SUM-RATE MAXIMIZATION AND SUM-POWER MINIMIZATION FOR 2-USER (2,2) DOWNLINK SYSTEMS USING GENERALIZED ZERO-FORCING

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ABSTRACT

This paper analyzes a downlink system where a 2-antenna base station is sending independent signals to 2 2-antenna mobile users simultaneously in the same physical (time and frequency) channel. To multiplex the signals in spatial domain, we consider the use of orthogonal space-division multiplexing (OSDM) or broadly known as generalized zero-forcing (GZF) that allows the users to be completely separated before decoding. The main contribution of this paper is that we derive the optimal array processing solutions for both the sum-rate maximization problem subject to a fixed transmit power constraint, and the sum-power minimization problem subject to a fixed rate constraint. The capacity and signal-to-noise ratio (SNR) regions for the OSDM system are also derived, and results for dirty-paper coding (DPC) and time-division systems are provided for comparison.

1. INTRODUCTION

In the context of mobile communications, broadcast channel refers to the channel of a point-to-multipoint transmission, which arises from a downlink scenario where a base station is sending independent signals to many mobile stations. Recently, the studies on investigating the capacity (either the sum-rate or the capacity region) of broadcast channels have been receiving considerable attention (e.g., [1]–[5]). Though dirty-paper coding (DPC) is the optimal (in the sense of maximizing the system capacity) way to operate the channel, in practice, it is much preferred to utilize the channel using only linear signal processing. One such approach is to employ multiple antennas at both the transmitter and the receivers for generalized beamforming (e.g., [6]–[11]). In particular, recent emphases tended to focus on the new paradigm of orthogonal space-division multiplexing (OSDM) or broadly known as generalized zero-forcing (GZF) [8]–[11].

For systems using OSDM or GZF, the antennas at the transmitter and the receivers are jointly optimized in order that the base station beams or projects the users onto disjoint spaces. Specifically, nulls are placed at the signal outputs (but not the antennas) of the unintended mobile receivers. Therefore, at each mobile station, highly complex joint decoding is not required. Unfortunately, all of the previously proposed GZF solutions are by and large ad hoc, and none of them has truly solved the optimal *linear* transmission strategy, either for sum-rate maximization or sum-power minimization.

Aiming to derive the optimal linear codec for multiple-input multiple-output (MIMO) antenna broadcast channels, this paper considers a 2-user (2,2) system[1]. In this setting, we shall solve analytically the optimal linear solution for the problem of 1) sum-rate maximization subject to the fixed power constraint and 2) sum-power minimization subject to the fixed rate constraint. We further show that the optimization boils down to solving a quadratic equation and choosing the root that maximizes the sum-rate or minimizes the sum-power. In this paper, we will assume that the channel state information (CSI) is known at both the transmitter and the receivers.

The remainder of the paper is organized as follows. Section II introduces the system model of a 2-user (2,2) system. Section III attempts to solve the optimal transmitter and receiver processing of the 2-user system. In Section IV, simulation setup and results are provided. Finally, we conclude the paper in Section V.

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[1] We use the notation K-user (n_T, n_R) to denote a broadcast system that an n_T-element transmitter is communicating with K mobile receivers each with n_R antennas.

2. SYSTEM MODEL

The system configuration of a 2-user (2,2) system in downlink is shown in Figure 1. The data symbol, z_1, of mobile user 1, before transmitted from the base station antennas, is post-multiplied by a complex antenna vector

$$\mathbf{t}_1 = \left[t_1^{(1)}\ t_2^{(1)}\right]^T \tag{1}$$

where $t_1^{(1)}$ and $t_2^{(1)}$ denote the transmit antenna weights of the symbol z_1 at the base station antennas, and the superscript T denotes the transposition. The weighted symbols, $\mathbf{t}_1 z_1$ and $\mathbf{t}_2 z_2$, of all users are then summed up and are finally transmitted from the antennas. Defining the transmitted signal vector as $\mathbf{x}$, the transmitted signal vector can be written as $\mathbf{x} = \mathbf{t}_1 z_1 + \mathbf{t}_2 z_2$.

