NON-REGENERATIVE MIMO RELAYING WITH CHANNEL STATE INFORMATION

Olga Muñoz, Josep Vidal, Adrián Agustín

Signal Processing and Communications Group, Technical University of Catalonia (UPC)
E-mail:{olga,pepe,agustin }@gps.tsc.upc.es

ABSTRACT

This paper deals with the design of non-regenerative relays in cooperative transmission schemes. The conventional non-regenerative approach is the amplify and forward (AF) approach, where the signal received at the relay is simply amplified and retransmitted. In this paper, we propose an alternative design which maximizes the capacity for non-regenerative cooperative transmission, when channel state information is available at the relay station. Therefore, the relay units are not simple amplify and forward units, but still they do not require neither to demodulate nor remodulate the symbols transmitted by the base station as in regenerative relaying schemes. We compare the performance so obtained with the performance for the conventional AF approach, and also with the performance of regenerative relays and direct non-cooperative transmission.

1. INTRODUCTION

Cooperation among users at the physical layer level has shown to be a promising approach for capacity and/or range increase. In these schemes, the signals received from the source and the relay station are combined at the destination. Therefore, cooperative schemes can be seen as a generalization of the typical multihop approach where a relaying terminal retransmits the symbols from the base station or central controller (thus providing range extension). The main advantage of cooperative schemes, with respect to classical relaying strategies, is that cooperation creates a "virtual" MIMO system that may offer significant capacity gains in fading channels with respect to existing systems.

There are two different approaches for cooperative transmission, according to the role played by the relaying terminal: the amplify and forward (AF) scheme [7] and the decode and forward scheme (DF) [5]. The most simply approach is the AF approach which is a non-regenerative approach where the relay amplifies and retransmits the signal received from the source. The most complex approach is the DF scheme where the relay station decodes the received signal and retransmits the decoded and regenerated symbols. The DF is also known as regenerative approach.

This work has been carried out in the framework of the EC-funded project ROMANTIK (IST-2001-32549) and supported by Spanish and Catalan Government Grants: TEC2004-04526, TIC2003-05482, TIC-2001-0633-C03-01, TIC2001-2356, and 2001SGR-00268.

In this paper, we focus on the design of non-regenerative relays as the AF approach. We assume, however, some additional intelligence at the relay station, that makes the relay able to carry out some further signal processing. Therefore, the relay units are not simple amplify and forward units, however they do not require neither to demodulate nor remodulate the symbols transmitted by the base station as in the DF approach.

The design of non-regenerative relays can be done under different optimization criteria as for instance maximum SNIR, minimum Mean Square Error or minimum Bit Error Rate for a given available transmit power. The criteria considered here is the maximization of capacity under different levels of channel state information (CSI). Three different degrees of channel state information can be considered at the relay station. The channel state information at the relay can be only information about the first hop channel (between the source and relay) which is simple, or information about the first and second hop channel (between relay and destination) which it would be possible if both user and relay share a previous dialogue. Finally, we could consider that the relay has knowledge about all the links involved at the communication, including the direct channel. Despite this is a more unrealistic situation, it is useful for comparison purposes. In this paper we consider the last two cases.

2. SIGNAL MODEL FOR NON-REGENERATIVE COOPERATIVE SCHEMES

The scenario under analysis is a TDD downlink scenario. The source transmits during the first time slot and the relay transmits during the second time slot. The source, relay and destination are assumed to have *M*, *R* and *N* antennas respectively. *R*>1 is feasible for fixed post lamps acting as relays.

For non-regenerative relays, the signal received at the destination, during the downlink (DL) slot and the relay link (RL) slot, can be modeled as:

$$\begin{bmatrix} \mathbf{y}_d^{(DL)} \\ \mathbf{y}_d^{(RL)} \end{bmatrix} = \begin{bmatrix} \mathbf{H}_0 \\ \mathbf{H}_2\mathbf{G}\mathbf{H}_1 \end{bmatrix} \mathbf{x}_s + \begin{bmatrix} \mathbf{I}_N & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_2\mathbf{G} & \mathbf{I}_N \end{bmatrix} \begin{bmatrix} \mathbf{n}_d^{(DL)} \\ \mathbf{n}_r^{(DL)} \\ \mathbf{n}_d^{(RL)} \end{bmatrix} \quad (1)$$

