Cooperative Transmission for Random Access Wireless Networks

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***Abstract*— [1]We propose a novel high-throughput medium access scheme for wireless networks that is suitable for bursty sources. We view the wireless network as a spatially distributed antenna, with antenna elements linked via the wireless channel. When there is a collision, the packets involved in the collision are saved in a buffer. In the slots following the collision, a set of nodes, designated as relays, form an alliance and bounce off the signal that they received during the collision slot. By processing the originally collided packets and the signals forwarded by the relays, the destination node can formulate and solve a multiple-input multiple-output problem, the inputs of which are the original packets. The spatial diversity introduced via the cooperative relaying enables us to effectively deal with the wireless channel without any bandwidth expansion nor additional antenna hardware. The proposed scheme maintains the benefits of ALOHA systems in the sense that all nodes share access to media resources efficiently and without extra scheduling overhead, and enables efficient use of network power.**

I. Introduction

It is expected that future wireless network will need to accommodate bursty /multimedia traffic. Fixed bandwidth allocation schemes are inefficient for such traffic. Simple medium access schemes for bursty sources include random access methods, an example of which is the slotted ALOHA [1]. By allowing users to transmit in an uncoordinated fashion, slotted ALOHA does not require overhead for connection establishment before each transmission and provides high efficiency. However, its throughput is limited under heavy traffic load, mainly because the packets that collide are totally discarded. This problem was recently tackled by the network assisted diversity multiple access (NDMA) approach [3]. According to [3], the collision can be resolved by combining the collided data with delayed retransmissions of those packets. However, the main assumption in that approach is that the channel coefficients are uncorrelated between adjacent slots, which is rather unrealistic.

We here investigate a new means of diversity for collision resolution, which is generated through cooperative retransmissions. The concept of cooperative diversity was first proposed in [4], [6]. In [6] users form pairs, and the members of each pair transmit a linear combination of their own signal and the signal transmitted by their counterpart during the previous slot. In [4], each packet requires two slots for transmission. Each node transmits its own packet in the first slot and a relay node forwards this packet in the second slot. These schemes are still based on the fixed bandwidth allocation where certain channel/bandwidth is assigned to each node for the purpose of transmitting its packets or relaying other packets. Also, in those approaches, the nodes are not sharing the channel. Therefore, such schemes would not provide efficiency in data networks with bursty traffic.

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In the scheme proposed here, the users send packets in a bursty manner over a common additive white Gaussian noise channel. The users do not coordinate their transmissions until a collision has been detected at the receiver. The collision will trigger the start of a cooperative transmission epoch, during each slot of which one randomly selected relay node will bounce off the packets they received during the collision slot. By processing the originally collided packets and the signals forwarded by the relays, the destination node can recover the original packets. The scheme does not require extra scheduling overhead. It combines time and spatial diversity through cooperative transmission, thus is robust in slowly varying wireless channel. While employing a single transmit/receive antenna at each node, the diversity advantage is achieved without sacrificing bandwidth resources. The protocol is also energy efficient since in each slot of the cooperative transmission epoch only one relay node at a time is required to transmit.

In the sequel, we describe the proposed method is detail. We also provide diversity analysis along with that of the NDMA scheme for comparison purposes.

II. The Proposed Medium Access Plan

Throughput this paper we will consider a small-scale slotted multi-access system, where each node can hear from the BS/AP on a control channel. Therefore, we do not deal with synchronization issues, near-far effects, or link delays. All transmitted packets have the same length, each packet requires one time unit / slot for transmission and all transmitters are synchronized. Only non-regenerative relays are used, and no decoding will be performed by the relay.

All nodes operate in a half duplex mode, i.e., they cannot receive and transmit using the same channel resources due to the strong self-interference from the transmitter on the received signal. Every node is equipped with only one antenna, and each node in the system obtains feedback from the base station (BS) /access point (AP) specifying whether the packet was transmitted successfully.

The scheme to be outlined next is described in the context of cellular networks or wireless LAN, where a set of nodes, denoted by $\mathcal{J} = \{1, 2, \cdots, J\}$, communicate with the BS /access point. Thus, all transmissions initiated by a source node $i \in \mathcal{J}$ are directed to a single destination $d \notin \mathcal{J}$. However, the scheme could be modified to support ad-hoc networks, where a node is designated to be a leader based on its available battery and computational power.

