OPTIMUM MULTIFLOW DMT WITH CYCLIC PREFIX

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ABSTRACT

We consider the design of optimum discrete multitone (DMT) systems supporting multiple services with potentially differing quality of service (QoS) requirements. With the konwledge of channel and colored interference at the receiver input of the DMT system, our goal is to minimize the transmitted power while holding QoS specifications for different users. With redundancy in the form of cyclic prefix, optimum bit loading scheme, subchannel assigning scheme and transceiver design are found for DMT system. As with our previous study involving zero padding redundancy, the key conclusions are: (i) The optimum transceiver is unaffected by changing service characteristics, and depends only on the channel and interference conditions. (ii) The QoS requirements, the number of users and the number of subchannels assigned to the different users only affect bitloading and subchannel assignment.

1. INTRODUCTION

Discrete Multi-tone (DMT) modulation has proved effective in reliable high rate data transmission over frequency selective communication channels. It has been established as standards in various applications, like ADSL, HDSL for wireline communications [1], and IEEE 802.11a for fixed wireless communications in the form of Orthogonal Frequency Division Multiplexing (OFDM). DMT systems sinultaneously support multiple service flows as voice, data and video. Each of these streams in general has different Quality of Service (QoS) requirements as quantified by such parameters as bit rates and symbol eror rates (SER). To maintain high levels of performance, appropriate allocation of bandwidth and rates among the various services becomes an important problem. In particular, in such wireline applications as ADSL and VDSL, where channel conditions do not undergo substantial changes after the initial setup, these channel conditions are fedback to the transmitter, and used to achieve optimum bit loading in a manner to be specified in the sequel. Recently several authors such as [7], [2], [8] have sought to further improve performance by using channel conditions to optimize the transmitter and receiver as well, although only in a single user context. Given the proliferation of multiflow systems expected in the future, we consider here transceiver optimization, together with optimum bitloading in a multiuser context.

In our previous work, [5], we had considered transceiver optimization assuming a zero padding redundancy and general linear redundancy removal. This paper extends the results of [5] to the case where the redundancy used is a cyclic prefix redundancy. This also thus extends the result of [2] which considers the optimization with cyclic prefix redundancy, in the single user context.

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Figure 1 depicts a DMT system. M-parallel incoming data streams are passed through M-point block transform G_0, followed by a parallel-to-serial conversion. We assume that the channel and the equalizer are known, and that the equalizer/channel combination is FIR of length κ. If the channel noise is white, then the knowledge of the equalizer implies the knowledge of the colored noise statistics at the equalizer output. Throughout we assume that the second order statistics of effective disturbance at the equalizer output is available. A cyclic prefix redundancy of length κ is added to the channel input to infuse resistance to channel induced intersymbol interference (ISI). At the receiver the redundancy is removed, followed by a serial-to-parallel conversion, then another M-point block transform S_0. In traditional OFDM, the input transform is Inverse Discrete Fourier Transform (IDFT) operation, and the output transform is DFT operation.

We consider in this paper orthogonal DMT system with cyclic prefix. r service flows are supported, of which k-th flow is assigned n_k subchannels, requires bit rate of t_k and sustain an SER of η_k. As the zero-padding counterpart [4] [5], for cyclic prefix DMT system considered here, our goal is to choose proper transform matrices G_0 and S_0, to assign subchannels to each service flow, and to allocate bit rates to each subchannel in an optimum way such that the QoS specifications are achieved and total transmitted power is minimized. The optimization problem differs from that in [5] as under zero padding redundancy the transmitted power equals the power at the output of the P/S block in figure 1. In the cyclic prefix case, however, the actual transmitted power is greater.