where $\mathbf{x}_s$ is the signal transmitted by the source, $\mathbf{H}_0$ denotes the $N \times M$ channel matrix between the source and the destination (direct channel), $\mathbf{H}_1$ denotes the $R \times M$ channel matrix between the source and the relay station (first hop channel), $\mathbf{G}$ is a $R \times R$ linear combining matrix at the relay, $\mathbf{H}_2$ denotes the $N \times R$ channel matrix between the relay and the destination (second hop channel) and $\mathbf{I}_N$ denotes the $N\times N$ identity matrix. Finally $\mathbf{n}_d^{(DL)}$ and $\mathbf{n}_r^{(DL)}$ are the noise vector received at destination and the relay during the DL slot, while $\mathbf{n}_d^{(RL)}$ is the noise vector at the destination. We will assume isotropic transmission from the source in the signal model. The reason is that for decentralized schemes the information should arrive to the destination and also to potential relay stations about which the source has no a priori information. Both the source (base station) and the relay are constrained in their transmit power.

The instantaneous capacity for a single cooperative connection using non-regenerative relays can be written as a function of matrix $\mathbf{G}$ in the following way:

$$I_{AF} = \tfrac{1}{2}\log\det\left(\mathbf{I}_{2N} + \frac{P_{BS}}{M\sigma^2}\mathbf{H}\mathbf{H}^H\begin{bmatrix}\mathbf{I}_N & \mathbf{0}\\ \mathbf{0} & (\mathbf{I}_N+\mathbf{H}_2\mathbf{G}\mathbf{G}^H\mathbf{H}_2^H)^{-1}\end{bmatrix}\right) \quad (2)$$

with

$$\mathbf{H}\mathbf{H}^H = \begin{bmatrix}\mathbf{H}_0\mathbf{H}_0^H & \mathbf{H}_0\mathbf{H}_1^H\mathbf{G}^H\mathbf{H}_2^H\\ \mathbf{H}_2\mathbf{G}\mathbf{H}_1\mathbf{H}_0^H & \mathbf{H}_2\mathbf{G}\mathbf{H}_1\mathbf{H}_1^H\mathbf{G}^H\mathbf{H}_2^H\end{bmatrix} \quad (3)$$

The factor ½ comes from the fact that the vector signal is actually transmitted in two time instances so the efficiency drops by a half when the units are in bits per second. As a consequence, a high spatial reuse of the relay slot is necessary in order to obtain a potential capacity gain [1, 2]. Spatial reuse means the following. Assuming a TDMA strategy for the DL, with the base station (BS) serving K users, firstly, the K DL transmissions are allocated. At the end of the frame, K' simultaneous retransmissions from the corresponding relays are allocated in a single slot, with $K' \le K$. Therefore, the effective capacity of a single cooperative connection has to be multiplied by a factor $K/(K+1)$ instead of a factor 1/2 corresponding to the relay slot non-reuse case. If the relays are placed at enough distance among them according to the maximum relay power, the interference generated in the relay slot can be negligible. In the following we will assume a high spatial reuse, that it is to say, $K/(K+1)$ close to 1.

For the conventional AF approach matrix $\mathbf{G}$ is given by $\mathbf{G} \propto \mathbf{I}_R$. The problem addressed in the following is the design of a different matrix gain at the relay, when several antennas are available, in order to maximize the instantaneous capacity under different levels of channel state information.

3. CHANNEL STATE INFORMATION ABOUT $\mathbf{H}_1$ AND $\mathbf{H}_2$ AT THE RELAY

We consider first that the relay has knowledge about the first and second hop channel but not about the direct hop channel. If we do not have information about the direct hop channel we cannot maximize the mutual information, but we are still able to maximize the upper bound of the mutual information. From the Hadamard's inequality [4] $\det\left(\begin{bmatrix}\mathbf{X}_{11} & \mathbf{X}_{12}\\ \mathbf{X}_{21} & \mathbf{X}_{22}\end{bmatrix}\right) \le \det(\mathbf{X}_{11})\det(\mathbf{X}_{22})$ (with equality if an only if $\mathbf{X}_{12}=\mathbf{0}$ and $\mathbf{X}_{21}=\mathbf{0}$ provided that both $\mathbf{X}_{11}$ and $\mathbf{X}_{22}$ are both positive definite), it follows that the determinant in (2) is upper bounded by:

