\mathbf{R}_j$, the total throughput, C, is the sum of the mutual information (MI) for all transceiver pairs

$$\sum_{k=1}^{K}\mathcal{E}\left[\log\left|\mathbf{I}_r+\eta_k\mathbf{H}_k\mathbf{R}_k\mathbf{H}_k^H\left(\sigma^2\mathbf{I}_r+\sum_{j\neq k}\eta_{kj}\mathbf{H}_{kj}\mathbf{R}_j\mathbf{H}_{kj}^H\right)^{-1}\right|\right]$$

We £rst introduce the following lemma.

Lemma 1 *If $\mathbf{H}$ is a $r\times t$ matrix with $i.i.d.$ zero mean unit variance entries, $\mathbf{R}$ is a $t\times t$ Hermitian and positive semidefinite matrix with trace a, then $E[\mathbf{HRH}^H]=a\mathbf{I}$.*

While Lemma 1 holds for a deterministic matrix $\mathbf{R}$, it can be trivially extended to cases where $\mathbf{R}$ is a Hermitian and positive semide£nite *random* matrix with the same trace constraint, as long as it is *independent* of $\mathbf{H}$.

3. NETWORK THROUGHPUT IN THE ABSENCE OF CSI AT THE TRANSMITTER

In [3], it was shown that in the absence of CSI at the transmitter in MIMO ad hoc networks, the optimal signaling shoud put equal power on all antennas (i.e., using the optimal interference free transmission [2]) with weak interference; while for strong interference, the transmitter should operate in a singular mode: it puts all its power on a single antenna (which is equivalent to transmitting identical information through all antennas). We establish in this section that with both channel blind transmission schemes, the asymptotic throughput is limited solely by the receive antenna size r.

3.1. Interference-free mode

Operating in the interference-free mode, the transmit vector from the kth transmitter has a covariance matrix $\mathbf{R}=\frac{P}{t}\mathbf{I}_t$. The total throughput C is therefore

$$\sum_{k=1}^{K}\mathcal{E}\left[\log\left|\mathbf{I}_r+\frac{\eta_k P}{t}\mathbf{H}_k\mathbf{H}_k^H\left(\sigma^2\mathbf{I}_r+\frac{P}{t}\sum_{j\neq k}\eta_{kj}\mathbf{H}_{kj}\mathbf{H}_{kj}^H\right)^{-1}\right|\right]$$

As $K\to\infty$, by the law of large number (LLN) and the fact that η_{kj} and $\mathbf{H}_{kj}$ are independent (A3), we have

$$\lim_{K\to\infty}\frac{1}{K-1}\sum_{j\neq k}\eta_{kj}\mathbf{H}_{kj}\mathbf{H}_{kj}^H \quad = \quad \mathcal{E}\left[\eta_{kj}\mathbf{H}_{kj}\mathbf{H}_{kj}^H\right]=\bar{\eta}t\mathbf{I}_r$$

Therefore

$$\begin{aligned}
C &\stackrel{K\to\infty}{=} \sum_{k=1}^{K}\mathcal{E}\left[\log\left|\mathbf{I}_r+\frac{\eta_k P}{t}\mathbf{H}_k\mathbf{H}_k^H\left(\sigma^2\mathbf{I}_r+P(K-1)\bar{\eta}\mathbf{I}_r\right)^{-1}\right|\right]\\
&\leq \sum_{k=1}^{K}\log\left|\mathcal{E}\left[\left(\mathbf{I}_r+\frac{1}{\sigma^2+P\bar{\eta}(K-1)}\frac{\eta_k P}{t}\mathbf{H}_k\mathbf{H}_k^H\right)\right]\right|\\
&= \sum_{k=1}^{K}\log\left|\mathbf{I}_r+\frac{\bar{\eta}P}{\sigma^2+P\bar{\eta}(K-1)}\mathbf{I}_r\right|\\
&= rK\log\left(1+\frac{\bar{\eta}P}{\sigma^2+P\bar{\eta}(K-1)}\right)\\
&\stackrel{K\to\infty}{=} r \quad nats/s/Hz \qquad (1)
\end{aligned}$$

where the inequality follows from Jensen's inequality ($\log|\cdot|$ is concave) and (1) follows from the fact

$$\lim_{x\to\infty}x\log\left(1+\frac{1}{x}\right)=1$$

3.2. Singular mode

Without loss of generality, assume that each transmitter puts all the power on its £rst antenna element. Consequently, the covariance matrix $\mathbf{R}$ is a singular matrix with $\mathbf{R}_{11}=P$ and all zero elements elsewhere. Therefore, the throughput C is

$$\sum_{k=1}^{K}\mathcal{E}\left[\log\left|\mathbf{I}_r+\eta_k\mathbf{H}_k\mathbf{R}\mathbf{H}_k^H\left(\sigma^2\mathbf{I}_r+\sum_{j\neq k}\eta_{kj}\mathbf{H}_{kj}\mathbf{R}\mathbf{H}_{kj}^H\right)^{-1}\right|\right]$$