Suppose that K packets have collided in the n-th slot. Once the collision has been detected, the system enters a *cooperative transmission epoch* (CTE). The BS /access point will send a control bit to all nodes indicating the beginning of a CTE and will continue sending this bit until the CTE is over. Therefore one bit per time slot is needed on the control channel to indicate whether the channel is available for new transmissions, or there is an ongoing CTE.

The CTE consists of $\hat{K}-1$ slots, with $\hat{K} \geq K$. The nodes that are not involved in the collision nor act as relays will remain silent until the CTE is over. During slot $n+k, 1 \leq k \leq \hat{K}-1$, one node is selected to act as relay and bounce off the signal mixture that it received during the n-th slot (collision slot). The selection of that relay node, i.e., r, can be based on a predetermined order, for example, $r = mod(n+k, J)+1$. Due to the half duplex assumption, if the chosen node happens to be a source node, it will simply retransmit its own packet. Thus, only one relay is active during each of the slots of the CTE.

In each slot of the CTE, the BS receives either a mixture of the packets that collided during slot n, or retransmission of a signal from a single source node. When $k > \hat{K}$, the broadcast channel will reset the control bit to terminate current CTE and all related transmissions thereafter. A feedback about whether the collided packets are received successfully will be sent over the control channel to the corresponding users once the packets are recovered. The nodes themselves are responsible for adjusting their behavior on re-sending failed packets based on the feedback they receive.

The parameter $\hat{K}$ plays a crucial role in the proposed approach. As it will be shown in the sequel, the collided packets can be recovered only if $\hat{K}$ is at least equal to K. In case of deep fading, where some relay transmissions can be too corrupted to be of use, $\hat{K}$ will need to be greater than K, so that the BS receives enough usable mixtures of the collided packets. The selection of $\hat{K}$ will be discussed in later in this paper.

We should emphasize that although each node should be connected to the BS/AP, the nodes do not have to be connected to each other. In other words, we do not require a fully connected network. If some relays can only hear from a subset of the source nodes, the mixing matrix will have zeros in the corresponding entries. As long as the mixing matrix maintains full rank the packets would still be recoverable. Loss of rank of the mixing matrix can be addressed by a CTE extension.

III. System Model of Cooperative Transmission

We consider a flat fading channel. The extension to a frequency selective channel is relatively simple and can be done along the lines of [11]. Let n denote the collision slot, and let the packet transmitted by the i-th node at slot n consists of N symbols $\boldsymbol{x}_i(n) \triangleq [x_{i,0}(n), \cdots, x_{i,N-1}(n)]$.

Let $\mathcal{S}(n) = \{i_1, \cdots, i_K\}$ be the set of sources, and $\mathcal{R}(n) = \{r_1, \cdots, r_{\hat{K}-1}\}$ the set of nodes that will serve as relays during the CTE. During n-th slot, the signal heard by the the BS and also by all non-source nodes is:

$$\boldsymbol{y}_r(n) = \sum_{i \in \mathcal{S}(n)} a_{ir}(n)\boldsymbol{x}_i(n) + \boldsymbol{w}_r(n) \tag{1}$$

where $r \in \{d\} \bigcup \mathcal{R}(n), r \notin \mathcal{S}(n)$, with $a_{ir}(n)$ denoting the channel coefficient between the i-th source node and the receiving node; $\boldsymbol{w}_r(n)$ representing noise; and $\{d\}$ denoting the destination node.

