EXPLOITING THE FINITE-ALPHABET PROPERTY FOR COOPERATIVE RELAYS

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ABSTRACT

We consider in this paper the design of a cooperative relay strategy by exploiting the £nite-alphabet property of the source. Assuming a single source-sink pair with L relay nodes all communicating in orthogonal channels, we derive necessary conditions for optimal relay signaling that minimizes the error probability at the sink node. The derived conditions allow us to construct an iterative algorithm to £nd the distributed relay signaling that is at least locally optimal. As a byproduct, one can show that the so-called decode-and-forward (DF) relay scheme does not satisfy the necessary condition hence is not optimal in its error probability performance. Indeed, numerical examples show that the proposed scheme provides substantial performance improvement over both DF and the amplify-and-forward approach.

1. INTRODUCTION

In wireless networks, a severe limiting factor is multipath induced channel fading. One of the most effective methods in mitigating fading is to exploit diversity. Examples include spatial diversity when multiple antennas are used in the transceivers, multipath diversity in frequency selective channels, and temporal diversity in fast fading channels. More recently, a new diversity resource has attracted increased attention, especially in the context of wireless *ad hoc* networks [1, 2]. There, multiple nodes collaborate in transmitting their information, and provide the channel diversity due to the independence of channel fading for different users. This is generally referred to as the cooperative diversity.

In this paper, we focus on the relay network consisting of a single source-sink pair and L relay nodes and exploit the £nite-alphabet (FA) property of the source to fully realize the cooperative diversity. The FA property is almost ubiquitous to all existing digital wireless networks regardless of their contents/applications. This is drastically different from existing cooperative diversity schemes [2] that use repetition or space-time based diversity but do not exploit the FA source structure of the message. As to be demonstrated, the FA property provides rich information and structure that, when exploited, can improve the performance by a signi£cant margin.

Recognizing the equivalence between cooperative relay with FA source and the distributed multiple hypotheses testing problem, we establish in this paper necessary conditions for relay signaling that minimizes the decoding error at the sink node. This has been explored in [3] where quantized signals from multiple antennas are fused to form a £nal decision for independently faded binary frequency shift keying (BFSK) signals. Distinctive in the current work, in addition to considering a general FA source instead of BFSK, is that the relay output are assumed to also go through fading channels between relay nodes to the sink node. Hence the proposed relay scheme can be considered as a distributed joint source-channel coding (JSCC) for cooperative relay. The derived necessary conditions can be used to search for relay signaling schemes that minimize the decoding error. One can also establish that the commonly used decode-and-forward (DF) is not optimal as it does not satisfy the necessary conditions.

The paper is organized as follows. Section 2 describes the system model and problem formulation. In Section 3, we derive the necessary conditions for minimum error probability cooperative relay strategies. Numerical examples are presented in Section 4 to show the performance gain over existing strategies. Finally, we conclude in Section 5.

2. STATEMENT OF THE PROBLEM

Consider a wireless relay network which includes one source node, L relay nodes and one sink node (Fig. 1). The source node broadcasts a signal S to all the relay nodes. We assume S is drawn from a FA set $\mathcal{S} = \{s_0, \cdots, s_{M-1}\}$ with prior probabilities $\{\pi_0, \cdots, \pi_{M-1}\}$. The received signal at the l^{th} relay node is,

$$X_l = \alpha_{1l} S + n_{1l}, \quad l = 1, \cdots, L \tag{1}$$

where α_{1l} and n_{1l} are respectively the channel coef£cient and noise for the channel between the source node and l^{th} relay node. The l^{th} relay node sends a relay signal U_l to the sink node based on its received signal X_l,

$$U_l = \gamma_l(X_l) \qquad l = 1, \cdots, L \tag{2}$$

We assume, without loss of generality, that U_l belongs to a FA set $T = \{0, 1, \cdots, N-1\}$. The received signal at the sink node from the l^{th} relay is

$$Y_l = \alpha_{2l} U_l + n_{2l} \qquad l = 1, \cdots, L \tag{3}$$

where α_{2l} and n_{2l} are respectively the channel coe£cient and noise for the channel between l^{th} relay node and sink node. Upon collecting $\mathbf{y} = \{Y_1, \cdots, Y_L\}$, the sink node makes a £nal decision

$$U_0 = \gamma_0(\mathbf{y})$$

where $U_0 \in \{0, \cdots, M-1\}$. The interpretation of the above FA set in the context of packet transmission will become clear in Section 4.