$$\left\{\begin{array}{l}\det\left(\mathbf{I}_N + \frac{P_{BS}}{M\sigma^2}\mathbf{H}_0\mathbf{H}_0^H\right)*\\ \det\left(\mathbf{I}_N + \frac{P_{BS}}{M\sigma^2}\mathbf{H}_2\mathbf{G}\mathbf{H}_1\mathbf{H}_1^H\mathbf{G}^H\mathbf{H}_2^H(\mathbf{I}_N+\mathbf{H}_2\mathbf{G}\mathbf{G}^H\mathbf{H}_2^H)^{-1}\right)\end{array}\right\} \quad (4)$$

that leads to an upper bound of the mutual information. This upper bound can be maximized with respect $\mathbf{G}$ with independence of the direct channel $\mathbf{H}_0$, since only the second term is a function of $\mathbf{G}$.

We will assume that the relay is able to extract information of channel $\mathbf{H}_1$ and $\mathbf{H}_2$. Therefore, our purpose will be to find matrix $\mathbf{G}$ that maximizes the following term:

$$\log\det\left(\mathbf{I}_N + \frac{P_{BS}}{M\sigma^2}\mathbf{H}_2\mathbf{G}\mathbf{H}_1\mathbf{H}_1^H\mathbf{G}^H\mathbf{H}_2^H(\mathbf{I}_N+\mathbf{H}_2\mathbf{G}\mathbf{G}^H\mathbf{H}_2^H)^{-1}\right) \quad (5)$$

Note that maximizing (5) is equivalent to maximize the mutual information in the classical relay channel. By the singular value decomposition theorem, the channels can be written as: $\mathbf{H}_1 = \mathbf{U}_1\mathbf{\Lambda}_1^{1/2}\mathbf{V}_1^H$ $\quad \mathbf{H}_2 = \mathbf{U}_2\mathbf{\Lambda}_2^{1/2}\mathbf{V}_2^H$, with $\mathbf{U}_1$, $\mathbf{V}_1$, $\mathbf{U}_2$ and $\mathbf{V}_2$ unitary matrices and $\mathbf{\Lambda}_1$ and $\mathbf{\Lambda}_2$ diagonal matrices. Denoting $\tilde{\mathbf{G}} = \mathbf{V}_2^H\mathbf{G}\mathbf{U}_1$, and applying the inverse lemma to the noise matrix and the commutative property of the determinant, we arrive that (5) can be written as:

$$\log\det\left(\mathbf{I}_R + \frac{P_{BS}}{M\sigma^2}\mathbf{\Lambda}_2^{1/2}\tilde{\mathbf{G}}\mathbf{\Lambda}_1\tilde{\mathbf{G}}^H\mathbf{\Lambda}_2^{1/2}(\mathbf{I}_R+\mathbf{\Lambda}_2^{1/2}\tilde{\mathbf{G}}\tilde{\mathbf{G}}^H\mathbf{\Lambda}_2^{1/2})^{-1}\right) \quad (6)$$

The restriction under the relay power can be written as:

$$trace\left(\frac{P_{BS}}{M}\tilde{\mathbf{G}}\Lambda_1\tilde{\mathbf{G}}^H + \sigma^2\tilde{\mathbf{G}}\tilde{\mathbf{G}}^H\right) \le P_{RS} \quad (7)$$

By the Hadamard determinant theorem [4] the matrix inside the determinant should be diagonal to maximize (6). A simple solution to achieve this is to consider matrix $\tilde{\mathbf{G}}$ diagonal. Then, computing the elements of $\tilde{\mathbf{G}}$ reduces to solve the following scalar problem where the unknowns are $|g_r|^2 \quad r=1,\dots,R$:

$$\begin{aligned}&\text{Minimize } -\sum_{r=1}^{R}\log\left(1+\frac{P_{BS}}{M\sigma^2}\frac{\lambda_{1,r}\lambda_{2,r}|g_r|^2}{1+\lambda_{2,r}|g_r|^2}\right)\\ &\text{subject to } \quad -|g_r|^2 \le 0 \quad r=1,\dots,R\\ &\qquad \sum_{r=1}^{R}\left(\frac{P_{BS}}{M}\lambda_{1,r}+\sigma^2\right)|g_r|^2 = P_{RS}\end{aligned} \quad (8)$$