Again, let $K\to\infty$ and we have, from LLN,

$$\lim_{K\to\infty}\frac{1}{K-1}\sum_{j\neq k}\eta_{kj}\mathbf{H}_{kj}\mathbf{R}\mathbf{H}_{kj}^H \quad = \quad \bar{\eta}\mathcal{E}\left[\mathbf{H}_{kj}\mathbf{R}\mathbf{H}_{kj}^H\right]$$

From Lemma 1, we have $\mathcal{E}\left[\mathbf{H}_{kj}\mathbf{R}\mathbf{H}_{kj}^H\right]=P\mathbf{I}_r$ From this, we can show in a similar fashion that

$$\lim_{K\to\infty}C=r \qquad nats/s/Hz$$

In both cases, r is the limiting network throughput. Thus, To achieve non-zero per user throughput, one needs to scale up r to the same order of K.

4. NETWORK THROUGHPUT WITH CSI

Consider the kth transmitter-receiver pair whose channel matrix is $\mathbf{H}_k$. With $\mathbf{H}_k$ available at the kth transmitter, it is reasonable to expect that better throughput may result. In particular, since the transmitter can fully utilize its multiple antennas for interference suppression/avoidance, one expects that the achievable throughput also depends on the number of transmit antennas. We show in the following that this is indeed the case. By limiting the transmitter processing to simple beamforming, we obtain an asymptotic throughput of $t+r+2\sqrt{tr}$ which scales both in t and in r.

Consider, for the kth user, one uses a beamforming vector $\sqrt{P}\mathbf{c}_k$, with $\|\mathbf{c}_k\| = 1$, which is determined solely using the channel matrix $\mathbf{H}_k$; i.e., $\mathbf{c}_k = \mathbf{c}_k(\mathbf{H}_k)$. The network throughput is now

$$C \overset{K\to\infty}{=} \sum_{k=1}^{K} \mathcal{E}\left[\log\left|\mathbf{I}_r + \frac{\eta_k P}{\sigma^2 + P\bar{\eta}(K-1)}\mathbf{H}_k\mathbf{c}_k\mathbf{c}_k^H\mathbf{H}_k^H\right|\right]$$

where we again used Lemma 1 along with the fact that $\mathbf{c}_k$ is independent of $\mathbf{H}_{kj}$ for $j \neq k$. Implied in the above throughput is a block fading channel model: each channel state realization is assumed to have suf£cient dwelling time for capacity achieving coding. We comment here that the singular mode described in Section 3 is a special case of the beamforming scheme with $\mathbf{c}_k = [1, 0, \cdots, 0]^T$ which is channel independent. With the knowledge of $\mathbf{H}_k$, one expects a better $\mathbf{c}_k$ that maximizes the mutual information may result. Using $\log|\mathbf{I} + \mathbf{AB}| = \log|\mathbf{I} + \mathbf{BA}|$, we have

$$\begin{aligned} &\log\left|\mathbf{I}_r + \frac{\eta_k P}{\sigma^2 + P\bar{\eta}(K-1)}\mathbf{H}_k\mathbf{c}_k\mathbf{c}_k^H\mathbf{H}_k^H\right| \\ &= \log\left(1 + \frac{\eta_k P}{\sigma^2 + P\bar{\eta}(K-1)}\mathbf{c}_k^H\mathbf{H}_k^H\mathbf{H}_k\mathbf{c}_k\right) \end{aligned}$$

Clearly, maximizing the quadratic term $\mathbf{c}_k^H\mathbf{H}_k^H\mathbf{H}_k\mathbf{c}_k$ subject to a norm constraint yields a beamforming vector $\mathbf{c}_k$ that coincides with the eigenvector corresponding to the largest eigenvalue of the matrix $\mathbf{H}_k^H\mathbf{H}_k$, denoted by $\mathbf{v}_1$. We now try to quantify the network throughput of this simple beamforming approach. First, $\mathbf{c}_k^H\mathbf{H}_k^H\mathbf{H}_k\mathbf{c}_k = \mathbf{v}_1^H\mathbf{H}_k^H\mathbf{H}_k\mathbf{v}_1 = \lambda_1^{(k)}$ where $\lambda_1^{(k)}$ is the maximum eigenvalue of $\mathbf{H}_k^H\mathbf{H}_k$. To £nd the corresponding mutual information, we can show

Theorem 1 $C = E[\lambda_1^{(k)}]\ nats/s/Hz$.

