# MESSAGE PROPAGATION IN A COOPERATIVE NETWORK WITH ASYNCHRONOUS RECEPTIONS

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## ABSTRACT

We consider a wireless network in which a single source transmits its message with the help of multiple *cooperative* relays. In our model the source broadcasts the message and the relays retransmit it as soon as they are able to produce an inference of acceptable quality. If the network is dense, multiple nodes will quasisynchronously retransmit the source message, acting as groups of cooperative relays thereby increasing the range of transmission. In this paper, we analyze the behavior of such cooperative networks with respect to parameters such as the source and relay transmission powers by deriving a dynamical system model using a continuum asymptote as the number of relay nodes goes to infinity. The methodology developed in this paper can be used to analyze the performance of other cooperative protocols.

## 1. INTRODUCTION

We consider a network with *cooperative transmissions*, assuming that a single source starts a transmission session with the intent of either broadcasting the message to every other node or reaching a specific destination node. In case where there is a specific destination, we assume that only a selected group of intermediate nodes will pass the message (the selection can be done by using local and source-destination position information).

Cooperative methods have received considerable attention recently [1–4]. In most of them [1–3] the cooperating nodes transmit packets and use orthogonal channels. This has the following disadvantages: (i) As the number of cooperating users increases, unless the bandwidth is infinite, cooperative transmission is not always providing power gains sufficient to compensate for the bandwidth overhead and this complicates the transmission scheduling problem; (ii) Allocating resources to different users usually requires a central control unit.

In [6] we proposed a method where the source sends messages containing several symbols and, the receivers use a local rule based on the Generalized Likelihood Ratio Test (GLRT) to process the message and retransmit. This is similar to [4] which also uses a local rule for retransmission without enforcing orthogonality, but different from [6], [4] considers symbol by symbol relaying.

After the source transmission, only the group of nodes that detects the presence of the message and meets a certain retransmission criterion relays the message. These nodes will be called the level-1 nodes and their transmissions will be overlapping and quasi-synchronous. After the level-1 nodes' transmissions, a second set of nodes (*i.e., level-2* nodes) receives the message and re-

transmits. The level-3, level-4,  $\cdots$  nodes are defined similarly. The retransmissions continue until every relay node retransmits once. Note that in our cooperation protocol, each level of nodes transmit in a different time interval simply as a result of the local rule. In fact, the rule naturally leads the self formed groups to use orthogonal time slots for transmission while orthogonality is not enforced for the transmissions within the same level. Hence, there is no need for centralized control.

In [7] we analyzed the network behavior using a simple deterministic channel model. Specifically, we assumed that each node was receiving the sum power from the nodes transmitting in the previous level; the transmission was based on the deterministic rule that a node would cooperate if and only if the power received from the previous level was crossing a predetermined threshold. In [7], this simple model was sufficient to determine that in order to guarantee that the message is passed to the entire network, it is necessary to choose appropriately the source, the relay power and the threshold used locally to decide on the retransmission.

However, as shown in [6] the channel model is naturally random and so is the outcome of the local rule. In this paper, we incorporate the random channel model developed in [6] and analyze the dynamics of our cooperative network protocol, extending the approach used in [7]. The results are presented as the solution of a nonlinear dynamical system. To obtain the results, we consider random networks and their continuum asymptote where the relay density goes to infinity while the total relay power is fixed.

In Section 2, we summarize previous results based on [6]. In Section 3, the network behavior is analyzed under two different retransmission criteria. In Section 4, we present numerical evaluations of the derived dynamical system.

## 2. CHANNEL STATISTICS

In [6], we determined a transmission and reception model as well the GLRT structure for the cooperative relay strategy described in the introduction. As mentioned before, based on the result of their local GLRT, the nodes retransmit the source message. As argued in [6], the aggregate effect of each level is to create an equivalent random multipath channel whose response convolves the transmitted message. This implies that the received signal at the *i*-th node is

$$\mathbf{r}^{(i)} = \mathbf{C} \, \mathbf{h}^{(i)} + \mathbf{w}^{(i)},\tag{1}$$

where **C** is a Toeplitz (convolution)  $M \times L$  matrix; M is the length of observed data and L is the channel order and the first column of **C** is the message vector **c**. Compared to [6], we assume that the GLRT estimates correctly the time window where the message is centered and therefore the vector  $\mathbf{h}^{(i)}$  can be ap-

This work is supported by National Science Foundation under grant ITR CCR - 0428427.