Given a flat fading channel, at the m-th mobile receiver ($m = 1, 2$), the signal at each receive antenna is a noisy superposition of the two transmitted signals perturbed by fading. As a result, we have

$$\mathbf{y}_m = \mathbf{H}_m \mathbf{x} + \mathbf{n}_m \tag{2}$$

where $\mathbf{y}_m = [y_1^{(m)}\ y_2^{(m)}]^T$ is the received signal vector with element $y_\ell^{(m)}$ representing the received signal at the ℓ-th antenna of the m-th mobile station, $\mathbf{n}_m$ is the noise vector with elements assumed to have Gaussian distribution with zero mean and variance of σ_n^2, and $\mathbf{H}_m$ denotes the channel matrix from the base station to the m-th mobile station, given by

$$\mathbf{H}_m = \begin{bmatrix} h_{1,1}^{(m)} & h_{1,2}^{(m)} \\ h_{2,1}^{(m)} & h_{2,2}^{(m)} \end{bmatrix} \tag{3}$$

where $h_{\ell,k}^{(m)}$ denotes the fading coefficient from the base station antenna k to the receive antenna ℓ of the m-th mobile station. We model $h_{\ell,k}^{(m)}$'s statistically by independent zero-mean complex Gaussian random variables with unit variance (i.e., $\mathrm{E}[|h_{\ell,k}^{(m)}|^2] = 1$), so the amplitudes are Rayleigh distributed and their phases are uniformly distributed from 0 to 2π.

An estimate of the transmitted symbol, z_m, can be obtained by combining the received signal vector at the m-th mobile station. This is done by

$$\hat{z}_m = \mathbf{r}_m^\dagger \mathbf{y}_m \tag{4}$$

where $\mathbf{r}_m = [r_1^{(m)}\ r_2^{(m)}]^T$ is the receive antenna weight vector of the m-th mobile station, and the superscript $\dagger$ denotes the conjugate transposition. Consequently, we can write the 2-user (2,2) antenna system as [9]–[11]

$$\hat{z}_m = \mathbf{r}_m^\dagger \left[\mathbf{H}_m(\mathbf{t}_1 z_1 + \mathbf{t}_2 z_2) + \mathbf{n}_m\right] \quad m = 1, 2. \tag{5}$$

3. GZF FOR 2-USER (2,2) DOWNLINK SYSTEMS

In GZF systems, it is required that the users are completely separated before decoding and each mobile user receives no co-channel interference. To this end, we want to have

$$\mathbf{r}_1^\dagger \mathbf{H}_1 \mathbf{t}_1, \mathbf{r}_2^\dagger \mathbf{H}_2 \mathbf{t}_2 > 0 \tag{6}$$

and

$$\mathbf{r}_1^\dagger \mathbf{H}_1 \mathbf{t}_2 = \mathbf{r}_2^\dagger \mathbf{H}_2 \mathbf{t}_1 = 0. \tag{7}$$

In addition, optimization is usually performed considering also the unit-norm constraint $\|\mathbf{t}_1\| = \|\mathbf{t}_2\| = 1$. With these constraints, we can choose to minimize the required (transmit) sum-power that achieves given rates for the users or to maximize the sum-rate of the system subject to a given fixed transmit power. But before we discuss these solutions (see Sections 3.1 and 3.2), let us first simplify the constraints (7).

It can be easily shown that (7) actually implies the receive signal processing, $\mathbf{r}_1$ and $\mathbf{r}_2$, to be maximal-ratio combining (MRC) for optimality (regardless of sum-rate maximization or sum-power minimization). That is,

$$\mathbf{r}_1 = \mathbf{H}_1 \mathbf{t}_1, \text{ and } \mathbf{r}_2 = \mathbf{H}_2 \mathbf{t}_2. \tag{8}$$

As a result, the constraints (7) will become

$$\begin{cases} \mathbf{t}_1^\dagger \mathbf{H}_1^\dagger \mathbf{H}_1 \mathbf{t}_2 = 0, \\ \mathbf{t}_2^\dagger \mathbf{H}_2^\dagger \mathbf{H}_2 \mathbf{t}_1 = 0. \end{cases} \tag{9}$$