where $\lambda_{1,r}$ and $\lambda_{2,r}$ are the r-th eigenvalue of the first and second hop channel respectively and g_r the r-th diagonal component of matrix $\tilde{\mathbf{G}}$. This is a standard convex optimization problem (the objective function and the inequality constraint functions are convex while the equality constraint function is affine) which can be solved by means of the Karush-Kuhn-Tucker conditions [3] to obtain the optimum value for $|g_r|^2 \quad r = 1,\ldots,R$ which is:

$$|g_r|^2 = \frac{1}{\frac{P_{BS}}{M}\lambda_{1,r} + \sigma^2} p_r \tag{9}$$

with:

$$p_r = \left[\sqrt{\frac{P_{BS}\lambda_{1,r}}{\mu M \lambda_{2,r}} + \left(\frac{P_{BS}\lambda_{1,r}}{2M\lambda_{2,r}}\right)^2} - \frac{P_{BS}\lambda_{1,r}}{2M\lambda_{2,r}} - \frac{\sigma^2}{\lambda_{2,r}}\right]^+ \tag{10}$$

where $[x]^+ = \begin{cases} x & if\ x > 0 \\ 0 & if\ x \le 0 \end{cases}$. The last term in (10) appears also in the expression for the optimal power assignment in a conventional MIMO system [6] when CSI is available at the transmitter. This last term penalizes the 'bad' modes of the second hop channel. Note, that the sum of the other three terms is always greater or equal to zero and enhances those modes of the first hop channel whose ratio with respect to the corresponding mode in the second hop channel is greater. Note, also, that for those channel eigenvalues which are zero (with independence if they are from the first or second hop channel) the assigned power will be zero. This means that for N=1 (with M and R>1), as it could be intuitively expected, the relay should retransmit only the best first hop eigenmode. Then, all the available relay power is assigned to this eigenmode, which is retransmitted using a matched filter to the single second hop channel eigenmode, since $\mathbf{G} = \mathbf{V}_2 \tilde{\mathbf{G}} \mathbf{U}_1^H$.

The proposed approach has been compared in terms of ergodic capacity with other strategies as non-cooperative and cooperative approaches. For cooperative approaches both conventional non-regenerative (AF) and regenerative relaying schemes (decode and forward unconstrained code DF-UC) have been considered. Different situations regarding mean SNR at the involved links have been studied. The simulations have been carried out considering flat fading Rayleigh channels in all links.

Figure 1 shows the ergodic capacity versus the mean signal to noise ratio in the second hop channel (SNR_2) for non-cooperative transmission (Non-Coop), decode and forward-unconstrained code (DF-UC), conventional AF approach (AF) and the proposed approach which is a non-regenerative relaying approach with signal processing (Non-Reg-SP). For this later scheme, the matrix gain has been obtained maximizing the bound of the instantaneous capacity, but the results shown correspond to the actual values of the capacity so obtained. High reuse of the relay channel has been assumed in this and forthcoming plots.

We have considered a downlink scenario, where, in order to implement cellular reuse of the relay link slot, the relay unit should be placed close to the mobile user unit [1]. The signal to noise ratio in the direct channel (SNR_0) is fixed to 10dB and two situations are considered regarding the signal to noise ratio for the first hop channel: In scenario A) SNR_0 and SNR_1 are the same, which implies that relay and destination have similar propagation conditions to the base station. In scenario B) we assume a relay station with better propagation conditions to the base station than the ones between base station and destination receiver. The mean SNRs considered are therefore A) SNR_1=SNR_0=10dB and B) SNR_1=SNR_0+5dB=15dB. Regarding the number of antennas, M=2, R=2 and N=1 antennas are considered at the source, the relay and the destination respectively.

Note that when the mean signal to noise ratio in the first hop channel (SNR_1) is similar to the one in the direct channel (SNR_0), AF and DF offer similar results. In this case, the non-regenerative relay with signal processing offers the best performance. When the mean signal to noise ratio in the first hop channel (SNR_1) is 5 dB greater than the signal to noise ratio in the direct channel (SNR_0), the DF-UC approach offers the best approach (since in this case the capacity of this scheme is not limited by the source-relay link [1,5]), followed by the non-regenerative signal processing case. In all situations the proposed approach offers a performance improvement over conventional AF (where each antenna of the relay amplifies and retransmits the received signal $\mathbf{G} \propto \mathbf{I}_R$) and also over the conventional non-cooperative transmission scheme.