proximated by a vector of limited order (denoted by *L*). Using this assumption, let  $h^{(i)}[n]$  be the *n*-th coefficient of the baseband complex equivalent aggregate channel for the *i*-th node. Assuming that the transmission between each node pair is affected by independent zero mean small scale fading, and by deterministic large scale path loss  $d^{\beta}$ , the mean and the covariance for  $h^{(i)}[n]$  are as follows:  $\mathbb{E}\{h^{(i)}[n]\} = 0$ , and

$$R_{hh}^{(i)}[l_1, l_2] = \left(\sum_{k \in S_i} \frac{P_k}{d_{i,k}^{2\beta}}\right) \Psi(l_1, l_2; \Delta) = K_i \Psi(l_1, l_2; \Delta) \quad (2)$$

where  $S_i$  denotes the set of nodes from which the *i*-th node received contributions, i.e. the previous level nodes. The transmission power of the *k*-th node is denoted by  $P_k$  and  $d_{ik}$  is the distance between *i*-th and *k*-th nodes. In order to derive the result above, we assumed that the relaying times  $\tau_k$  dictated by the local GLRT rule are uniformly distributed with mean  $\mu_{q_k}$  and variance  $\sigma^2$  within a level. In  $\mu_{q_k}$ ,  $q_k$  denotes the index of the level in which the *k*-th node is located. In (2), the parameter  $\Delta := \sqrt{3}\sigma$ . In summary, the channel coefficients are zero mean. Assuming that the observation vector  $\mathbf{r}^{(i)}$  is centered around  $\mu_{q_i}/T$  and p(t) is the transmission pulse shaping filter of duration T (with normalized power), the covariance  $\mathbb{E}{\mathbf{h}^{(i)}[l_1]\mathbf{h}^{*(i)}[l_2]}$  is a scaled version of the following function,

$$\Psi(l_1, l_2; \Delta) = \frac{1}{2\Delta} \int_{-\Delta}^{\Delta} p(l_1 T - t) p(l_2 T - t) dt.$$
 (3)

The reader is referred to [6] for further details on the derivation of (2) and (3).

## 3. ANALYSIS OF NETWORK BEHAVIOR

In this section, we assume that the channel coefficients (1) are circular complex Gaussian with zero mean and covariance given by (2). That is,

$$\mathbf{h}^{(i)} \sim \mathcal{CN}\left(\mathbf{0}, K_i \Psi\right). \tag{4}$$

In a dense network this assumption is plausible even when the fading is not Rayleigh because the response is the sum of several randomly faded replicas. Due to the fact that  $\mathbf{h}^{(i)}$  is random, the sets of nodes belonging to any level is random and, therefore the parameter  $K_i$  in (4) which depends on  $S_i$  [c.f. (2)] is random as well. The transition from one level to the other can be modelled as a Markov chain with a large number of possible states. In order to simplify the analysis, we use a continuum approximation valid for a large number of nodes. This brings us two advantages: 1) similar to our previous work in [7] we are able to substitute sums with definite integrals over bounded regions; 2) we can use the law of large numbers and substitute random quantities asymptotically by their mean.

The signal attenuation model used in (2),  $P_k/d_{ik}^\beta$  is not appropriate for the continuum analysis, because it does not hold when the distance between nodes is very small [8]. Hence, we will consider constant power for the near-field  $d \leq d_0$  for some small  $d_0$ . Let f(x, y) be the path-loss function at the location (x, y),

$$f(x,y) = \begin{cases} \frac{1}{(x^2+y^2)^{\beta}} & (x^2+y^2) \ge d_0^2\\ \frac{1}{d_0^{2\beta}} & 0 \le (x^2+y^2) \le d_0^2. \end{cases}$$

First we analyze the protocol under the retransmission criterion that  $|\mathbf{h}^{(i)}|^2$  exceeds a given threshold  $\tau$ . This requires the

channel knowledge at the node while the received data is described by (1). We consider the correct reception model in Section 3.4. Also, the protocol uses two different levels of power; the source power  $P_s$  and the relay power  $P_r$ . Note that the superscript that denotes the node index will be substituted by the coordinates (x, y)of the point. In the notation that indicates the channel vector  $\mathbf{h}_i(x, y)$ we will instead add the suffix *i* to denote the index of the level that generated  $\mathbf{h}_i(x, y)$  as its aggregate channel response with respect to the node in the point (x, y).

Finally, note that because we assume that the small scale fading coefficients are independent and zero mean and because the  $\mathbf{h}_i(x, y)$  are Gaussian, this implies that the channels observed in two different points are uncorrelated and therefore independent.