Now, we use the eigenvalue decomposition (EVD) to write $\mathbf{H}_1^\dagger \mathbf{H}_1 = \mathbf{U}_1 \boldsymbol{\Lambda} \mathbf{U}_1^\dagger$ and $\mathbf{H}_2^\dagger \mathbf{H}_2 = \mathbf{U}_2 \boldsymbol{\Sigma} \mathbf{U}_2^\dagger$. Further, we let $\mathbf{v} = \mathbf{U}_1^\dagger \mathbf{t}_1 = [v_1\ v_2]^T$ and $\mathbf{w} = \mathbf{U}_2^\dagger \mathbf{t}_2 = [w_1\ w_2]^T$. Thus, the constraints can be rewritten as

$$\begin{cases} \mathbf{v}^\dagger \mathbf{P} \mathbf{w} & = 0 \\ \mathbf{w}^\dagger \mathbf{Q} \mathbf{v} & = 0 \\ \|\mathbf{v}\| & = 1 \\ \|\mathbf{w}\| & = 1 \end{cases} \tag{10}$$

where $\mathbf{P} \triangleq \boldsymbol{\Lambda} \mathbf{U}_1^\dagger \mathbf{U}_2$ and $\mathbf{Q} \triangleq \boldsymbol{\Sigma} \mathbf{U}_2^\dagger \mathbf{U}_1$.

Note that we have now four equations (10) and four unknowns, v_1, v_2, w_1, w_2. We are able to find all the possible solutions for GZF. After some manipulations, we have

$$\begin{gathered} (Q_{12}P_{22}^* - Q_{22}P_{21}^*)\left(\tfrac{v_2}{v_1}\right)^2 + \\ (Q_{12}P_{12}^* - Q_{22}P_{11}^* - Q_{21}P_{21}^* + Q_{11}P_{22}^*)\left(\tfrac{v_2}{v_1}\right) + \\ (Q_{11}P_{12}^* - Q_{21}P_{11}^*) = 0, \end{gathered} \tag{11}$$

$$|v_1|^2 + |v_2|^2 = 1, \tag{12}$$

$$\frac{w_2}{w_1} = -\frac{P_{11} + P_{21}\left(\frac{v_2}{v_1}\right)^*}{P_{12} + P_{22}\left(\frac{v_2}{v_1}\right)^*}, \tag{13}$$

$$|w_1|^2 + |w_2|^2 = 1. \tag{14}$$

In the equations, P_{ij} and Q_{ij} are, respectively, the (i,j)-th elements of the matrices, $\mathbf{P}$ and $\mathbf{Q}$, and the superscript $*$ denotes the complex conjugate operation.

Therefore, by solving the roots of the equations (11)–(14), we can find $\mathbf{v}$ and $\mathbf{w}$ that ensure GZF. Then, the transmit antenna weights can be found by $\mathbf{t}_1 = \mathbf{U}_1\mathbf{v}$ and $\mathbf{t}_2 = \mathbf{U}_2\mathbf{w}$, and hence the receive antenna weights.

3.1. Sum-rate Maximization

To understand the capacity of the GZF system, we consider the sum-rate of the system, i.e.,

$$C_{\text{sum-rate}} = \log_2(1+\gamma_1) + \log_2(1+\gamma_2) \tag{15}$$

where γ_1 and γ_2 are, respectively, the signal-to-noise ratio (SNR) of the users 1 and 2. Because of the orthogonality of the users, we can write

$$\gamma_1 = \frac{\epsilon P_T}{\sigma_n^2}\mathbf{t}_1^\dagger \mathbf{H}_1^\dagger \mathbf{H}_1 \mathbf{t}_1, \ \gamma_2 = \frac{(1-\epsilon)P_T}{\sigma_n^2}\mathbf{t}_2^\dagger \mathbf{H}_2^\dagger \mathbf{H}_2 \mathbf{t}_2 \tag{16}$$

where P_T is the total transmit power, ϵ is the power distribution coefficient, and σ_n^2 is the noise power.