The major conclusions of this paper are as follows. Just as in the case of zero padding redundancy, the optimum transceiver depends only on the knowledge of the channel and equalizer, and does not depend on either the number of flows or the QoS requirements of each flow. The latter only affect bit loading an subchannel assignment. In such wireline applications as DSL, ADSL or VDSL, once the initial connection has been established, the channel conditions do not change substantially, and at most suffer very slow drift. Thus once the estimates of the channel conditions have been fed back to the transmitter after the initial setup, for all practical purposes the transceiver does not have to be changed during the life of a given connection. Only bitloading and subchannel selection will be affected as the number of services and their QoS requirements change.

2. PROBLEM FORMULATION

Assume there are M data streams $x_0(n), x_1(n), \ldots, x_{M-1}(n)$ being fed into the DMT system that supports r users, k-th user takes n_k data streams for its transmission. Assume that that with the subchannels indexed from $\{1, \cdots, M\}$, $\mathcal{I}_k$ indexes the subchannels assigned to the k-th user, and that b_j is the number of bits/symbol assigned to the j-th subchannel. Then the bit rate constraint on the k-th user translates to

$$\sum_{j\in\mathcal{I}_k} b_j = t_k, \tag{2.1}$$

where t_k is a specified integer. For large bit per symbol constellation schemes, the ouput SNR required to achieve an SER of η is approximately $\sigma_{\hat{x}}^2/\sigma_e^2 = d2^{\zeta b}$ where $\sigma_{\hat{x}}^2$ and σ_e^2 are the signal and noise power respectively, [6], the constant d is determined by the modulation scheme used and the desired SER, η, and ζ depends on the modulation scheme employed (in the case of PAM, $\zeta = 2$) and b is the number bits per symbol.

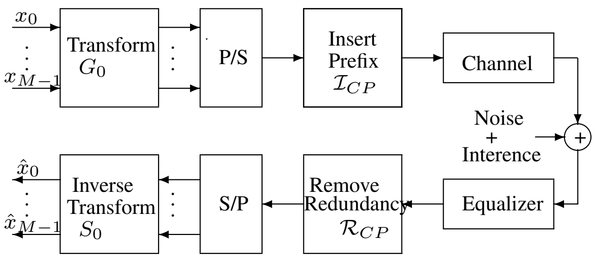

Fig. 1. DMT communication system.

The equalized FIR channel is assumed known for transceiver design purpose, with length κ, it can be specified as

$$C(z) = c_0 + c_1 z^{-1} + \ldots + c_\kappa z^{-\kappa} \tag{2.2}$$

Cyclic prefixing DMT requires that block vector length increase at least κ so as to be immune to inter-block interence (IBI). Define, $N = M + \kappa$. Call the $N \times N$ blocked version of $C(z)$, $\mathcal{C}(z)$.

$$\mathcal{C}(z) = \begin{bmatrix} c_0 & z^{-1}c_{N-1} & z^{-1}c_{N-2} & \ldots & z^{-1}c_1 \\ c_1 & c_0 & z^{-1}c_{N-1} & \ldots & z^{-1}c_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ c_{N-1} & c_{N-2} & \ldots & c_1 & c_0 \end{bmatrix} \tag{2.3}$$

with $c_i = 0$, for $i \geq \kappa$. With redundancy insertion and removal matrices expressed by

$$\mathcal{I}_{CP} = \begin{bmatrix} 0 & I_\kappa \\ I_M & \end{bmatrix} \qquad \mathcal{R}_{CP} = \begin{bmatrix} 0 & I_M \end{bmatrix}, \tag{2.4}$$

it can be easily verified that the composite of $\mathcal{I}_{CP}$, $\mathcal{C}(z)$ and $\mathcal{R}_{CP}$ is an $M \times M$ constant matrix, denoted by C_L. We assume that C_L is nonsingular.