During the $(n+k)$-th slot, a relay will retransmit its own signal if it was a source during slot n, or the signal it received during slot n, after scaling. Let $c(n+k)$ represent the scaling constant, which is selected so that the transmit power is maintained within the constraints of the relay's transmitter. The BS will then receive:

$$\boldsymbol{z}_d(n+k) = \begin{cases} a_{rd}(n+k)\boldsymbol{x}_r(n) + \boldsymbol{w}_d(n+k), \\ \qquad r \in \mathcal{R}(n) \bigcap \mathcal{S}(n) \\ a_{rd}(n+k)c(n+k)\boldsymbol{y}_r(n) + \boldsymbol{w}_d(n+k) \\ \qquad r \in \mathcal{R}(n), \quad r \notin \mathcal{S}(n) \end{cases} \tag{2}$$

where $\boldsymbol{z}_d(n+k)$ is a $1 \times N$ vector; $\boldsymbol{w}_d(n+k)$ denotes the noise vector at the access point.

Let us define matrices $\boldsymbol{X}$, whose rows are the signals sent by source nodes i.e., $\boldsymbol{X} = [\boldsymbol{x}_{i_1}^T(n), \cdots, \boldsymbol{x}_{i_K}^T(n)]^T$, and $\boldsymbol{Z}$, whose rows are the signals heard by the destination node during slots $n, n+1, \cdots, n+\hat{K}-1$, i.e., $\boldsymbol{Z} = [\boldsymbol{z}_d^T(n), \boldsymbol{z}_d^T(n+1), \cdots, \boldsymbol{z}_d^T(n+\hat{K}-1)]^T$ with $\boldsymbol{z}_d(n) = \boldsymbol{y}_d(n)$. Let us assume that among the $\hat{K}-1$ nodes, the first l nodes are relay nodes with $0 \leq l \leq \hat{K}-1$ and the remaining are source nodes, i.e. $(r_{l+1}, \cdots, r_{\hat{K}-1}) \subseteq \mathcal{S}(n)$.

The received signal at the destination can be written in matrix form as:

$$\boldsymbol{Z} = \boldsymbol{H}\boldsymbol{X} + \boldsymbol{W} \tag{3}$$

where

$$\begin{aligned} \boldsymbol{H} &= \begin{bmatrix} 1 & \mathbf{0}_{1\times l} & \mathbf{0}_{1\times \hat{K}-l-1} \\ \mathbf{0}_{l\times 1} & \boldsymbol{B} & \mathbf{0}_{l\times \hat{K}-l-1} \\ \mathbf{0}_{\hat{K}-l-1\times 1} & \mathbf{0}_{\hat{K}-l-1\times l} & \mathbf{I}_{\hat{K}-l-1} \end{bmatrix} \begin{matrix} \boldsymbol{A} \\ \boldsymbol{A}_r \end{matrix} ; \\ \boldsymbol{W} &= \boldsymbol{W}_1 + \begin{bmatrix} \mathbf{0}_{1\times N} \\ \boldsymbol{B}\boldsymbol{W}_2 \\ \mathbf{0}_{\hat{K}-l-1\times N} \end{bmatrix} \end{aligned} \tag{4}$$

$\boldsymbol{A}$ is a $(l+1) \times K$ channel coefficient matrix with $A_{1n} = a_{i_n d}$ and $A_{mn} = a_{i_n r_{m-1}}(n)$ for $1 < m \leq l+1$; The $(\hat{K}-l-1) \times K$ matrix $\boldsymbol{A}_r = [\mathbf{0}_{(\hat{K}-l-1)\times(K-\hat{K}+l+1)} \quad \boldsymbol{E}]$ contains channel gains for those source nodes that are selected during the CTE to retransmit their own packets, with $\boldsymbol{E} = Diag[a_{r_{l+1},d}(n+l+1), \cdots, a_{r_{\hat{K}-1},d}(n+\hat{K}-1)]$; $\boldsymbol{W}_1 = [\boldsymbol{w}_d^T(n), \cdots, \boldsymbol{w}_d^T(n+\hat{K}-1)]^T$ denoting the noise received by destination; If $l \geq 1$, $\boldsymbol{B} = Diag[c(n+1)a_{r_1 d}(n+1), \cdots, c(n+l)a_{r_l,d}(n+l)]$ and $\boldsymbol{W}_2 = [\boldsymbol{w}_{r_1}^T(n), \cdots, \boldsymbol{w}_{r_l}^T(n)]^T$ representing noise at the relay nodes; otherwise, they are both empty matrices.

