JOINT DESIGN OF INTERPOLATION FILTERS AND FREQUENCY EQUALIZERS IN DMT SYSTEMS FOR XDSL APPLICATIONS

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ABSTRACT

We formulate the joint design of interpolation filter and frequency equalizer in DMT systems as the problem for minimization of the mean square error between the input data symbol and its output estimate. Then an iterative algorithm is proposed for solution. Each iteration requires only solving two quadratic minimizations; the algorithm also ensures convergence. Simulations for VDSL applications demonstrate that an increment in transmission rate can be achieved compared with the conventional design of the interpolation filter and frequency equalizer separately.

1. INTRODUCTION

In xDSL communications employing the discrete multitone (DMT) modulation, as noted in [1], the receiver design for xDSL applications commonly uses an interpolation filter to compensate for the timing offset and a fixed frequency equalizer (FEQ) to equalize the channel distortions. The interpolation filter and the FEQ are conventionally designed separately [2, 3]. Recently, the interpolation filter and the FEQ are jointly designed in [4] based on the max-min criterion, its approach, however, fails to provide a simple algorithm for solution and only the brute-force exhaustive searching is adopted. It further constrains the interpolation filter to be the type of Kaiser window filter and ignores the noise effect in order to reduce the realization complexity; these constraints, however, may degrade the receiver performance. In this paper, instead of using the max-min criterion, the minimum mean square error is used to formulate the joint design of the interpolation filter and the FEQ. Then a simple iterative algorithm is presented for solution. Each iteration requires only to solve two quadratic minimizing problems; the algorithm also ensures convergence. Simulations for VDSL [5] applications demonstrate that for short channel a substantial increment in transmission rate can be achieved compared with the conventional design.

This work was supported by National Science Council, Taiwan, under NSC93-2213-E-009-040.

2. PROBLEM FORMULATION

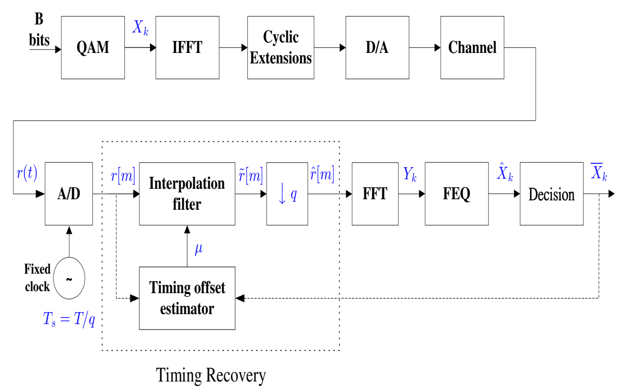

Fig. 1. The block diagram of a DMT system

The block diagram of a DMT system is shown in Fig. 1. Let N, an even integer, denote the number of carriers in the DMT system. The transmitted data $X_0, \ldots, X_{N-1}$ with $X_{N/2} = 0$ and $X_k = X^*_{N-k}$ for $k = 1, \ldots, N/2 - 1$ where the superscript * denotes the complex conjugation, are modulated by the IFFT, supplemented by cyclic prefix/suffix extensions and windowing, and finally converted by the digital-to-analog (D/A) converter into analog signals through the channel; the receiver, including the analog-to-digital (A/D) converter, the timing recovery block, the FFT, the FEQ, and the decision detector, identifies the transmitted data. The timing recovery mainly consists of a timing offset estimator, an interpolation filter, and a downsampling converter. We assume that the timing offset estimator obtains a correct estimation; our purpose is to jointly design the interpolation filter and the FEQ such that the mean-square error (MSE) between the transmitted data X_k and the FEQ output $\hat{X}_k$ averaged over $k = 0, \ldots, N/2 - 1$ is minimized. The followings develop the formulation of the criterion in terms of the interpolation filter and FEQ parameters.

2.1. Received Data Sample

Assume that the frame synchronization has been achieved such that the redundant cyclic prefix/suffix extensions have been removed. Then, the received signal can be approximately represented [4] by the following equation,

$$r(t) = \sum_{k=1-N/2}^{N/2-1} X_k G_k e^{j(2\pi k/N)(t/T)} + n(t), \quad (1)$$

for $0 \le t \le (N-1)T$ where G_k is the channel frequency response to signal $e^{j(2\pi k/N)t}$, $n(t)$ is an added white Gaussian noise, and T denotes the input sample period. Denote T_s as the sampling period of the A/D converter; it is given by $T_s = T/q$ where q, an integer, is called as the oversampling factor which is commonly 1 or 2 for xDSL applications. The sample time for the m-th data is $mT_s - \mu T_s$ where μ is the normalized timing offset, thus $-0.5 \le \mu < 0.5$. Denote $r[m] = r((m-\mu)T_s)$, then using (1), we have

$$r[m] = \sum_{k=1-N/2}^{N/2-1} X_k G_k e^{j(2\pi k/qN)(m-\mu)} + n[m]. \quad (2)$$

Hence, in one symbol period we obtain qN samples.