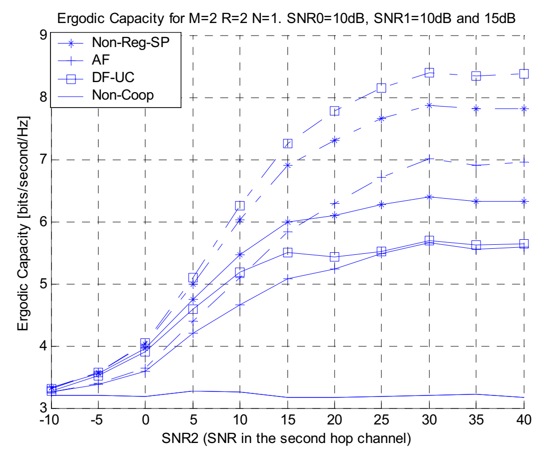

Figure 1. Ergodic capacity as a function of mean SNR_2 (2^{nd} hop channel), for M=2, R=2, N=1. Solid lines correspond to scenario A: SNR_0=SNR_1=10 dB. Dashdot lines correspond to scenario B: SNR_0=10dB, SNR_1=15dB.

4. CHANNEL STATE INFORMATION ABOUT H_0, H_1 AND H_2 AT THE RELAY

In previous section a procedure to maximize capacity making use of the knowledge about $\mathbf{H}_1$ and $\mathbf{H}_2$ was proposed. Nevertheless, the approach in previous section does not maximize the mutual information itself but a bound of mutual information. Assuming knowledge about the direct channel is available, maximization of mutual information itself can be accomplished. Despite in practice it may be difficult to have knowledge about $\mathbf{H}_0$ at the relay station, the results obtained are interesting and useful for comparison purposes.

General expression for mutual information with non-regenerative relays can be rewritten as follows:

$$I_{AF} = \log\det\left(\mathbf{I}_N + \frac{P_{BS}}{M\sigma^2}\mathbf{H}_0\mathbf{H}_0^H\right) + I \qquad (11)$$

with I:

$$I = \log\det\left(\mathbf{I}_N + \frac{P_{BS}}{M\sigma^2}\mathbf{H}_2\mathbf{G}\mathbf{A}\mathbf{G}^H\mathbf{H}_2^H(\mathbf{I}_N + \mathbf{H}_2\mathbf{G}\mathbf{G}^H\mathbf{H}_2^H)^{-1}\right) \qquad (12)$$

where

$$\mathbf{A} = \mathbf{H}_1\left[\mathbf{I}_M - \mathbf{H}_0^H\left(\mathbf{I}_N + \frac{P_{BS}}{M\sigma^2}\mathbf{H}_0\mathbf{H}_0^H\right)^{-1}\mathbf{H}_0\frac{P_{BS}}{M\sigma^2}\right]\mathbf{H}_1^H \qquad (13)$$

Solution for matrix **G** that maximizes capacity is similar to the one obtained in previous section $\mathbf{G} = \mathbf{V}_2\tilde{\mathbf{G}}\mathbf{U}_1^H$ with $\tilde{\mathbf{G}}$ a diagonal matrix. The difference, however, relays in the fact that $\mathbf{U}_1$ contains the eigenmodes of matrix **A**.

Note that in (13) we are performing an operation which is similar to a projection of the first hop channel onto the orthogonal subspace of the direct hop channel. Actually, for high signal to noise ratio in the direct hop channel the matrix in brackets in (13) is indeed a projection matrix, which means we project onto the orthogonal subspace of channel $\mathbf{H}_0$. For low signal to noise ratio in the direct hop channel, we allow a certain amount of the signal component in the direct hop channel subspace to be transmitted through the relay channel.

Figure 2 shows the cumulative function of mutual information for the conventional AF and the proposed approach without and with knowledge of direct channel $\mathbf{H}_0$. Without knowledge, matrix **G** is obtained by maximizing the bound for mutual information as it was described in section 3. When knowledge about $\mathbf{H}_0$ is available, the mutual information itself is maximized as described in this section. The performance so obtained in this case is the achievable best one.