To compute $E[\lambda_1^{(k)}]$, notice that $\mathbf{H}_k$ being a channel matrix of complex Gaussian $i.i.d.$ entries, $\mathbf{H}_k^H\mathbf{H}_k$ is essentially a sample covariance matrix of a vector random variable $\mathbf{h} \sim \mathcal{CN}(0, \mathbf{I})$. From [4], for large t, r,

$$\mathcal{E}\left[\lambda_1^{(k)}\right] \approx (\sqrt{t} + \sqrt{r})^2 \qquad (2)$$

While this asymptotic throughput is still independent of the transmit power, one can improve the throughput by scaling up t or r or both. The fact that transmitting along the singular direction that has the largest SNR (largest eigenvalue of $\mathbf{H}_k^H\mathbf{H}_k$) yields the maximum throughput is not surprising: since the channel matrices are assumed to be independent, the interference power are evenly distributed among all subspaces when K is large. As such, sending information along only the strongest eigenmode can limit the total interference power while maximizing the signal power. Next, we generalize the beamforming idea and present the asymptotically optimal transmitting scheme. Consider that the transmit vector for the kth transmitter has a covariance matrix $\mathbf{R}_k$ with $trace(\mathbf{R}_k) = P$. The throughput is now

$$\sum_{k=1}^{K} \mathcal{E}\left[\log\left|\mathbf{I}_r + \eta_k\mathbf{H}_k\mathbf{R}_k\mathbf{H}_k^H\left(\sigma^2\mathbf{I}_r + \sum_{j\neq k}\eta_{kj}\mathbf{H}_{kj}\mathbf{R}_j\mathbf{H}_{kj}^H\right)^{-1}\right|\right]$$

Again, invoke the asymptotic assumption ($K \to \infty$) and use Lemma 1, we can get

$$C \overset{K\to\infty}{=} \sum_{k=1}^{K} \mathcal{E}\left[\log\left|\mathbf{I}_t + \mathbf{R}_k\frac{\eta_k}{\sigma^2 + P\bar{\eta}(K-1)}\mathbf{H}_k^H\mathbf{H}_k\right|\right]$$

Hence the asymptotically optimal $\mathbf{R}_k$ corresponds to simply water£lling for the combined channel covariance matrix $\frac{\eta_k}{\sigma^2+P\bar{\eta}(K-1)}\mathbf{H}_k^H\mathbf{H}_k$. In other words, if we de£ne

$$\hat{\mathbf{H}}_k = \left(\frac{\eta_k}{\sigma^2 + P\bar{\eta}(K-1)}\right)^{1/2}\mathbf{H}_k \qquad (3)$$

Then $\mathbf{R}_k$ should be chosen through single user water£lling corresponding to the channel matrix $\hat{\mathbf{H}}_k$ [2]. Notice this is different than simply scaling the water£lling solution for $\mathbf{H}_k$: the water£lling level is determined by the inverse of the eigenvalues of $\hat{\mathbf{H}}_k^H\hat{\mathbf{H}}_k$ hence depends on the scaling factor in a nonlinear fashion.

5. NUMERICAL EXAMPLES

In this section, we use numerical examples to study the network throughput of a MIMO *ad hoc* network. In particular, we demonstrate that, with CSI available at the transmitter, substantially larger network throughput can be achieved than the channel-blind approach. This is in sharp contrast to the single user MIMO systems where CSI provides a constant yet typically insigni£cant gain over the blind transmitter for a well behaved channel matrix.

Throughout this section, we assume that $\mathbf{H}_{kj}$, consists of $i.i.d.$ complex Gaussian entries, implying a rich scattering environment with Rayleigh ¤at fading channels. The path loss/shadowing coef£cient η_{kj} is $lognormal$ distributed appropriately normalized to be unit mean (hence the path loss is assumed to be absorbed through appropriately scaling the noise variance). We plot the sum (network) throughput as a function of K for two different parameter sets:

1. $t = r = 16$, $P = 2$, $\sigma^2 = 1$. The result is in Fig. 1.
2. $t = r = 16$, $P = 10$, $\sigma^2 = 1$. The result is in Fig. 2.

We simulate the throughput by averaging, for each case, over 50 independently generated channel matrices for each transmitter-receiver pair.

Remarks

- In all cases, knowing the CSI at the transmitter (the water£lling and beamforming approaches) improves substantially the network throughput over the channel-blind transmission schemes ('interference-free' and the singular modes).

- Both channel-blind transmission schemes have asymptotic (K large) throughput that is close to $r\ nats/s/Hz$, or $(r \log_2 e)\ bits/s/Hz$. Further, this asymptotic value is independent of the transmit power (compare Figs 1 and 2).