2.2. Interpolation, Downsampling, and FEQ

The interpolation filter is used to compensate for the timing offset μT; it is commonly realized by an FIR filter [6, 7] with its coefficients varying with μ. Assume the FIR interpolation filter is of degree M with its coefficients denoted by $h_\mu[l], l = 0, \ldots, M$, then its output $\widetilde{r}[m]$, excited by the received data $r[m]$ which is replaced by (2), is given by

$$\widetilde{r}[m] = \sum_{k=1-N/2}^{N/2-1} X_k G_k H_{\mu,k} e^{j(2\pi k/qN)(m-\mu)} + v[m], \quad (3)$$

where $v[m] = \sum_{l=0}^{M} n[m-l] h_\mu[l]$ and $H_{\mu,k} = \sum_{l=0}^{M} h_\mu[l] e^{-j(2\pi/qN)kl}$.

The interpolation filter coefficient is commonly characterized by a polynomial of μ such that the well-known Farrow structure [8] can be employed for efficient realization. Assume this polynomial is of degree P, then $h_\mu[l] = \sum_{p=0}^{P} a_{l,p}\mu^p$. Denote $\omega_k = 2\pi k/(qN)$, thus $H_{\mu,k}$, the frequency response of the interpolation filter at ω_k, can be expressed in a compact form as

$$\begin{aligned} H_{\mu,k} &= \sum_{m=0}^{M}\sum_{p=0}^{P} a_{m,p}\mu^p e^{-j\omega_k m} \\ &= \boldsymbol{a}^T(\boldsymbol{\mu} \otimes \boldsymbol{\omega}_k) \end{aligned} \quad (4)$$

where the superscript T denotes the transposition, $\otimes$ represents the right Kronecker product, $\boldsymbol{a} = [a_{0,0}, \cdots, a_{M,0}, a_{0,1}, \cdots, a_{M,1}, \cdots, a_{0,P}, \cdots, a_{M,P}]^T$, $\boldsymbol{\mu} = [\mu^0, \mu^1 \cdots, \mu^P]^T$, and $\boldsymbol{\omega}_k = [1, e^{-j\omega_k}, \cdots, e^{-jM\omega_k}]^T$.

The output, after down-sampling by q, is obtained directly by $\widehat{r}[m] = \widetilde{r}[qm]$,

$$\widehat{r}[m] = \sum_{k=1-N/2}^{(N/2)-1} X_k G_k H_{\mu,k} e^{j\omega_k(qm-\mu)} + v[qm] \quad (5)$$

for $0 \le m \le N-1$. These data $\hat{r}[m]$ are transformed by FFT, yielding

$$Y_k = X_k G_k H_{\mu,k} e^{-j\omega_k\mu} + V_k, \; k = 0, \ldots, N-1 \quad (6)$$

Multiplying Y_k by the FEQ Q_k and using (4), we obtain the X_k estimate, $\hat{X}_k = Y_k Q_k$,

$$\begin{aligned} \hat{X}_k &= X_k G_k H_{\mu,k} e^{-j\omega_k\mu} Q_k + V_k Q_k \\ &= X_k G_k Q_k [\boldsymbol{a}^T(\boldsymbol{\mu} \otimes \boldsymbol{\omega}_k)] e^{-j\omega_k\mu} + V_k Q_k \end{aligned} \quad (7)$$

2.3. The Average MSE Criterion

The average of the mean-square error between the transmitted data and estimated output for the $N/2$ tones is used as the criterion J, given by

$$J = \sum_{k=0}^{N/2-1} C_k \mathrm{E}[|X_k - \widehat{X}_k|^2] \quad (8)$$

where $\mathrm{E}[\cdot]$ denotes the expectation and C_k the weighting factor for the kth tone. Substituting (7) into (8), rearranging, we obtain J as a function of the interpolation filter coefficients $\boldsymbol{a}$ and FEQ components $Q_k, k = 0, \ldots, N/2-1$,

$$\begin{aligned} J \;=\; & \sum_{k=0}^{N/2-1} C_k S_{x_k} \mathrm{E}[|1 - \boldsymbol{a}^T(\boldsymbol{\mu} \otimes \boldsymbol{\omega}_k) e^{-j\omega_k\mu} G_k Q_k|^2] \\ & + \sum_{k=0}^{N/2-1} C_k S_{n_k} |Q_k|^2 \mathrm{E}[|\boldsymbol{a}^T(\boldsymbol{\mu} \otimes \boldsymbol{\omega}_k)|^2] \end{aligned} \quad (9)$$

where $S_{x_k} = \mathrm{E}[|X_k|^2]$ and S_{n_k} is the power spectrum density of noise sample $n[k]$ at ω_k. Note that the above derivation is valid only if the input noise $n(t)$ is bandlimited in $[-1/(2T), 1/(2T)]$ when oversampling $q \ge 2$ is enabled; this assumption is reasonable because the antialiasing filter is commonly employed before the A/D converter.

The criterion J (9) is obviously a nonlinear function of both the FEQ components $Q_k, k = 0, \ldots, N/2-1$ and the interpolation filter coefficients $\boldsymbol{a}$. Conventional nonlinear optimization approaches may be applied for solution but they are often complicated. In the following section, a simple iterative algorithm is presented for solution.