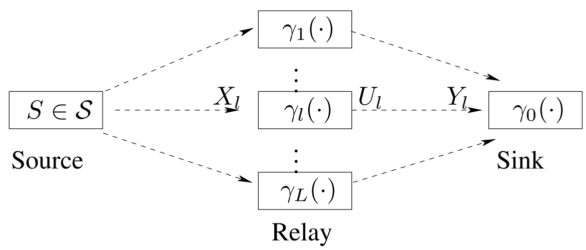

Fig. 1. A wireless relay network.

An error happens if $U_0 \neq S$. The goal is, therefore, to design the local relay scheme $\gamma_l(\cdot)$ for each relay node and decoding rule $\gamma_0(\cdot)$ at the sink node such that the overall error probability at the sink node is minimized.

From the distributed detection point of view, this relay system is an M-ary hypotheses testing system with each hypothesis corresponding to one of the input alphabet symbols; i.e., $H_i : S = s_i$. As we assume all the transmissions occur in orthogonal channels, the signals received at relay nodes can thus be assumed independent conditioned on the input source, i.e.,

$$p(X_1, \cdots, X_L | H_i) = \prod_{l=1}^{L} p(X_l | H_i), i = 0, \cdots, M-1 \tag{4}$$

Similarly, for the signals received at the sink node,

$$p(Y_1, \cdots, Y_L | U_1, \cdots, U_L) = \prod_{l=1}^{L} p(Y_l | U_l) \tag{5}$$

As usual, the sink node is assumed to employ the maximum *a posteriori* probability (MAP) decoding rule:

$$U_0 = \gamma_0(\mathbf{y}) = \arg \max_{i \in \{0,1,\cdots,M-1\}} \pi_i p(\mathbf{y} | H_i) \tag{6}$$

which can be obtained in a straightforward manner assuming knowledge of relay rules and other parameters. We note here that optimal decoding integrating the transmission has been investigated in the context of channel-aware decision fusion design (see, e.g., [4]). As such, we will only focus on the relay signaling design without further elaborating on the design of decoding rule.

3. NECESSARY CONDITIONS

We adopt a person-by-person optimal (PBPO) approach, i.e., we optimize $\gamma_l(\cdot)$ for the l^{th} relay node given £xed γ_k for all $k \neq l$ and a £xed decoding rule $\gamma_0(\cdot)$ at the sink node. We start by expanding the error probability with respect to $\gamma_l(\cdot)$. Denote by

$$\begin{aligned} \mathbf{u} &= [U_1, U_2 \cdots, U_L], \\ \mathbf{x} &= [X_1, X_2, \cdots, X_L], \end{aligned}$$

the probability of error at the sink node is

$$\begin{aligned} P_e &= 1 - \sum_{i=0}^{M-1} \pi_i P(u_0 = i | H_i) \\ &= 1 - \sum_{i=0}^{M-1} \pi_i \Bigg[\int_{\mathbf{y}} P(u_0 = i | \mathbf{y}) \times \\ &\qquad \sum_{\mathbf{u}} P(\mathbf{y} | \mathbf{u}) \int_{\mathbf{x}} P(\mathbf{u} | \mathbf{x}) p(\mathbf{x} | H_i) d\mathbf{x} d\mathbf{y} \Bigg] \\ &\triangleq 1 - P_D \end{aligned}$$

De£ne

$$\begin{aligned} \mathbf{u}^l &= [U_1, \cdots, U_{l-1}, U_{l+1}, \cdots, U_L], \\ \mathbf{x}^l &= [X_1, \cdots, X_{l-1}, X_{l+1}, \cdots, X_L] \end{aligned}$$

Using the conditional independence assumption (4) and (5), we can expand P_D with respect to $U_l = \gamma_l(X_l)$ as

$$\begin{aligned} P_D &= \sum_{i=0}^{M-1} \pi_i \Bigg[\int_{\mathbf{y}} P(u_0 = i | \mathbf{y}) \sum_{\mathbf{u}} P(\mathbf{y} | \mathbf{u}) \times \\ &\qquad \int_{\mathbf{x}} P(\mathbf{u}^l | \mathbf{x}^l) p(\mathbf{x}^l | H_i) P(U_l | X_l) p(X_l | H_i) d\mathbf{x}^l dX_l d\mathbf{y} \Bigg] \\ &= \int_{X_l} \sum_{j=0}^{N-1} P(U_l = j | X_l) \times \\ &\qquad \left[\sum_{i=0}^{M-1} \pi_i p(X_l | H_i) P(u_0 = i | U_l = j, H_i) \right] dX_l \end{aligned}$$

De£ne, for $l = 1, \cdots, L$, $j = 0, \cdots, N-1$

$$D_{lj}(X_l) = \sum_{i=0}^{M-1} \pi_i P(u_0 = i | U_l = j, H_i) p(X_l | H_i) \tag{7}$$

we can rewrite P_D as:

$$P_D = \int_{X_l} \sum_{j=0}^{N-1} P(U_l = j | X_l) D_{lj} dX_l$$

Thus, to maximize P_D, we set $P(U_l = j^* | X_l) = 1$ where j^* is the index that maximizes $D_{lj}(X_l)$. Hence we have,

Theorem 1 *The optimal relay rule for the l^{th} relay node must satisfy*

$$U_l = \gamma_l(X_l) = \arg \max_{j \in \{0,1,\cdots,N-1\}} D_{lj}(X_l) \qquad (8)$$

For $D_{lj}(\cdot)$ de£ned in (7).