The channel estimation and active user detection is done through the orthogonal ID sequence that are attached to each packet as in [3]. At the BS, the correlation of the received signal and the ID sequences is performed. The collision order K, can be detected by comparing the result of the correlation to a pre-defined threshold. The ID sequences are also used as pilots for channel estimation.

The CTE extends over $\hat{K}-1$ slots with $\hat{K} > K$. $\hat{K}$ is maintained and updated by the BS. Initially, the BS will set $\hat{K} = K$. If the channel conditions between relay and destination during a certain CTE slot is so bad that it impossible for the BS to collect information, the BS will increase $\hat{K}$ by one. The BS will continue updating $\hat{K}$ until enough information is gathered for resolving the packets.

Once the $\hat{K} \times K$ mixing matrix $\boldsymbol{H}$ is estimated, the transmitted packets can be obtained via a maximum likelihood (ML) decoder. An alternative way, which is computationally simpler, is to use linear decoder, for example, Zero Forcing (ZF) decoder.

IV. Diversity Advantage

To isolate the benefits of the space diversity introduced by the proposed scheme, we perform pairwise error probability (PEP) analysis under static channel, where temporal diversity is not available. In that case, simple retransmission of the collided packets by their initial senders will fail. In the derivation, we treat the transmitted symbols as nuisance parameters with unknown deterministic values.

The diversity order b, is defined as [9] $\lim_{SNR\to\infty} \frac{\log P_e(\boldsymbol{X}\to\hat{\boldsymbol{X}})}{\log SNR} = -b$, where $P_e(\boldsymbol{X} \to \hat{\boldsymbol{X}})$ is the probability that signal $\hat{\boldsymbol{X}}$ is the output of ML detection when the symbol block $\boldsymbol{X}$ was sent during the collision slot and the following CTE.

We assume that: (A1) The channel coefficients in matrices $\mathbf{A}$, $\mathbf{A}_r$ and $\mathbf{B}$ are independent and identically-distributed (i.i.d.) zero-mean, circularly-symmetric complex Gaussian random variables with variance σ_a^2; (A2) The power of transmitted symbols is σ_x^2; The noise is taken as a complex, zero-mean white Gaussian variable with variance σ_w^2; (A3) During the CTE, the gain $c = \sqrt{\frac{\sigma_x^2}{K\sigma_a^2\sigma_x^2+\sigma_w^2}}$ is applied at the relay nodes, so that the average energy for each relay transmitter is kept equal to σ_x^2 for a K fold transmission.

Proposition 1: Consider the scheme described by (3). Under assumptions (A1)-(A3) and for high SNR it holds:

$$P_e(\boldsymbol{X} \to \hat{\boldsymbol{X}}) \leq (SNR/4)^{-(l+1)r} \frac{\int_0^\infty \frac{f(\gamma_0,\gamma_i)}{\gamma_0^r \prod_{i=1}^l \gamma_i} d\gamma_0 d\gamma_i}{\prod_{j=1}^r \lambda_j^0 \quad \prod_{j=1}^r \lambda_j}{}^l \tag{5}$$

where λ_i, r are respectively the eigenvalues and rank of matrix $(\hat{\boldsymbol{X}} - \boldsymbol{X})(\hat{\boldsymbol{X}} - \boldsymbol{X})^H$; λ_i^0 are eigenvalues of matrix $\boldsymbol{D}_g(\hat{\boldsymbol{X}} - \boldsymbol{X})(\hat{\boldsymbol{X}} - \boldsymbol{X})^H \boldsymbol{D}_g^H$ with $\boldsymbol{D}_g = Diag[\boldsymbol{I}_{K-\hat{K}+l+1}, 2\boldsymbol{I}_{\hat{K}-l-1}]$;

$$\gamma_i \triangleq |a_{r_i d}|^2 / \left. \sum_{i=1}^{l} |a_{r_i d}|^2/\sigma_a^2 + K \right) \quad with \quad a_{r_0 d} = \sqrt{K}\sigma_a$$

and $f(\gamma_0, \gamma_i)$ is the joint probability density function of random variables γ_0 and $\gamma_i, i = 1, \cdots, l$.

Proof: See Appendix A.