We assume ISI free transmission or the perfect reconstruction (PR) condition: $\hat{\mathbf{x}}(n) = \mathbf{x}(n)$ for all n, or

$$S_0 C_L G_0 = I. \tag{2.5}$$

Under (2.5) $\sigma_{x_j}^2 = \sigma_{\hat{x}_j}^2$. Thus the if $j \in \mathcal{I}_k$ then the SER of η_k required by the k-th flow is met if

$$\sigma_{x_j}^2 = d_k 2^{\zeta_k b_j} \sigma_{e_j}^2. \tag{2.6}$$

Here $\sigma_{e_j}^2$ is the noise power at the output of the j-th subchannel, and d_k, ζ_k are determined by η_k and the modulation scheme employed by the k-th flow.

Now observe that the cyclic prefix redundancy ensures that the total transmitted power is in fact

$$P_B = \sum_{j=1}^{M} m_j \sigma_{x_j}^2 [G_0^H G_0]_{jj}$$

where $m_j = 1$ if $j \in \{1, \cdots, M-\kappa\}$ and $m_j = 2$ otherwise.

Thus total transmitted power is given by

$$P_B = \sum_{k=1}^{r} \sum_{j\in\mathcal{I}_k} m_j \sigma_{x_j}^2 [G_0^H G_0]_{jj} \tag{2.7}$$

Denote R_u to be the known autocorrelation matrix of the $N \times 1$ blocked vector $\mathbf{u}(n)$, of the noise at the equalizer output. We assume that R_u is known. Similarly R_w and R_v are the autocorrelation matrices of $\mathbf{w}(n)$, the $M \times 1$ blocked noise vector at the output of $\mathcal{R}_{CP}$, and $\mathbf{v}(n)$, $M \times 1$ blocked noise vector S_0, respectively. We have the relations

$$R_v = S_0 R_w S_0^H, \qquad R_w = \mathcal{R}_{CP} R_u \mathcal{R}_{CP}^H \tag{2.8}$$

Then to meet the SER requirement the total transmission power is given by

$$P_B = \sum_{k=1}^{r} \sum_{j\in\mathcal{I}_k} d_k 2^{\zeta_k b_j} m_j [G_0^H G_0]_{jj} [S_0 R_w S_0^H]_{jj}. \tag{2.9}$$

Thus, the optimization problem becomes: Given positive n_k, η_k, t_k, $M \times M$ positive definite Hermitian R_w, minimize (2.9) subject to (2.1), and (2.5), by selecting b_j (bit loading) $\mathcal{I}_k$ (subchannel assignment), and $M \times M$ nonsingular matrices S_0 and G_0.

3. OPTIMUM SELECTIONS

First we denote

$$T_0 = S_0 C_L \tag{3.10}$$
$$\hat{R}_w = C_L^{-1} R_w C_L^{-H}, \tag{3.11}$$

then

$$G_0^H G_0 = T_0^{-H} T_0^{-1} \tag{3.12}$$
$$S_0 R_w S_0^H = T_0 \hat{R}_w T_0^H \tag{3.13}$$

Consequently,

$$P_B = \sum_{k=1}^{r} \sum_{j\in\mathcal{I}_k} d_k 2^{\zeta_k b_j} m_j [T_0^{-H} T_0^{-1}]_{jj} [T_0 \hat{R}_w T_0^H]_{jj} \tag{3.14}$$

This objective function can be rewritten as

$$J(T_0) = \sum_{i=1}^{M} \alpha_i (e_i^T T_0 \hat{R}_w T_0^H)(e_i^T T_0^{-H} T_0^{-1} e_i) \tag{3.15}$$

where $\alpha_i = d_k 2^{\zeta_k b_j} m_{i,k} > 0, i \in \mathcal{I}_k$. Then we have the following characterization of a minimizing T_0.