- The asymptotic throughput for the simple beamforming approach is less the $t + r + 2\sqrt{tr}$. This is because of the fact that Eq. (2) is only true asymptotically (i.e., both r and t are suf£ciently large). Otherwise, the distribution of the largest eigenvalue of a sample covariance matrix is skewed toward smaller values, resulting in a smaller expected value. Increasing t and r will improve the accuracy of this approximation.

6. CONCLUSIONS

MIMO communications in an *ad hoc* network is studied in this paper. We demonstrated that the knowledge of CSI at the transmitter is instrumental in obtaining higher network throughput. Without CSI, we showed that the network throughput is fundamentally limited by the receiver antenna size. With CSI, the throughput scales as $t + r + 2\sqrt{tr}$ $nats/s/Hz$ with a simple beamforming approach hence improves when either t or r increases. Asymptotically optimal signaling for MIMO interference transmission with CSI was shown to be a simple water£lling solution.

We expect that, with stronger CSI assumption, better throughput may result. For example if a transmitter knows the channels for both its desired receiver and all other receivers that it interferes with, interference suppression beamforming may be used in limiting the interference power. This will be reported elsewhere.

An alternative way of utilizing the MIMO potential in MANET is to use channelized transmission, i.e., the total time-frequency is divided into orthogonal subchannels to allow interference-free MIMO communication in each subchannel. This, however, puts exacting demand on the medium access control (MAC). To accommodate the dynamic traf£c in a MANET, an adaptive MAC is needed to guarantee access to all active users while leave no idle channels for maximal bandwidth ef£ciency. This is a formidable task in an *ad hoc* network due to the lack of a central node. On the other hand, recognizing that multiple antennas at the transceivers provide inherent multiplexing capability due to their interference cancellation capability, it is imperative to study MIMO communication in *ad hoc* networks with interference transmission. Not only it may alleviate the need for a fully adaptive MAC layer or the effect of spectrum underutilization with £xed channel allocation, allowing simultaneous transmissions also helps exploit the multiuser diversity that may potentially improve upon the channelized MIMO approach. Analysis of channelized MIMO and its comparison with interference transmission will also be reported in the near future.

7. REFERENCES

[1] G.J. Foschini and M.J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Communications*, vol. 6, pp. 311–335, 1998.

[2] E. Telatar, "Capacity of multi-antenna Gaussian channels," *Eurapean Trans. Telecomm.*, vol. 10, pp. 585–595, Nov/Dec 1999.

[3] R.S. Blum, "MIMO capacity with interference," *IEEE J. Select. Areas Commun.*, vol. 21, June 2003.

[4] I.M. Johnstone, "On the distribution of the largest eigenvalue in principle components analysis," *Ann. Statist.*, vol. 29, no. 2, pp. 295–327, 2001.

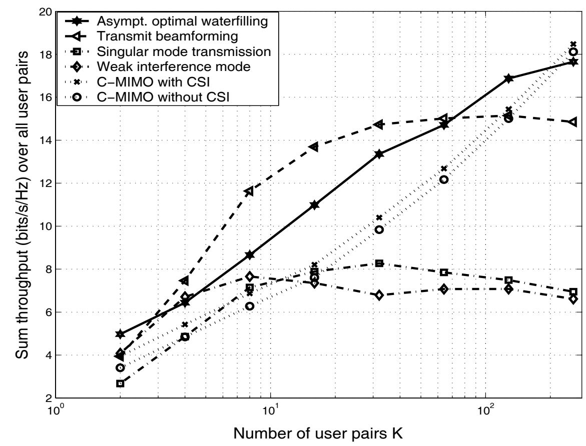

Fig. 1. The sum throughput of a MIMO *ad hoc* network with $t = r = 16$, $P = 2$, $\sigma^2 = 1$.

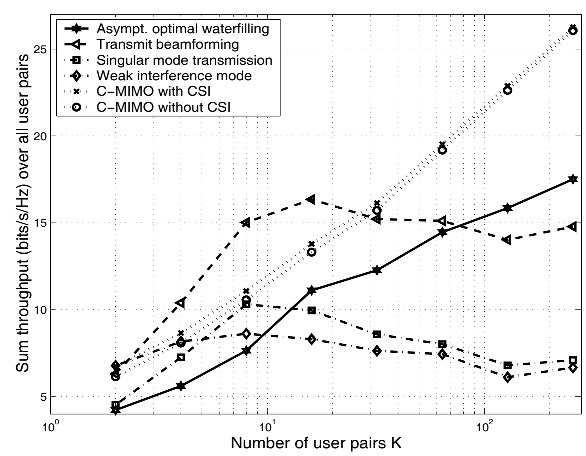

Fig. 2. The sum throughput of a MIMO *ad hoc* network with $t = r = 16$, $P = 10$, $\sigma^2 = 1$.