Before we discuss the above result, we provide PEP analysis for the NDMA scheme. According to NDMA, in case of $K-$fold collision, the collided users retransmit their packets in the subsequent $K-1$ slots. Let $\tilde{\boldsymbol{Z}}$ denote the received signal matrix at the destination; $\tilde{\boldsymbol{X}}$ and $\tilde{\boldsymbol{W}}$ denote the transmitted symbol matrix and the noise, respectively. The channel model for the NDMA scheme is given by:

$$\tilde{\boldsymbol{Z}} = \tilde{\boldsymbol{A}}\tilde{\boldsymbol{X}} + \tilde{\boldsymbol{W}} \tag{6}$$

where $\tilde{\boldsymbol{A}} \in \mathbb{C}^{K\times K}$ is the channel gain matrix during the K fold collision resolution epoch.

We assume that: (B1) The elements in the channel gain matrix $\tilde{A}_{ij} \sim \mathcal{N}(0, \sigma_a^2)$; (B2) The noises are independent with $\mathcal{N}(0, \sigma_w^2)$; (B3) The correlation matrix of channel coefficients $\boldsymbol{R}_{\tilde{A}} = E(\tilde{\boldsymbol{A}}\tilde{\boldsymbol{A}}^H)$ has rank m with $m \leq K$.

Proposition 2: For the NDMA scheme, under assumptions (B1)-(B3) and for high SNR, it holds:

$$P_e(\tilde{\boldsymbol{X}} \to \hat{\tilde{\boldsymbol{X}}}) \leq (SNR/4)^{-rm} \prod_{i=1}^{rm} \lambda_i^{-1}(\boldsymbol{S}) \tag{7}$$

where $\lambda_i(\boldsymbol{S})$ denotes the i-th singular value of matrix $\boldsymbol{S} = \boldsymbol{\Theta} \otimes (\hat{\tilde{\boldsymbol{X}}} - \tilde{\boldsymbol{X}})(\hat{\tilde{\boldsymbol{X}}} - \tilde{\boldsymbol{X}})^H/\sigma_x^2$, $\otimes$ denotes Kronecker product; r is the rank of matrix $(\hat{\tilde{\boldsymbol{X}}} - \tilde{\boldsymbol{X}})(\hat{\tilde{\boldsymbol{X}}} - \tilde{\boldsymbol{X}})^H$; and $\boldsymbol{\Theta}$ is a diagonal matrix containing the m non-zero eigenvalues of matrix of $E(\tilde{\boldsymbol{A}}\tilde{\boldsymbol{A}}^H)$ along its diagonal.

Proof: See Appendix B.

Remarks: The result for the proposed method (see (5)) indicates that, independent of the channel conditions, for any coding scheme that maintains $r \geq 1$ and any epoch involving K collided packets and l relays, diversity order equal or greater than $l+1$ can be achieved. More relays will result in higher diversity order. However, in the case where all relays are source nodes, the diversity gain becomes r. This can happen at high traffic loads, where most nodes are involved in collisions. They will simply retransmit their own packets one by one (like in TDMA) instead of using a relay. On the other hand, the diversity gain of NDMA scheme depends on the rank of the correlation matrix of channel coefficient [5]. If the channel varies slowly so that all the rows of the correlation matrix have basically the same value, the diversity factor will reduce to r.

V. Simulation Results

We considered that total number of users in the network is $J = 32$. To investigate the network performance under certain traffic load, a Bernoulli model was used, i.e., we performed a random experiment consisting of M repeated independent trials. In each trial, all users are statistically the same, and each user sends out their packets, with probability $\frac{\lambda}{J}$. Packets received at the BS with bit error rate higher than $P_e = 0.02$ were considered lost or corrupted. The throughput is defined as the average number of packets that are successfully transmitted in a time slot under traffic load λ. Therefore, if a CTE of length $\hat{K} - 1$ is needed to resolve k packets, then the instant throughput is $k/\hat{K}$. All transmitted packets were considered lost if the BS made an error in identifying the active users during the collision slot.