Theorem 3.1 *Let the SVD of $M \times M$, positive definite Hermitian $\hat{R}_w$ be*

$$\hat{R}_w = U\Lambda^2 U^H \tag{3.16}$$

with Λ real, diagonal and U unitary. Then for arbitrary $\alpha_i > 0$, for some unitary V, (3.15) is minimized by

$$T_0 = V\Lambda^{-1/2}U^H \tag{3.17}$$

and (3.15) becomes

$$J(T_0) = \sum_{i=1}^{M} \alpha_i \left[V\Lambda V^H\right]_{ii}^2 \tag{3.18}$$

Using the Arithmetic Mean-Geometric Mean (AM-GM) inequality which states that the Arithmetic Mean of a set of positive numbers is greater than or equal to its Geometric Mean, with equality holding if and only if all numbers are equal, we have

$$\begin{aligned} &P_B \geq P_{BOPT} \\ &= \sum_{k=1}^{r} d_k n_k \left[2^{\zeta_k t_k} \prod_{j\in\mathcal{I}_k} m_j [T_0^{-H} T_0^{-1}]_{jj} [T_0 \hat{R}_w T_0^H]_{jj}\right]^{\frac{1}{n_k}} \end{aligned} \tag{3.19}$$

with equality holding if and only if for all k and $i, j \in \mathcal{I}_k$

$$\begin{aligned} & 2^{\zeta_k b_j} m_j [T_0^{-H} T_0^{-1}]_{jj} [T_0 \hat{R}_w T_0^H]_{jj} \\ = \ & 2^{\zeta_k b_{i,k}} m_{i,k} [T_0^{-H} T_0^{-1}]_{ii} [T_0 \hat{R}_w T_0^H]_{ii} \end{aligned} \tag{3.20}$$

This in turn provides the optimum *bit loading rule*:

$$\begin{aligned} b_j &= \frac{t_k}{n_k} \\ &-\frac{1}{\zeta_k}\log_2\left[\frac{m_j [T_0^{-H} T_0^{-1}]_{jj} [T_0 \hat{R}_w T_0^H]_{jj}}{(\prod_{j\in\mathcal{I}_k} m_j [T_0^{-H} T_0^{-1}]_{jj} [T_0 \hat{R}_w T_0^H]_{jj})^{\frac{1}{n_k}}}\right] \end{aligned} \tag{3.21}$$

Thus, to minimize P_B, it is sufficient to find unitary matrix V that minimizes

$$\sum_{k=1}^{r} d_k n_k 2^{\zeta_k t_k} [\prod_{j\in\mathcal{I}_k} m_j [V\Lambda V^H]_{jj}^2]^{\frac{1}{n_k}} \tag{3.22}$$

Denote $\beta_k = d_k n_k 2^{\zeta_k t_k / n_k}$, $a_j = [V\Lambda V^H]_{jj}$, then we need to minimize the following objective function

$$J_M(a_0, \ldots, a_{M-1}) = \sum_{k=1}^{r} \beta_k \prod_{j\in\mathcal{I}_k} m_j^{\frac{1}{n_k}} a_j^{\frac{2}{n_k}}. \tag{3.23}$$

Notice that the expression in (3.23) is independent of the actual selection of b_j. This conceptual separation indicates that T_0 can be found without determining b_j.

Now two factors affect the value of J_M. First of course the choice of a_j. Even given a_j, J_M the choice of $\mathcal{I}_k$, i.e. the way in which the a_j are arranged between the products affects the value of J_M. Given a choice of a_j call the choice of $\mathcal{I}_k$, that leads to the smallest J_M, as representing an *optimum arrangment*.

Before proceeding, we need a few results from the theory of majorization, [3].

