# COMBATING SYNCHRONIZATION ERRORS IN COOPERATIVE RELAYS

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**ABSTRACT**

Cooperative relays have recently been proposed and studied for mobile ad hoc networks. It has been shown that under perfect symbol synchronization, parallel relays with space-time modulation can yield more than 10dB power savings over conventional serial relays in a highly mobile environment. In this paper, we analyze the effect of synchronization errors on the performance of parallel relays, and also study several effective methods to reduce the negative impact of synch errors. Our study shows that when the synch errors are much smaller than a symbol interval, the performance of parallel relays deteriorates gracefully. We have also found that well designed space-time coding techniques such as TR-STC and ST-OFDM can be highly effective in combating large synch errors with only marginal reduction of data rate.

## 1. INTRODUCTION

Cooperative relays are an important physical layer concept for mobile wireless ad hoc networks to achieve higher throughput, lower energy consumption and/or longer lifetime. A variety of cooperative relays have recently been proposed and studied in a variety of ways, e.g., [1], [2], [3]. In [1], it is shown that under perfect synchronization, a chain of mobile parallel relays with space-time modulation can yield more than 10dB power savings than a chain of conventional serial relays in a highly mobile environment. Related results are also reported in [2] and [3]. A further effort on the networking aspect of parallel relays is reported in [4].

Although achieving symbol synchronization between mobile relays is physically feasible for narrowband single- and multi-carrier systems ($20\mu s$ narrowband symbol spans $6000m$), and it is highly desirable to achieve optimal or near optimal performance with realistic complexity of the receiver, research in this area is still in its infancy. Therefore, there is a clear need to understand the effect of synch errors and to develop techniques to combat synch errors.

Mietzner and Hoeher [5] recently reported an analysis of the performance of the Alamouti code in the presence of synch errors. But their assumption about the sampling time at the receiver is incorrect when synch errors are not multiples of a symbol interval. Our recent study of the well-known symbol synchronizer and sampler, called early-late gate, suggests that the correct assumption of the sampling time at the receiver is half way between two ideal sampling instants associated with two superimposed baseband signals (assuming a two-transmitter system). In this paper, we provide an analysis of a similar system as discussed in [5] but under the correct sampling model.

To combat synch errors, Li [6] proposed a space-time coded transmission scheme for each relay. However, the data rate of the scheme becomes very low when the number of transmitters becomes large. In this paper, we show that much better transmission schemes are available to make the bit-error-rate robust against synch errors without a major reduction of the data rate. Indeed, with time-reversed space-time codes (TR-STC) or space-time orthogonal frequency division multiplexing (ST-OFDM), synchronization errors between multiple mobile transmitting nodes may become virtually non-existent at a receiving node while the data rate is only reduced marginally.

## 2. PROBLEM FORMULATION AND MODELING

Consider a single section of a chain (or route) of regenerative parallel relays where there are multiple transmitting nodes and multiple receiving nodes [1]. Each transmitting node transmits a stream of symbols towards all receiving nodes. Each receiving node performs data detection independent of other receiving nodes. The receiving nodes that have performed data detection (and error correction) successfully will then become the transmitting nodes in the next section of the chain. The physical layer (signal processing) operation at each receiving node essentially consists of the equalization of a (virtual) multiple-input single-output (MISO) channel. If the symbol carrier (or subcarrier) is narrow-band and all nodes can synchronize well with each other at the symbol level, then the space-time block codes designed for frequency-flat channels can be readily applied to such MISO channels. With only two transmitting nodes and two receiving nodes in each section, a diversity factor equal to four per section can be achieved. More generally, with $N$ nodes in each transmission tier, diversity equal to $N^2$ per section is achievable [1].

But if the nodes cannot synchronize well with each other, performance deterioration is expected. An important question now is: how does the performance deteriorate as the synch errors increase? Furthermore, we want to know the answer to: what are the good solutions when the synch errors are large? These two questions will be addressed in this paper.

Note that when there is a synch error, a receiving node receives a superposition of multiple baseband signals from multiple transmitting nodes, and these baseband signals are not aligned (synchronized) with each other. In this case, the sampler at the receiving node is bound to be confused as the optimal sampling time of one component of the received signal is not the optimal sam-

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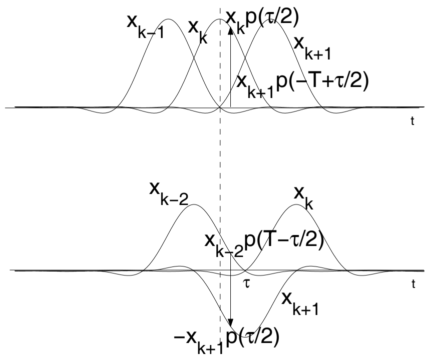


**Fig. 1**. Two baseband signals from two transmitters that arrive at a receiver, where each symbol is pulse-shaped by a raised cosine function.

pling time of the other(s). In [5], the authors simply assume that the actual sampling at the receiver is at the ideal time for one of these components. This is an incorrect assumption as we discuss next.