### 3.1. Random Network

Let  $S = \{(x_i, y_i) : i = 1 \dots N\}$  be the set of relay locations with  $(x_0, y_0) = (0, 0)$  corresponding to the source. After the source transmits with power  $P_s$  based on our model,  $\forall (x, y) \in S$ , the channel coefficient are such that

$$K_0(x,y) = P_s f(x,y)$$
  

$$\mathbf{h}_0(x,y) \sim \mathcal{CN}(\mathbf{0}, K_0(x,y)\Psi)$$
(5)

The locations of the level-1 nodes are the set

$$S_1 = \{(x, y) \in S : |\mathbf{h}_0(x, y)|^2 \ge \tau\}.$$

Note that  $S_1$  is a random set because  $\{|\mathbf{h}_0(x, y)|^2 \ge \tau\}$  is a random event. Also, as mentioned in the previous section, the channel coefficients are independent for different (x, y). We can easily generalize the results for level-*n*;  $K_n(x, y)$  and  $\mathbf{h}_n(x, y)$ are as follows for  $n \ge 1$ ,

$$K_n(x,y) = P_r \sum_{(x_i,y_i)\in\mathcal{S}_n} f(x-x_i,y-y_i)$$
(6)  
$$\mathbf{h}_n(x,y) \sim \mathcal{CN}(\mathbf{0},K_n(x,y)\Psi)$$

The locations of the level-n nodes are given by

$$\mathcal{S}_n = \{(x, y) \in \mathcal{S} \setminus \bigcup_{i=1}^{n-1} \mathcal{S}_i : |\mathbf{h}_{n-1}(x, y)|^2 \ge \tau\}.$$
 (7)

Note that a node transmits the message only once. An important question in the considered cooperative protocol is how do the network parameters such as  $P_s$ ,  $P_\tau$ ,  $\tau$  affect the network behavior. To be able to answer such questions, we need to understand how the sets  $S_1, S_2, \cdots$  evolve as the message moves forward. For this purpose we will consider the continuum model described next.

#### 3.2. Continuum Model

Let S denote the region where the nodes lie. Define  $A := \text{Area}(\mathbb{S})$ . Let  $\rho = N/A$  be the density [nodes/unit area] of relays. In the continuum model we are interested in the high density asymptote. That is, the number of relay nodes N goes to infinity, while A and the *total relay power*  $P_rN$  are fixed. This implies that the *relay power per unit area*  $\bar{P}_r := P_rN/A = P_r\rho$  is also fixed. Let  $P_n(x, y)$  be the probability of that the node  $(x, y) \in S_n$ . After the source transmission, we have

$$\kappa_{0}(x,y) = P_{s}f(x,y)$$

$$\mathbf{h}_{0}(x,y) \sim \mathcal{CN}(\mathbf{0},\kappa_{0}(x,y)\Psi)$$

$$P_{1}(x,y) = Pr\{|\mathbf{h}_{0}(x,y)|^{2} \ge \tau\} = Pr\{(x,y) \in \mathcal{S}_{1}\}$$

$$(8)$$

We can derive  $\kappa_1(x, y)$  as follows.

**Lemma 1** As  $N \rightarrow \infty$ ,  $K_1(x, y) \rightarrow \kappa_1(x, y)$  in probability, where

$$\kappa_1(x,y) = \iint_{\mathbb{S}} \bar{P}_r P_1(u,v) f(x-u,y-v) du dv.$$

**Proof** Since  $P_r = \overline{P}_r A/N$ , we can rewrite  $K_1(x, y)$  as

$$K_1(x,y) = \bar{P}_r \frac{A}{N} \sum_{i=1}^N g(x - x_i, y - y_i) \mathbf{1}_{\{(x_i, y_i) \in S_1\}},$$

where  $\mathbf{1}_{\{E\}}$  is the indicator function. We assume that initially the nodes are uniformly distributed. Then, as  $N \to \infty$ , using the *law of large numbers* and averaging over the random variable  $\mathbf{1}_{\{(x_i, y_i) \in S_1\}}$  and over the nodes random location;

$$\kappa_{1}(x,y) = AP_{r}\mathbb{E}\{f(x-x_{i},y-y_{i})\mathbf{1}_{\{(x_{i},y_{i})\in\mathcal{S}_{1}\}}\}$$
  
=  $A\bar{P}_{r}\mathbb{E}\{\mathbb{E}\{f(x-x_{i},y-y_{i})\mathbf{1}_{\{(x_{i},y_{i})\in\mathcal{S}_{1}\}}|(x_{i},y_{i})\}\}$   
=  $A\bar{P}_{r}\mathbb{E}\{f(x-x_{i},y-y_{i})P_{1}(x_{i},y_{i})\}.$ 

Since the nodes are uniformly distributed,

$$K_1(x,y) \rightarrow \iint_{\mathbb{S}} \bar{P}_r P_1(u,v) f(x-u,y-v) du dv$$

where the convergence is in probability.