The fact that we use the PBPO criterion implies that the derived conditions are only necessary but not suf£cient conditions for optimality. Recognizing that the necessary conditions for the relay function $\gamma_l(\cdot)$ is coupled with the decoding rule, we propose the following iterative algorithm in search of relay and decoding rules that are at least local optimum in minimizing the error probability.

Iterative algorithm

1. Initialize the local relay strategies for each relay node $\gamma_l^{(0)}, l = 1, \cdots, L$ and set the iteration index $r = 1$;

2. Obtain the optimal decoding rule $f^{(r)}$ using (6) for £xed local relay rules $\gamma_l^{(r-1)}, l = 1, \cdots, L$;

3. For each l, obtain the PBPO local relay rule $\gamma_l^{(r)}$ using (8) given the £xed local relay rules for the other relay nodes and the £xed decoding rule;

4. Obtain the error probability $P_e^{(r)}$ at the sink node given the relay rules $\boldsymbol{\gamma}^{(r)} = \{\gamma_1^{(r)}, \cdots, \gamma_L^{(r)}\}$ and decoding rule $f^{(r)}$ and compare it with $P_e^{(r-1)}$. If the difference is less than a prescribed value, stop. Otherwise, set $r = r + 1$ and go to Step 2.

An important distinction between the current work and that of [5] is that we are considering an M-ary hypotheses testing problem with general vector input (i.e., packet). As such, one does not have the luxury of equating the local relay rule to a single quantizer; i.e., one does not have a scalar quantization problem but needs to quantize a multi-dimensional $(M-1)$ suf£cient statistic. Thus convergence checking by comparing relay rules is generally not viable.

4. PERFORMANCE EVALUATION

In this section, through a number of numerical examples, we demonstrate the performance advantage of our approach over some existing strategies, namely DF and amplify-and-forward (AF) [2]. As we are concerned with packet relay, all the variables, including S, X_l, U_l, and Y_l are now vectors. For DF, each relay simply makes its own decision using an MAP rule:

$$U_l = \arg \max_{i \in \{0,\cdots,M-1\}} \pi_i p(X_l | H_i)$$

This is clearly different than that speci£ed in Theorem 1 where the relay output is coupled with the decoding rule at the sink node as well as other relay rules. For AF, the output is simply a scaled version of the input, i.e., $U_l = cX_l$ where the scaling factor c is determined so that all schemes have the same average power constraint.

In the following, we assume the channels between the source node and the relay nodes are identically distributed Rayleigh fading channels, and so are the channels between the relay nodes and the sink node. The packet sent from the source node are assumed to be a K bit codeword drawn from M different codewords with equal probability. Hence $M \leq 2^K$. We also assume the local decision at each relay node is K bits, thus the relay output alphabet is $N = 2^K$. We de£ne SNR_s as the common signal-to-noise ratio (SNR) of each source-relay channel and SNR_r as the common SNR of each relay-sink channel.

We consider £rst a simple model of two parallel relay links with a repetition coded binary source. With binary sources, one can use the JSCC approach [5] to £nd a relay rule in the form of likelihood ratio quantizers. To simplify the design, we approximate the fading channels using binary symmetric channels (BSC) with properly computed crossover probability using the fading parameters. The error probability plots are given in Fig. 2 where we vary SNR_s for £xed SNR_r and Fig. 3 where we vary SNR_r for a £xed SNR_s. Some remarks are in order.

- In all cases, the proposed method has minimum error probability.
- SNR_s and SNR_r have different impact on the error probability performance. From Fig. 2, the proposed scheme is uniformly better than others for £xed SNR_r and varying SNR_s. However, for £xed SNR_s and varying SNR_r, the advantage of the proposed scheme over DF diminishes at low SNR. This can be explained as follows. The fact that the proposed scheme is better than DF for the repetition coded scheme is because in the DF scheme, the relay output is restricted to be binary, while in our scheme, one can use $2^K = 16$ codewords in the relay signaling. However, as the relay channel SNR decreases, the inherent adaptivity as discussed in [5] dictates that more redundancy be built in the relay node output. In the extreme case (very low SNR), the relay node produces a binary output for maximum redundancy to combat channel noise, resulting in a similar performance with that of DF. The leveling of DF at high SNR is because the relay node take a hard decision for DF and is therefore susceptible to decoding error that is limited by the £xed SNR at the source to relay nodes.
- The JSCC with BSC approximation suffers only small degradation compared with the proposed scheme, making it appealing in practice for *binary* sources.