Definition 3.1 *Consider two sequences $x = \{x_i\}_{i=1}^n$ and $y = \{y_i\}_{i=1}^n$ with $x_i \geq x_{i+1}$ and $y_i \geq y_{i+1}$. Then we say that y majorizes x, denoted as $x \prec y$, if $\sum_{i=1}^k x_i \leq \sum_{i=1}^k y_i$ holds for $1 \leq k \leq n$, with equality at $k = n$. We say that y weakly supermajorizes x, denoted $x \prec^W y$, if $\sum_{i=j}^n x_i \geq \sum_{i=j}^n y_i, 1 \leq j \leq n$.*

Fact 3.1 *If H is an $n\times n$ Hermitian matrix with diagonal elements $h = \{h_i\}_{i=1}^n$ and eigenvalues $\lambda = \{\lambda_i\}_{i=1}^n$, then $h \prec \lambda$.*

Definition 3.2 *A real valued function $\phi(z) = \phi(z_1, \ldots, z_n)$ defined on a set $\mathcal{A} \subset R^n$ is said to be Schur concave on $\mathcal{A}$ if $x \prec y$ on $\mathcal{A} \Rightarrow \phi(x) \geq \phi(y)$. ϕ is strictly Schur concave on $\mathcal{A}$ if strict inequality $\phi(x) > \phi(y)$ holds when x is not a permutation of y. Further if $x \prec^W y$ then also $\phi(x) > \phi(y)$.*

We will now state a theorem that results in a test for strict Schur concavity. We denote $\phi_{(k)}(z) = \frac{\partial\phi(z)}{\partial z_k}$.

Lemma 3.1 *Let $\phi(z)$ be a scalar real valued function defined and continuous on $\mathcal{D} = \{(z_1, \ldots, z_n) : z_1 \geq \ldots \geq z_n\}$, and twice differentiable on the interior of $\mathcal{D}$. Then $\phi(z)$ is Schur concave on $\mathcal{D}$ if $\phi_{(k)}(z)$ is increasing in k.*

The following Lemma together with Lemma 3.1 proves that J_M is in fact Schur concave under optimal arrangements.

Lemma 3.2 *Consider for integers $p, q \geq 2$,*

$$f = (\alpha \prod_{k=0}^{p-1} m_k a_k)^{2/p} + (\beta \prod_{l=0}^{q-1} m_l h_l)^{2/q}$$

with $\alpha, \beta, a_k, h_l > 0$. Suppose for some i, j

$$a_i > h_j \text{ and } \frac{\partial f}{\partial a_i} > \frac{\partial f}{\partial h_j}. \tag{3.24}$$

Then

$$\begin{aligned} g \ &= \ (\alpha \prod_{k=0,k\neq i}^{p-1} a_k . h_j \prod_{k=0}^{p-1} m_k)^{2/p} \\ &+ \ (\beta \prod_{k=0,k\neq j}^{q-1} h_k . a_i \prod_{k=0}^{q-1} m_k)^{2/q} < f. \end{aligned} \tag{3.25}$$

Further if $a_i > a_{i+1}$, then $\partial f/\partial a_i < \partial f/\partial a_{i+1}$, if $h_i > h_{i+1}$, then $\partial f/\partial h_i < \partial f/\partial h_{i+1}$.

The importance of concluding that $J_M(a_0, \ldots, a_{M-1})$ to be Schur concave resides in the simple and explicit solution of T_0 [4], i.e., for a suitable permutation matrix P,

$$T_0 = PU^H \tag{3.26}$$

minimizes $J_M(a_0, \ldots, a_{M-1})$.

In multiuser scenario even with uneven subchannel distribution, we can show the optimum transceiver obtained above remains the same as that in single user case of [2]. Indeed the optimum receiver is

$$S_0 = PU^H C_L^{-1} = (PU^H C_L^{-1} R_w^{1/2}) R_w^{-1/2} \tag{3.27}$$

and using (3.11), (3.16), we have

$$(PU^H C_L^{-1} R_w^{1/2})(R_w^{1/2} C_L^{-H} C_L^{-1} R_w^{1/2})(PU^H C_L^{-1} R_w^{1/2})^H$$
$$= PU^H U \Lambda^2 U^H U \Lambda^2 U^H U P^H = P \Lambda^4 P^H \quad (3.28)$$

which is a diagonal matrix.

Indeed G_0 and S_0 only require the knowledge of the channel and R_u. Only the permutation matrix is affected by QoS requirements and used only to distribute the subchannels between the users.