Analogously, we can prove that  $K_n(x, y)$  in (7) tends to a deterministic value. We can define the following quantities.

$$\kappa_n(x,y) = \iint_{\mathbb{S}} \bar{P}_r P_n(u,v) f(x-u,y-v) du dv$$
  

$$\mathbf{h}_n(x,y) \sim \mathcal{CN}(\mathbf{0}, \kappa_n(x,y)\Psi)$$
  

$$\rho_n(x,y) = \prod_{i=1}^{n-1} (1 - Pr\{|\mathbf{h}_i(x,y)|^2 < \tau\})$$
  

$$P_{n+1}(x,y) = Pr\{|\mathbf{h}_n(x,y)|^2 > \tau\}\rho_n(x,y) \qquad (9)$$

where  $\rho_n(x, y)$  is the probability that a node located (x, y) is not in any of the  $S_i$ ,  $i = 1 \dots n-1$ . The set of equations (9) provides the system of integral-difference equations whose solution specifies the statistics of the network dynamics. The only function missing is the term  $Pr\{|\mathbf{h}_n(x, y)|^2 > \tau\}$  which is done next.

**Remark:** A similar analysis can be done for systems in which the nodes in a given level transmit in orthogonal channels (as in TDMA or FDMA) or use a space-time code. Under these scenarios, the receiver achieves the maximal ratio combining gains. The analysis of such networks with a simple deterministic channel model is done in [7]. When each node has a separate orthogonal channel, considering random signals does not complicate the problem, because in that case  $|\mathbf{h}_n(x,y)|^2 \rightarrow \kappa_n(x,y)$  as node density increases (the proof follows from the *law of large numbers*). Hence, the analysis in [7] is also valid under such scenarios.

# **3.3. Calculation of** $Pr\{|\mathbf{h}_n(x,y)|^2 > \tau\}$

The random variable  $\mathbf{t} := |\mathbf{h}_n(x, y)|^2$  is a quadratic function of Gaussian random variables. We can easily find the characteristic function of  $\mathbf{t}$ ,

$$\Phi_0(jw) = \frac{1}{|\mathbf{I} + jw\kappa_n(x,y)\Psi|} = \prod_{i=1}^M \frac{1}{(1 + jw\lambda_i\kappa_n(x,y))^{a_i}}$$

where  $\Psi$  is such that  $\{\Psi\}_{i,j} = \Psi(i,j;\Delta)$  and  $\Psi(i,j;\Delta)$  is defined in 3. Let  $\lambda_i$  be the *i*-th distinct eigenvalue of  $\Psi$  with corresponding multiplicity of  $a_i$  for  $i = 1 \dots M$ . Let  $\sum_{i=1}^{M} a_i := L$ .

**Lemma 2** The probability  $Pr\{\mathbf{t} > \tau\}$  is

$$Pr\{|\mathbf{h}_{n}(x,y)|^{2} > \tau\} = \sum_{i=1}^{M} \sum_{k=1}^{a_{i}} \frac{A_{ik}}{(k-1)!} \Gamma(k, \frac{\tau}{\lambda_{i}\kappa_{n}(x,y)}),$$

where  $\Gamma(a, x) = \int_x^\infty e^{-t} t^{a-1} dt$ . If we assume that  $\Psi$  has distinct eigenvalues, then the above expression simplifies to

$$Pr\{|\mathbf{h}_{n}(x,y)|^{2} > \tau\} = \sum_{i=1}^{L} A_{i1}e^{-\tau/\kappa_{n}(x,y)\lambda_{i}}.$$
 (10)

**Proof** We omit the proof for brevity. Note that  $A_{ik}$  is obtained from partial fraction expansion of  $\Phi_0(jw)$ .

For any n,  $P_n(x, y)$  can be calculated in a similar way.