Given $x = v_2/v_1$ and $y = w_2/w_1$ are the roots of the equations (11)–(14) to ensure GZF, the sum-rate of the system can be expressed as

$$\begin{aligned} C_{\text{sum-rate}}(\epsilon) = \log_2 &\left(1 + \frac{\epsilon P_T}{\sigma_n^2}\cdot\frac{\lambda_1+\lambda_2|x|^2}{1+|x|^2}\right) \\ + \log_2 &\left(1 + \frac{(1-\epsilon P_T)}{\sigma_n^2}\cdot\frac{\sigma_1+\sigma_2|y|^2}{1+|y|^2}\right) \end{aligned} \tag{17}$$

where λ_i and σ_j are, respectively, the i-th diagonal entry of $\mathbf{\Lambda}$ and the j-th diagonal entry of $\mathbf{\Sigma}$. The maximum achievable sum-rate using GZF can be obtained by optimizing the value of ϵ and it can be shown to be achieved when

$$\epsilon_{\text{opt}} = \frac{1-a+b}{2} \tag{18}$$

where

$$a = \frac{\sigma_n^2}{P_T}\cdot\frac{1+|x|^2}{\lambda_1+\lambda_2|x|^2} \ \text{ and } \ b = \frac{\sigma_n^2}{P_T}\cdot\frac{1+|y|^2}{\sigma_1+\sigma_2|y|^2}. \tag{19}$$

The maximum achievable sum-rate is given by

$$C_{\text{sum-rate}} = \log_2\frac{(1+a+b)^2}{4ab} \tag{20}$$

and the capacity achievable by the users for a given total transmit power P_T is

$$C_{\text{GZF}}(P_T, \mathbf{H}_1, \mathbf{H}_2) = \bigcup_{0\le\epsilon\le 1}\left\{(R_1,R_2) : \begin{array}{c} R_1 \le \log_2\left(1+\frac{\epsilon}{a}\right) \\ R_2 \le \log_2\left(1+\frac{1-\epsilon}{b}\right)\end{array}\right\}. \tag{21}$$

3.2. Sum-power Minimization

For most applications, the users' data-rates of interests are pre-determined and they may not match to the maximum available capacity of the channel. Rather, it is more desirable to minimize the total transmit power for support of the given data-rates. Mathematically, it is required to solve the transmit antenna weights $\mathbf{t}_1$ and $\mathbf{t}_2$ (or x and y) to minimize the total required power that makes every user satisfy a given SNR requirement (γ_1 or γ_2). That is to

$$\min_{x,y} P_T = P_1 + P_2 \tag{22}$$

where

$$P_1 = \frac{\gamma_1\sigma_n^2(1+|x|^2)}{\lambda_1+\lambda_2|x|^2} \ \text{ and } \ P_2 = \frac{\gamma_2\sigma_n^2(1+|y|^2)}{\sigma_1+\sigma_2|y|^2}. \tag{23}$$

Since x and y are the roots that satisfy the constraints (11)–(14), the minimization can be done by choosing the roots that gives the minimum of P_T.

In addition, we can also find the SNR region achievable by the users for a given total transmit power P_T, which is given by

$$SNR(P_T, \mathbf{H}_1, \mathbf{H}_2) = \{(\gamma_1,\gamma_2) : P_1 + P_2 \le P_T\}. \tag{24}$$

4. SIMULATION RESULTS

In this section, computer simulation results for a particular channel realization are provided. Figure 2 shows the rate region of the 2-user (2,2) system using GZF for various total power P_T/σ_n^2 (from 0 to 24 dB). As can be seen, higher rates can be achieved for both users if more transmit power is available. Interestingly, we can also see that as P_T/σ_n^2 increases, the shape of the capacity region tends to become more rectangular. This is reasoned by the fact that the two users are made orthogonal by GZF. Similar results can be observed in the SNR region in Figure 3.

In Figure 4, results for GZF, time-division and broadcast regions are provided for comparison. It can be seen that in terms of the system sum-rate, the optimum linear GZF solution gives result close to the maximum sum-rate offered by the channel (using dirty-paper coding (DPC)) while obviously has better result compared to a time-division system.