The results are shown for R=2 (solid lines) and R=4 (dashdot lines) antennas at the relay. Note that increasing the number of antennas at the relay increases the impact of using $\mathbf{H}_0$ to design the gain matrix **G.** Increasing R means to increase the number of degrees of freedom. As a consequence, we design the gain matrix **G** able to orthogonalize the equivalent relay channel and direct channel, and still achieve a high capacity in the orthogonal subspace of the direct hop channel $\mathbf{H}_0$. This can be observed in the figure, where the cumulative function of the instantaneous capacity upper bound has been depicted for R=2 and R=4. The upper bound has been calculated using (4) with **G** calculated as explained in the current section (taking into account $\mathbf{H}_0$). The upper bound performance is not achievable since it requires that $\mathbf{H}_0\mathbf{H}_1^H\mathbf{G}^H\mathbf{H}_2^H(\mathbf{I}_N + \mathbf{H}_2\mathbf{G}\mathbf{G}^H\mathbf{H}_2^H)^{-1} = \mathbf{H}_2\mathbf{G}\mathbf{H}_1\mathbf{H}_0^H = \mathbf{0}$ (to accomplish equality in Hadamard's inequality). Nevertheless, the actual performance when CSI about $\mathbf{H}_0$ is available (lines with diamonds figure 2) approaches the upper bound when the number of relaying antennas increases.

Further results have been obtained, which are not included here due to the lack of space. When the number of antennas at the relay is the same than the number of antennas at the destination, results have shown that the improvement obtained from the use of CSI reduces. Nevertheless, whenever the number of antennas at the relay station is greater than the number of antennas at the destination (and so some channel eigenvalues are zero in this case) a significant capacity improvement can be obtained optimizing matrix **G** compared with the conventional amplify and forward approach.

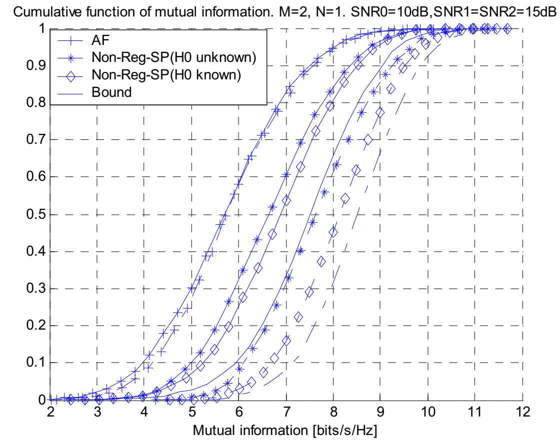

Figure 2. Cumulative function of mutual information for M=2, N=1, R=2 (solid line) and R=4 (dashdot line). SNR_0=10dB, SNR_1= SNR_2=15dB.

5. CONCLUSIONS

In this paper, the use of CSI at the relay station has been considered for the optimum design of the gain matrix for non-regenerative relays. An increase in capacity can be obtained when the first and second hop channel are known at the relay station, despite this is not the capacity-maximizing solution, only achievable when also the direct hop channel is known. For such a case the solution has also been obtained, showing that matrix **G** should perform a role to 'orthogonalize' the direct and relaying path in order to maximize capacity. Finally, results demonstrate that loss due to no CSI about $\mathbf{H}_0$ is not significant.

6. REFERENCES

[1] A.Agustín, O. Muñoz, J. Vidal "A Game Theoretic Approach for Cooperative MIMO Schemes with Cellular Reuse of the Relay Slot", ICASSP 04*,* April 2004.

[2] G. Scutari, S. Barbarossa, D. Ludovici, "Cooperation Diversity in Multihop Wireless Networks using Opportunistic Driven Multiple Access", SPAWC 2003, Rome, Italy, June 2003.

[3] S. Boyd, L. Vandenberghe, *Convex Optimization*, Cambridge University Press, 2004.

[4] R.A. Horn and C.R. Johnson, *Matrix Analysis*, Cambridge University Press, 1985.

[5] J. Laneman, D.N.C. Tse and G. W. Wornell, "Cooperative Diversity in Wireless Networks: Efficient Protocols and Outage Behavior", submitted to *IEEE Trans. on Information Theory*, January 2002.

[6] I.E. Telatar, "Capacity of multi-antenna Gaussian channels", *European Transactions on Telecommunications*, vol. 10, no. 6, pp. 585-595, Nov.-Dec. 1999.

[7]. A. Wittneben, B. Rankov, "Impact of Cooperative Relays on the Capacity of Rank-Deficient MIMO Channels", IST Summit 2003.