### 3.4. Retransmission Criterion in AWGN

In this section, we deal with the more realistic retransmission model than in (1). That is,

$$\mathbf{r}_n(x,y) = \mathbf{C} \, \mathbf{h}_n(x,y) + \mathbf{w}_n(x,y),$$

where  $\mathbf{w}_n(x, y) \sim \mathcal{CN}(0, \sigma^2 \mathbf{I})$  represent the additive white Gaussian noise (AWGN) samples and  $\mathbf{C}$  is defined as in (1). The index n indicates the level that produced the received signal. The definition of the set of n-th level nodes (7) is replaced by

$$\mathcal{S}_n = \{ (x, y) \in \mathcal{S} \setminus \bigcup_{i=1}^{n-1} \mathcal{S}_i : \mathbf{r}_n^H(x, y) \mathbf{C} (\mathbf{C}^H \mathbf{C})^{-1} \mathbf{C}^H \mathbf{r}_n(x, y) \ge \tau \}$$

Define  $\mathbf{z}_n(x, y) = (\mathbf{C}^H \mathbf{C})^{-1/2} \mathbf{C}^H \mathbf{r}_n(x, y)$ , then

$$egin{array}{rcl} {}_n(x,y) &\sim & \mathcal{CN}(\mathbf{0},K_n(x,y)\mathbf{F}+\sigma^2\mathbf{I}) \ \mathbf{F} &= & (\mathbf{C}^H\mathbf{C})^{1/2}\Psi(\mathbf{C}^H\mathbf{C})^{1/2}. \end{array}$$

Under continuum approximation  $K_n(x, y) \rightarrow \kappa_n(x, y)$  (Section 3.2). Since  $\mathbf{z}_n(x, y)$  is a Gaussian random vector, the results obtained in the previous section can be adapted here easily replacing the matrix  $\kappa_n(x, y)\Psi$  with the matrix  $\kappa_n(x, y)\mathbf{F} + \sigma^2 \mathbf{I}$ .

## 4. NUMERICAL EVALUATIONS

In spite of the several simplification made, the relationships expressed among the functions  $P_n(x, y)$ ,  $\rho_n(x, y)$  and  $\kappa_n(x, y)$  are integral-difference equations that are hard to solve analytically. We solved them numerically and gained intuition on how the network parameters  $P_s$ ,  $P_r$  and the threshold  $\tau$  affect the dynamics of the network.

Based on the numerical solutions, we come to the following conclusions on how the message propagation depends on the ratio  $\tau/\bar{P}_r$ . At small values of the  $\tau/\bar{P}_r$ , the message propagates in the network when initiated with sufficient source power  $(P_s > P_{s0})$ . On the other hand, for large values of  $\tau/\bar{P}_r$ , the message propagation is bounded to a region, and dies out eventually independent of what is the value of  $P_s < \infty$ . This is made evident by the fact that  $\forall n$  the function  $P_n(x, y) \to 0$  for (x, y) beyond a certain



Fig. 1. Transmissions continue.

radius from the source. Because  $P_n(x, y)$  represents the probability that the node in point (x, y) will transmit as part of the level  $n, P_n(x, y) \to 0$  means that the point (x, y) will never transmit. Figures 1 and 2 shows two different regions of operation. Here, we assume that the network lies in a disc and the source is at its center. Due to the symmetry, we plot  $P_n(r) \triangleq P_n(\sqrt{x^2 + y^2})$ , where r is the distance to the source.

We can also analyze the behavior of the network when there exists a false alarm, i.e. a node that erroneously detects the presence of a message and retransmits it. The question is if alarm propagates catastrophically among all relays. This case can be easily analyzed with the same model by simply replacing the source power  $P_s$  in the initial conditions (8) by the relay power  $P_r$ . That is, the derived dynamical system equations can be used with different initial condition, i.e.  $\kappa_0 = P_r f(x, y)$ .

In order to prevent such false message propagation, we suggest a simple power control policy. Since for small values of the  $\tau/\bar{P}_r$ , there exists a threshold for source power to initiate a flow, the protocol can be designed such that  $P_r$  is sufficiently smaller than  $P_{s0}$ .

Due to above proposition, by designing the source power and relay power one can prevent the false alarm considerably. This will require to increase the network density in order to keep the same performance for the probability of detection.

## 5. CONCLUSION

In this paper, we analyzed the behavior of networks with cooperative transmissions. The analysis is based random channel models and the idea of continuum approximation, which models the networks with high node density. We believe the techniques used in this paper can be useful in the analysis of other cooperative protocols.



Fig. 2. Transmissions die out.

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