Techniques for symbol synchronization and sampling at the receiver are available in [7]. The early-late gate is a simple and practical method that is used widely. Let $\tau_1$ be the delay of one signal arriving at the receiver, and $\tau_2$ be the delay of the second signal arriving at the receiver. If $\tau = \tau_1 - \tau_2$ is a multiple of the symbol interval $T$, the sampling time decided by the gate is ideal for both signals. Without loss of generality, we now assume $0 \le |\tau| \le T$. We have observed that the sampling time decided by the gate is about $\tau/2$ ahead of the ideal time for one signal and $\tau/2$ behind the ideal time for the other. The pulse shape $p(t)$ we used is the raised-cosine pulse shape given in [8]. This pulse crosses zero at all integer multiples of $T$.

We now use the above sampling model to describe the sampled signal at the receiver. As illustrated in Figure 1, it can be shown that the sampled signal at the receiver at the sample index $k$ is given by:

$$y(k) \quad = \quad \sum_{i=1}^{2} \sum_{\ell=-1}^{1} x_i(k-\ell) h_i p((-1)^i \frac{\tau}{2} + \ell T) + n(k)$$

where $x_i(k)$ is the transmitted symbol from transmitter $i$ at time $k$, $h_i$ is the (flat) channel fading factor between transmitter $i$ and the receiver, and $i = 1, 2$. Here, we have used $p(t) = p(-t)$.

For a simple exposition of our study, we will neglect the contributions from the higher-order sidelobes of $p(t)$ beyond the first-order sidelobes. We also define $p_0 = p(T - \frac{\tau}{2}), p_1 = p(-\frac{\tau}{2})$ and $p_2 = p(-T - \frac{\tau}{2})$. Then, the effective impulse response of the (virtual) "two-input and one-output" discrete system is described by $\boldsymbol{h}_1 = [h_1(0), h_1(1), h_1(2)]^T = [h_1 p_0, h_1 p_1, h_1 p_2]^T$ and $\boldsymbol{h}_2 = [h_2(0), h_2(1), h_2(2)]^T = [h_2 p_2, h_2 p_1, h_2 p_0]^T$. Note that the elements in each $\boldsymbol{h}_i$ are not statistically independent of each other.

## 3. PERFORMANCE OF THE ALAMOUTI SYSTEM WITH SYNCH ERRORS

Assume that two transmitting nodes use the Alamouti code to transform a (common) sequence of symbols into two coded sequences for transmission. Then, every two consecutive samples received at a node can be described by: $\boldsymbol{y} = X\boldsymbol{h} + \boldsymbol{n}$ where $\boldsymbol{y} = [y(k), y(k+1)]^T$, $\boldsymbol{h} = [\boldsymbol{h}_1^T, \boldsymbol{h}_2^T]^T$, $X = [X_1, X_2]$, and

$$X_1 = \begin{bmatrix} 0 & x(k) & x(k+1) \\ x(k) & x(k+1) & 0 \end{bmatrix}, \qquad (1)$$

$$X_2 = \begin{bmatrix} 0 & x^*(k+1) & -x^*(k) \\ x^*(k+1) & -x^*(k) & 0 \end{bmatrix}, \qquad (2)$$

and the term $\boldsymbol{n}$ includes both the interference and the common noise, i.e.,

$$\boldsymbol{n} = \begin{bmatrix} h_1(0)x(k-1) - h_2(0)x^*(k-2) \\ h_1(2)x(k+2) + h_2(2)x^*(k+3) \end{bmatrix} + \boldsymbol{n}_0 \qquad (3)$$

It is known that the interfering symbols $x(i)$ are not Gaussian-distributed. But in order to obtain an approximate performance evaluation, we will assume them to be Gaussian with zero mean and variance $\sigma_s^2$. The common noise vector $\boldsymbol{n}_0$ is white Gaussian with zero mean and the variance $\sigma^2$. Then, the composite noise vector $\boldsymbol{n}$ is Gaussian with zero mean and the covariance matrix $C = diag(\sigma_1^2, \sigma_2^2)$ where $\sigma_1^2 = (|h_1(0)|^2 + |h_2(0)|^2)\sigma_s^2 + \sigma^2$ and $\sigma_2^2 = (|h_1(2)|^2 + |h_2(2)|^2)\sigma_s^2 + \sigma^2$. Note that the diagonal elements of $C$ are generally not equal and are also dependent on the channel fading factors $h_i$.