The users'ID sequences were selected based on a J-th order Hadamard matrix. The length of packets was fixed to $N = 424$ bits (equal to the length of an ATM cell). Each packet contained 4-QAM symbols. Maximum likelihood decoding was used at the receiver to fully explore the diversity advantage of the scheme. The wireless channel was simulated according to the sum-of-sinusoids simulation model for Rayleigh fading channels of [12]. At 5.2 GHz carrier frequency, Doppler frequency shift $f_d = 52$ Hz corresponds to relative vehicle speed of $v = 3$ m/s. Fig.1(a) show the throughput versus the traffic load, for $SNR = 15$dB and $SNR = 20$dB, respectively. As a result of high data transmission rate $R = 12M$ bps and the users' speed, the channel gains at neighboring time slots are correlated, rendering the throughput of the NDMA method worse than that of the proposed one. We also compared our method with the slotted ALOHA, which is not able to handle multi-packet transmission. The proposed scheme clearly outperforms both methods. Fig 1(b), corresponds to the same scenario except that now the data rate was 256 kbps, thus effectively considering a faster varying channel. It is interesting to note that the performance of the proposed method has not changed between the two aforementioned different channel conditions and thus is more robust in wireless fading environment. Although now it looks like that the NDMA achieves similar performance as the proposed one with non-correlated channel coefficients, we have to take into consideration that the proposed method consumes less power than the NDMA. Although NDMA scheme could have the collided users transmit one at a time during the collision resolution epoch, that would need more control overhead.

VI. Conclusions

We presented a novel medium access protocol for wireless networks based on a distributed transmission scheme. The spatial diversity introduced via the cooperative relaying enables us to effectively deal with the wireless channel without any bandwidth expansion nor additional antenna hardware. The proposed scheme maintains the benefits of ALOHA systems in the sense that all nodes share access to media resources efficiently and without extra scheduling overhead, and enables efficient use of network power. Several improvements can be made that address fairness, prioritization, optimum selection of relays; those will be subject of future research.

Appendix A: Proof of Proposition 1

Let us define $\boldsymbol{B}' = Diag[0, ca_{r_1 d}(n+1), \cdots, ca_{r_l,d}(n+l), 0, \cdots, 0]$. According to equation (3), the conditional probability of the receiver deciding erroneously in favor of $\hat{\boldsymbol{X}}$ when $\boldsymbol{X}$ was transmitted equals [7]:

$$
\begin{aligned}
& P_e(\boldsymbol{X} \rightarrow \hat{\boldsymbol{X}} | \boldsymbol{H}) \\
= \; & Q\left(\frac{\|\boldsymbol{H}(\hat{\boldsymbol{X}} - \boldsymbol{X})\|^2}{\sqrt{2\sigma_w^2 \left(\|\boldsymbol{H}(\hat{\boldsymbol{X}} - \boldsymbol{X})\|^2 + \|\boldsymbol{B}'^H \boldsymbol{H}(\hat{\boldsymbol{X}} - \boldsymbol{X})\|^2 \right)}} \right)
\end{aligned} \tag{A.1}
$$

where $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy$.

Using properties of Frobenius norm and applying the Chernoff bound [8] we get:

$$
P_e(\boldsymbol{X} \rightarrow \hat{\boldsymbol{X}} | \boldsymbol{H}) \leq \exp\left(-\frac{\|\boldsymbol{H}(\hat{\boldsymbol{X}} - \boldsymbol{X})\|^2}{4\sigma_w^2(\|\boldsymbol{B}\|^2 + 1)} \right) \tag{A.2}
$$

Taking into account that $\boldsymbol{B}$ is a diagonal matrix and $\sigma_w^2 \ll \sigma_x^2$ at high SNR, we notice that

$$
\|\boldsymbol{B}\|^2 + 1 = \sum_{i=1}^{l} \frac{\sigma_x^2 |a_{r_i d}|^2}{K\sigma_a^2\sigma_x^2 + \sigma_w^2} + 1 \approx \sum_{i=1}^{l} \frac{|a_{r_i d}|^2}{K\sigma_a^2} + 1 \tag{A.3}
$$