Next, we consider a more practical scenario where the packet is coded with a $(7, 4)$ Hamming code with $L = 2$ relay nodes. As shown in Figs. 4 and Figs. 5, the proposed approach again has the best performance.

5. CONCLUSION

We exploit the £nite-alphabet (FA) property of the source message in the design of a cooperative relay system. Aimed at minimizing the error probability at the sink node, we derive the necessary conditions for an optimal distributed signaling scheme for a FA source. An iterative algorithm is presented to search for the (possibly local) optimal signaling scheme. Numeric examples show that our approach provides substantial performance gain over two standard relay strategies, namely the amplify-and-forward and the decode-and-forward schemes.

6. REFERENCES

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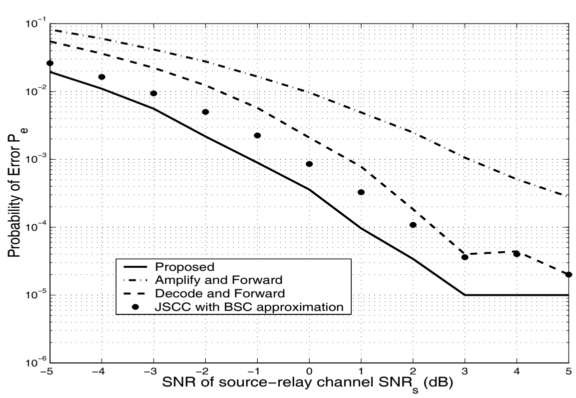

Fig. 2. Error probability versus SNR_s for $L = 2$, $M = 2$, $K = 4$, and $SNR_r = 0dB$.

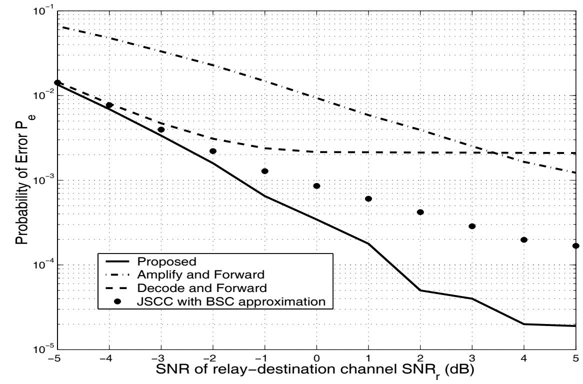

Fig. 3. Error probability versus SNR_r for $L = 2$, $M = 2$, $K = 4$, and $SNR_s = 0dB$.

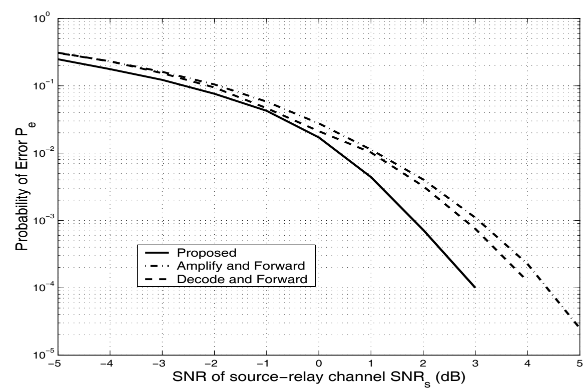

Fig. 4. Error probability versus SNR of source-relay channel for the case using $L = 2$ and $(7, 4)$ Hamming coded source input ($SNR_r = 5dB$).

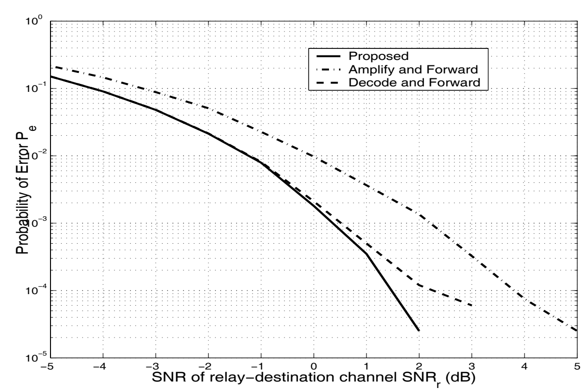

Fig. 5. Error probability versus SNR of relay-sink channel for the case using $L = 2$ and $(7, 4)$ Hamming coded source input ($SNR_s = 5dB$).