5. CONCLUSIONS

This paper has derived the optimal linear zero-forcing solution for a 2-user (2,2) downlink system in the case of both sum-rate maximization and sum-power minimization. Given the solutions, we have also derived the capacity and the SNR regions for the GZF downlink systems.

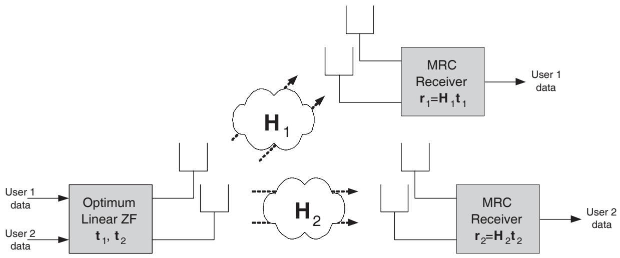

Fig. 1. The system configuration of a 2-user (2,2) downlink system.

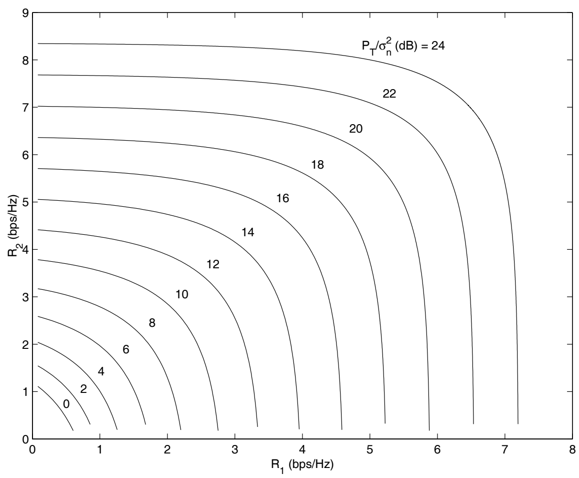

Fig. 2. The rate region for a given total transmit power or SNR P_T/σ_n^2.

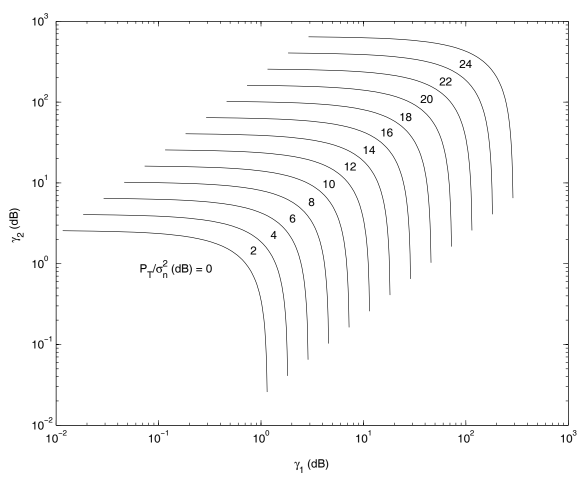

Fig. 3. The SNR region for a given total transmit power or SNR P_T/σ_n^2.

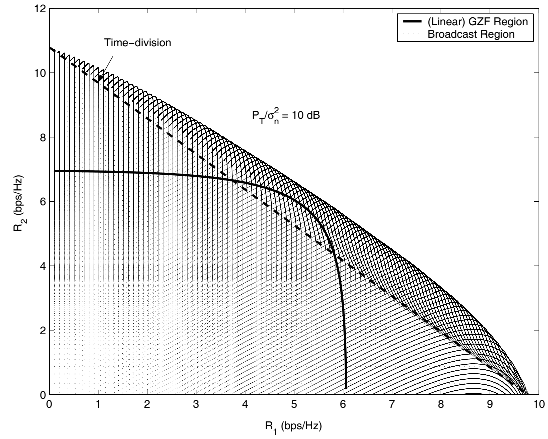

Fig. 4. The comparison of the capacity region for broadcast channel, time-division systems and GZF or OSDM systems.

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