To obtain a pair-wise error probability (PEP) of an optimal decoder, we define $P_e = P(X \to \hat{X}|\boldsymbol{h})$ and $E = \hat{X} - X$. Then, it can be shown that $P_e = Q\left(\sqrt{\frac{\boldsymbol{h}^H E^H C^{-1} E \boldsymbol{h}}{2}}\right)$ where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp(-\frac{t^2}{2}) dt$. The averaged PEP is $P(X \to \hat{X}) = E_{\boldsymbol{h}}\left(Q\left(\sqrt{\frac{\boldsymbol{h}^H E^H C^{-1} E \boldsymbol{h}}{2}}\right)\right)$.

Figure 2 shows the averaged PEP versus SNR for different values of the synch error $\tau$. In this figure, we assume that the BPSK symbol constellation is used, and $\hat{x}(k) - x(k) = 0$ and $\hat{x}(k+1) - x(k+1) = \pm 2$. It can be seen that when $\tau$ is much smaller than $T$, the performance degradation is not obvious. But as $\tau$ becomes closer to $T$, the degradation becomes large.

## 4. SPACE-TIME CODING TO COMBAT SYNCH ERRORS

As discussed previously, the synch errors make a frequency-flat system frequency-selective. Therefore, an effective approach to combating synch errors is to use space-time codes designed for

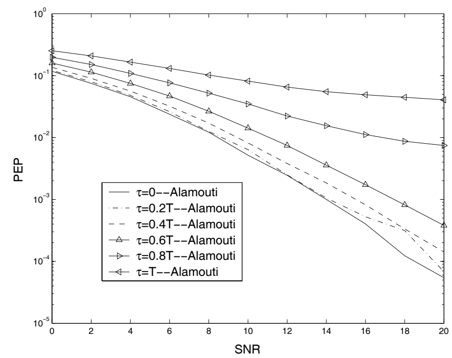


**Fig. 2**. PEP of the $2 \times 1$ Alamouti system with the synch error $\tau$.

frequency-selective channels. However, the samples of the channel impulse response caused by synch errors are not statistically independent, hence synch errors do not introduce additional multipath diversity.

### 4.1. Time-reverse space-time code (TR-STC)

In this section, we consider the time-reverse space-time codes [9]. In this case, the two neighboring blocks of symbols transmitted from transmitter 1 are

$[0, x(0), x(1), ..., x(N-1), 0, 0, -x(N), ... - x(2N-1), 0]$.

The blocks transmitted from transmitter 2 are the time-reversed version of that from transmitter 1 but with complex conjugation, i.e.,

$[0, x^*(2N-1), ...x^*(N), 0, 0, x^*(N-1), ..., x^*(0)]$.

Here, the 0's are added to get rid of the interference between blocks. When the length of the blocks is large enough, the reduction in the data rate is negligible. The samples received at a node from $t = 0$ to $t = N+3$ can be expressed as: $\boldsymbol{y} = X\boldsymbol{h} + \boldsymbol{n}_0$ where $\boldsymbol{h} = [h_1(2), h_1(1), h_1(0), h_2(2), h_2(1), h_2(0)]^T$, $X = \begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{bmatrix}$, and

$$X_{11} = \begin{bmatrix} x(0) & 0 & 0 \\ x(1) & x(0) & 0 \\ x(2) & x(1) & x(0) \\ \vdots & \vdots & \vdots \\ x(N-1) & x(N-2) & x(N-3) \\ 0 & x(N-1) & x(N-2) \\ 0 & 0 & x(N-1) \end{bmatrix}$$ and other $X_{i,j}$

are defined similarly from the corresponding blocks of symbols.