Let $\boldsymbol{r}_i$ denote the i-th row of matrix $\boldsymbol{A}_r$ and $\boldsymbol{a}_i$ is the i-th row of matrix $\boldsymbol{A}$. For the static channel case, it holds:

$$
\begin{aligned}
& \|\boldsymbol{H}(\hat{\boldsymbol{X}} - \boldsymbol{X})\|^2 = trace(\boldsymbol{H}(\hat{\boldsymbol{X}} - \boldsymbol{X})(\hat{\boldsymbol{X}} - \boldsymbol{X})^H \boldsymbol{H}^H) \\
= \; & \boldsymbol{g}\boldsymbol{D}_g(\hat{\boldsymbol{X}} - \boldsymbol{X})(\hat{\boldsymbol{X}} - \boldsymbol{X})^H (\boldsymbol{g}\boldsymbol{D}_g)^H \\
& + \sum_{i=2}^{l+1} \frac{\sigma_x^2 |a_{r_i d}|^2}{K\sigma_a^2\sigma_x^2 + \sigma_w^2} \boldsymbol{a}_i(\hat{\boldsymbol{X}} - \boldsymbol{X})(\hat{\boldsymbol{X}} - \boldsymbol{X})^H \boldsymbol{a}_i^H
\end{aligned} \tag{A.4}
$$

where $\boldsymbol{g} = [a_{i_1,d}, \cdots, a_{i_K,d}]$, which is only related to the channel between sources nodes and BS. $\boldsymbol{D}_g = Diag[\boldsymbol{I}_{K-\hat{K}+l+1}, 2\boldsymbol{I}_{\hat{K}-l-1}]$. Following [9], and performing SVD we obtain:

$$
\begin{aligned}
\frac{1}{\sigma_x^2}(\hat{\boldsymbol{X}} - \boldsymbol{X})(\hat{\boldsymbol{X}} - \boldsymbol{X})^H &= \boldsymbol{Q}^H \boldsymbol{\Lambda} \boldsymbol{Q}, \\
\frac{1}{\sigma_x^2}\boldsymbol{D}_g(\hat{\boldsymbol{X}} - \boldsymbol{X})(\hat{\boldsymbol{X}} - \boldsymbol{X})^H \boldsymbol{D}_g^H &= \boldsymbol{Q}_0{}^H \boldsymbol{\Lambda}_0 \boldsymbol{Q}_0
\end{aligned} \tag{A.5}
$$

where $\boldsymbol{Q}, \boldsymbol{Q}_0$ are unitary matrices and $\boldsymbol{\Lambda} = Diag[\lambda_1, \cdots, \lambda_K]$ and $\boldsymbol{\Lambda}_0 = Diag[\lambda_1^0, \cdots, \lambda_K^0]$ are real diagonal matrices.

For $i = 2, \cdots, l+1$, let $\boldsymbol{a}_i\boldsymbol{Q}^H = (\beta_{i1}, \cdots, \beta_{iK})$ and $\boldsymbol{g}\boldsymbol{Q}_0^H = (\alpha_1, \cdots, \alpha_K)$. Then we get

$$
\|\boldsymbol{H}(\hat{\boldsymbol{X}} - \boldsymbol{X})\|^2 \approx \sigma_x^2 \sum_{i=1}^{l} \frac{|a_{r_i d}|^2}{K\sigma_a^2} \sum_{j=1}^{K} \lambda_j |\beta_{i+1,j}|^2 + \sigma_x^2 \sum_{j=1}^{K} \lambda_j^0 |\alpha_j|^2 \tag{A.6}
$$

where the approximation is due to $\sigma_w^2 \ll \sigma_x^2$ at high SNR.

The fading coefficients are complex Gaussian random variables, thus β_{ij} and α_j are also complex Gaussian with zero mean and variance σ_a^2. Averaging (A.2) with respect to β_{ij} and α_j leads to (5).