4. SIMULATION RESULTS

We compare the transmitted power of the DFT-based DMT system and optimum transceiver DMT system with optimum bit allocation. For DFT-based DMT system, each user has its evenly distributed bits/symbols among its subchannels. We assume the equalized channel to be $C(z) = 1 + 0.5z^{-1}$, and $u(n)$ the noise at the equalizwer output has power spectral density shown in figure 2. Each subchannel employs PAM constellation. Above 13 dB improvement is observed over conventional DMT.

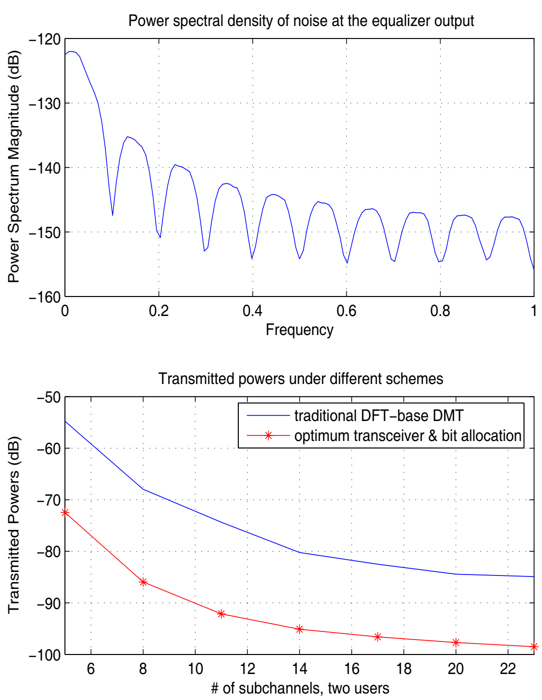

Fig. 2. Comparison of transmit power levels.

5. CONCLUSIONS

In this paper, we have presented optimum DMT transceiver design and bit loading scheme for minimizing the transmitted power, with different users having different QoS requirements. Unlike the similar problem for DMT system with zero padding, whose optimum input and out put transforms are unitary, in this case these transforms need no be unitary. An attractive feature of the solution is that the optimum transceiver depends only on the knowledge of the channel and equalizer, and does not depend on either the number of flows or the QoS requirements of each flow. The latter only affect bit loading an subchannel assignment.

6. REFERENCES

[1] J.S. Chow, J.C. Tu, J.M. Cioffi, "A discrete multitone transceiver system for HDSL applications", *IEEE Journal on Selected Areas in Communications*, pp 895-908, Aug 1991.

[2] Y.-P. Lin and S.-M. Phoong, "Optimal ISI-free DMT transceivers for distorted channels with colored noise", *IEEE Transactions on Signal Processing*, pp 2702 -2712, Nov 2001.

[3] A. W. Marshall and I. Olkin, *Inequalities: Theory of Majorization and its applications*, Academic Press, 1979.

[4] A. Pandharipande and S. Dasgupta, "Optimal DMT-based transceivers for multiuser communications", *IEEE Transactions on Communications*, pp 2038-2046, Dec 2003.

[5] A. Pandharipande and S. Dasgupta, "Optimum multiflow biorthogonal DMT with unequal subchannel assignment", *IEEE Transactions on Signal Processing*, to appear.

[6] J.G. Proakis, *Digital Communications*, McGraw-Hill, Third Edition, 1995.

[7] Y.-P. Lin and S.-M. Phoong, "Perfect discrete multitone modulation with optimal transceivers", *IEEE Transactions on Signal Processing*, pp 1702 -1711, June 2000.

[8] A. Scaglione, S. Barbarossa and G.B. Giannakis, "Filterbank transceivers optimizing information rate in block transmissions over dispersive channels", *IEEE Transactions on Information Theory*, pp 1019-1032, Apr 1999.