Following a similar procedure as used before, we have

$$P(X \to \hat{X}) = E_{\boldsymbol{h}}\left(Q\left(\sqrt{\frac{\boldsymbol{h}^H E^H E \boldsymbol{h}}{2\sigma^2}}\right)\right)$$

where $E = \hat{X} - X = \begin{bmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{bmatrix}$. It can be shown that $X$ is block-wise orthogonal. Furthermore, we can show that $E^H E = diag(T, T)$ where $T = E_{11}^H E_{11} + E_{21}^H E_{21} = E_{12}^H E_{12} + E_{22}^H E_{22}$. Define $\boldsymbol{h} = [\boldsymbol{h}_1^T, \boldsymbol{h}_2^T]^T$, $\boldsymbol{p}_1 = [p_2, p_1, p_0]^T$, and $\boldsymbol{p_2} = [p_0, p_1, p_2]^T$. Then, we have

$$\begin{aligned} \boldsymbol{h}^H E^H E \boldsymbol{h} &= \boldsymbol{h}_1^H T \boldsymbol{h}_1 + \boldsymbol{h}_2^H T \boldsymbol{h}_2 \\ &= |h_1|^2 \boldsymbol{p}_1^H T \boldsymbol{p}_1 + |h_2|^2 \boldsymbol{p}_2^H T \boldsymbol{p}_2 \\ &= (|h_1|^2 + |h_2|^2) \boldsymbol{p}_1^H T \boldsymbol{p}_1 \end{aligned} \quad (4)$$

Here, $|h_i|^2$ is exponentially distributed. Using an alternative form of the $Q$ function given in [10], we have

$$\begin{aligned} PEP &= \int_0^\infty \int_0^\infty \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \exp(-\frac{\boldsymbol{h}^H E^H E \boldsymbol{h}}{4\sigma^2 \sin^2\theta}) d\theta \\ &\quad \exp(-x_1) \exp(-x_2) dx_1 dx_2 \\ &= \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \frac{1}{(1 + \frac{c}{\sin^2\theta})^2} d\theta \\ &= \frac{1}{4}\left[2 - \frac{\sqrt{1+\frac{1}{c}}c(3+2c)}{(1+c)^2}\right] \end{aligned} \quad (5)$$

where $c = \frac{\boldsymbol{p}_1^H T \boldsymbol{p}_1}{4\sigma^2}$ is proportional to SNR. The Taylor expansion of the PEP at high SNR shows that the PEP is proportional to $\frac{1}{SNR^2}$. Thus the diversity of the system is two.

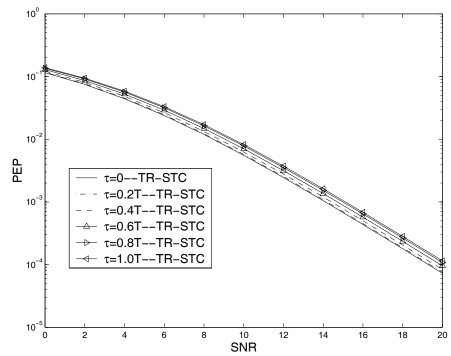


**Fig. 3**. PEP for a two-transmitter and one-receiver TR-STC system with the synch error $\tau$. $N = 20$. If $\tau$ is increased by a multiple of $T$, the PEP remains the same provided a proper number of zeros are padded between blocks.

The PEP for the TR-STC system (with $N = 20$) is illustrated in Figure 3. It can be seen that by using TR-STC, the performance can be improved significantly. The degradation in performance is very small even when $\tau$ is large.

The use of TR-STC for two transmitting relays can be easily extended to multiple transmitting relays using the general Orthogonal Space-Time Code (OSTC) [11]. That is, each symbol in OSTC is replaced by a block of symbols in TR-STC, and the conjugate of a symbol in OSTC corresponds to conjugation and time-reverse of a block of symbols in TR-STC.

### 4.2. Space-Time OFDM (ST-OFDM)

An alternative to the TR-STC code is the ST-OFDM code [9]. A major advantage of the ST-OFDM code is that a frequency selective channel is converted by ST-OFDM into multiple frequency flat channels. With a proper outer code applied together with ST-OFDM as an inner code, the full diversity of a frequency selective channel can be exploited as well. The detector for a ST-OFDM system is generally simpler than that for a TR-STC system.

At the receiver of this ST-OFDM system, the received samples at $k$-th subcarrier in blocks 1 and 2 (after the FFT) can be described as follows:

$$\begin{bmatrix} y_1(k) \\ y_2^*(k) \end{bmatrix} = \begin{bmatrix} H_1(k) & -H_2(k) \\ H_2^*(k) & H_1^*(k) \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2^*(k) \end{bmatrix} + \begin{bmatrix} n_1(k) \\ n_2^*(k) \end{bmatrix}$$

where $H_i(k) = \sum_{l=0}^{2} h_i(l) e^{-j\frac{2\pi}{N}kl}$. The PEP for the optimal detection of the corresponding two blocks of symbols (without an outer code) can be shown to be