Appendix B: Proof of Proposition 2

Eigendecompose $\boldsymbol{R}_{\tilde{A}}$ such that $\boldsymbol{R}_{\tilde{A}} = \boldsymbol{V}\boldsymbol{\Theta}\boldsymbol{V}^H$ where $\boldsymbol{\Theta} = Diag[\theta_1, \cdots, \theta_K]$ is the diagonal matrix of the eigenvalues of $\boldsymbol{R}_{\tilde{A}}$, and the columns of $\boldsymbol{V}$ are the corresponding eigenvectors. It is easy to see that $\tilde{\boldsymbol{A}}$ can be decorrelated as $\boldsymbol{A}_w = \boldsymbol{\Theta}^{-\frac{1}{2}}\boldsymbol{V}^H\tilde{\boldsymbol{A}}$. Therefore, we have $\tilde{\boldsymbol{A}} = \boldsymbol{V}\Theta^{\frac{1}{2}}\boldsymbol{A}_w$, where the entries of $\boldsymbol{A}_w$ are i.i.d. complex circular symmetric Gaussian with zero mean and variance 1.

Using properties of the Kronecker produce, the pair wise error probability for NDMA, conditioned on the channel, satisfies:

$$
\begin{aligned}
& P_e(\tilde{\boldsymbol{X}} \rightarrow \hat{\tilde{\boldsymbol{X}}} | \tilde{\boldsymbol{A}}) \\
\leq \; & \exp\left(-\frac{1}{4\sigma_w^2} trace(\tilde{\boldsymbol{A}}(\hat{\tilde{\boldsymbol{X}}} - \tilde{\boldsymbol{X}})(\hat{\tilde{\boldsymbol{X}}} - \tilde{\boldsymbol{X}})^H \tilde{\boldsymbol{A}}^H) \right) \\
= \; & \exp\left(-\frac{1}{4\sigma_w^2} trace(\boldsymbol{A}_w(\hat{\tilde{\boldsymbol{X}}} - \tilde{\boldsymbol{X}})(\hat{\tilde{\boldsymbol{X}}} - \tilde{\boldsymbol{X}})^H \boldsymbol{A}_w^H \boldsymbol{\Theta}) \right) \\
= \; & \exp\left(-\frac{1}{4\sigma_w^2} vec(\boldsymbol{A}_w^T)^T (\boldsymbol{\Theta} \otimes (\hat{\tilde{\boldsymbol{X}}} - \tilde{\boldsymbol{X}})(\hat{\tilde{\boldsymbol{X}}} - \tilde{\boldsymbol{X}})^H) vec(\boldsymbol{A}_w^H) \right)
\end{aligned} \tag{A.7}
$$

Taking into account that $rank(\boldsymbol{R}_{\tilde{A}}) = rank(\boldsymbol{\Theta}) = m$ and $rank(\boldsymbol{A} \otimes \boldsymbol{B}) = rank(\boldsymbol{A})rank(\boldsymbol{B})$, and following similar process as in (A.4)-(A.6), we arrive at the equations of Proposition 2.

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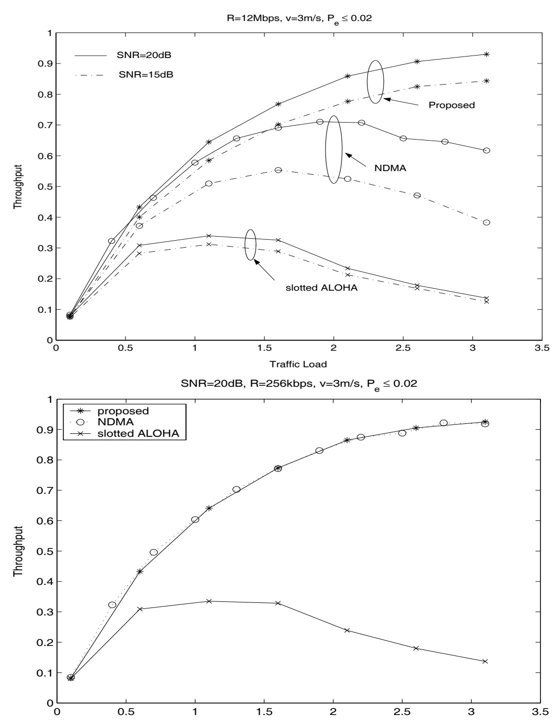

Fig. 1. Throughput versus traffic load (a) R=12Mbps. (b) R=256kbps.