$$\begin{aligned} PEP &= E_{\boldsymbol{h}}\left(Q\left(\sqrt{\sum_{k=0}^{N-1} \frac{\boldsymbol{e}(k)^H H(k)^H H(k) \boldsymbol{e}(\boldsymbol{k})}{2\sigma^2}}\right)\right) \\ &= E_{\boldsymbol{h}}\left(Q\left(\sqrt{\frac{|h_1|^2 c_1 + |h_2|^2 c_2}{2\sigma^2}}\right)\right) \end{aligned} \quad (6)$$

where $c_i = \sum_{k=0}^{N-1} \|\boldsymbol{e}(k)\|^2 \sum_{l=0}^{2} |p_i(l) e^{-j\frac{2\pi}{N}kl}|^2$, and $\boldsymbol{e}(k) = \boldsymbol{x}(k) - \hat{\boldsymbol{x}}(k)$. Due to the special structure of $\boldsymbol{p}_i$, we have $c_1 = c_2$, then

$$PEP = \frac{1}{4}\left[2 - \frac{\sqrt{1+\frac{1}{c_1}}c_1(3+2c_1)}{(1+c_1)^2}\right] \quad (7)$$

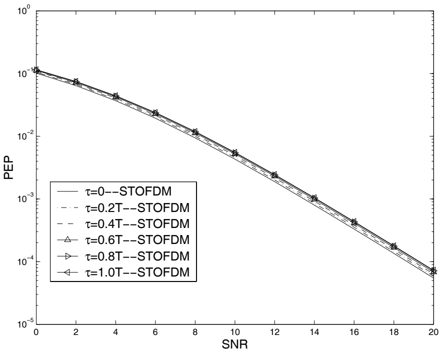


**Fig. 4**. PEP for a two-transmitter and one-receiver ST-OFDM system with the synch error $\tau$. $N = 20$. If $\tau$ is increased by a multiple of $T$, the PEP remains the same provided a proper number of zeros are padded between blocks.

It can be seen from Figure 4 and Figure 3 that the performance difference between TR-STC and ST-OFDM is very small. Both of them are robust to synch errors. However, by using ST-OFDM, the ML detector can be decomposed into $N$ independent ML detectors of much smaller and frequency-flat channels.

## 5. EXTENSION TO VERY LARGE SYNCH ERRORS

Until now, we have assumed that the timing error is $0 < \tau < T$. To consider a more general situation, let the timing error be $D = dT + \tau$, where $d$ is an integer and $0 < \tau < T$.

Now the effect of $D$ can be divided into two parts, one is from the fractional part $\tau$, which will effectively create ISI channels that we have analyzed in the previous sections. The other part is from the multiple of $T$, which will cause a $d$-symbol shift between the two transmitted sequences. In the following, we discuss the effect of the second part.

In the Alamouti system, two neighboring samples at the receiver are used to perform symbol detection. Due to the $d$-symbol shift, the inter-symbol interference remains strong. Thus, the performance degrades significantly when $d$ increases.

In the TR-STC system, if we insert $d$ zeros at the end of the first transmitted block, and $d$ zeros at the beginning of the second transmitted block, then we cancel the interference between blocks and the code matrix $X$ remains the same. Hence, the same performance as given by (5) can be achieved. A major cost here is a reduction of the data rate. However, if $d \ll N$, the rate deduction can be ignored.

In the ST-OFDM system, since the shift in time domain corresponds to a phase change in frequency domain, ST-OFDM still works as there is only a phase shift in the channel. Namely, the received samples become

$$\begin{bmatrix} y_1(k) \\ y_2^*(k) \end{bmatrix} = \begin{bmatrix} H_1(k) & -\hat{H}_2(k) \\ \hat{H}_2^*(k) & H_1^*(k) \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2^*(k) \end{bmatrix} + \begin{bmatrix} n_1(k) \\ n_2^*(k) \end{bmatrix}$$

where $\hat{H}_2(k) = H_2(k) \exp\left(-j2\pi \frac{kd}{N}\right)$. Orthogonality still holds and thus the performance is the same as that given by (7). The additional cost here is that we need to add $d$ more cyclic prefix symbols to each block, which reduces data rate.

## 6. CONCLUSION

We have analyzed the effects of synchronization errors on parallel relays with space-time modulation. Our results show that although the performance of parallel relays deteriorates gracefully when synchronization errors are small, the performance degradation is large when synchronization errors are large. However, by using time-reversed space-time codes or space-time OFDM, the performance of parallel relays remains robust to synchronization errors. A major cost is a data rate reduction which can be made small if the symbol block size is much longer than the timing error $\tau$. Another possible cost is the longer delay, on the order of the block length, which may be negligible depending on the upper bound on synch errors.

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