3. ACM ALGORITHM FOR SOLUTION

Investigating (9) closely we observe that both the FEQ components and the interpolation filter contribute, separately, to the cost function in a quadratic form. That is, J is a quadratic function of the FEQ components if the interpolation filter coefficients $\boldsymbol{a}$ are fixed and vice versa. Hence, the algorithm of alternating coordinates minimization (ACM) [9] is an effective way for solution.

The ACM algorithm for solving the minimization of (9) involves iterations of two alternating optimizing operations: in the l-th iteration, the first optimizing operation solves $\boldsymbol{a}^{(l)}$ to minimize (9) given a fixed $Q_k = Q_k^{(l-1)}$ for all k; the second optimizing operation solves $Q_k^{(l)}$ to minimize (9) given a fixed $\boldsymbol{a} = \boldsymbol{a}^{(l)}$. The algorithm may start either with the first optimizing operation given initial FEQ $Q_k^{(0)}$ or with the second optimizing operation given the initial interpolation filter $\boldsymbol{a}^{(0)}$, then the two optimizing operations continue alternatingly until the convergence of $\boldsymbol{a}^{(l)}$ and $Q_k^{(l)}$. Since the two operations all find $\boldsymbol{a}^{(l)}$ or $Q_k^{(l)}, k = 0, \ldots, N/2-1$ to minimize J, the obtained MSE J of each iteration is thus guaranteed non-increasing. Also J is non-negative and hence bounded from below, the algorithm, therefore, ensures convergence. Detailed derivations of two optimizing operations are described below.

3.1. First optimizing operation: solve $\boldsymbol{a}^{(l)}$ given $Q_k = Q_k^{(l-1)}, k = 0, \ldots, N/2-1$.

Since Q_k's are fixed, J (9) is a quadratic function of interpolation filter coefficients $\boldsymbol{a}$. Taking the gradient of J with respect to $\boldsymbol{a}$ and setting it to a zero vector yields the unique solution for $\boldsymbol{a}$,

$$\begin{aligned}\boldsymbol{a}^{(l)} &= \{\sum_{k=0}^{N/2-1} C_k|Q_k|^2(S_{x_k}|G_k|^2 + S_{n_k})\mathrm{Re}[\boldsymbol{R}_k]\}^{-1} \\ &\quad \{\sum_{k=0}^{N/2-1} C_k S_{x_k}\mathrm{Re}[G_k Q_k \boldsymbol{p}_k]\} \qquad (10)\end{aligned}$$

where $\mathrm{Re}[z]$ denotes the real part of z, and the matrix $\boldsymbol{R}_k$ and vector $\boldsymbol{p}_k$ are

$$\begin{aligned}\boldsymbol{R}_k &= \mathrm{E}[(\boldsymbol{\mu} \otimes \boldsymbol{\omega}_k)(\boldsymbol{\mu} \times \boldsymbol{\omega}_k)^H] \\ &= \mathrm{E}[\boldsymbol{\mu}\boldsymbol{\mu}^H] \otimes (\boldsymbol{\omega}_k \boldsymbol{\omega}_k^H) \qquad (11) \\ \boldsymbol{p}_k &= \mathrm{E}[\boldsymbol{\mu} e^{-j\omega_k \mu}] \otimes \boldsymbol{\omega}_k \qquad (12)\end{aligned}$$

with the superscript H standing for the transpose conjugate operation. Since the probability density function of μ is conventionally assumed to be uniform, closed-form formulas for evaluating $\mathrm{E}[\boldsymbol{\mu} e^{-j\omega_k \mu}]$ and $\mathrm{E}[\boldsymbol{\mu}\boldsymbol{\mu}^H]$ exist and can be obtained from the standard mathematical tables; hence the solution (10) can be simply realized.

3.2. Second optimizing operation: solve $Q_k^{(l)}, k = 0, \ldots, N/2-1$, given $\boldsymbol{a} = \boldsymbol{a}^{(l)}$.

When $\boldsymbol{a} = \boldsymbol{a}^{(l)}$, J (9) is also a simple quadratic function of Q_k and its solution for minimizing J can be similarly derived,

$$Q_k^{(l)} = \frac{S_{x_k} G_k^* \boldsymbol{p}_k^H \boldsymbol{a}}{(S_{x_k}|G_k|^2 + S_{n_k})\boldsymbol{a}^T \boldsymbol{R}_k \boldsymbol{a}}, \qquad (13)$$

for $k = 0, \ldots, N/2-1$. Thus its realization is also simple.

Note that the ACM algorithm, like most nonlinear optimization algorithms, may converge to a local minimum. Therefore, a sensible initial estimate may be required. One good initial estimate of the interpolation filter is the Lagrange interpolator [6]; another good initial FEQ is $Q_k^{(0)} = S_{x_k} G_k^* / (S_{x_k}|G_k|^2 + S_{n_k})$ for all k, which is the FEQ designed for minimizing J when the interpolation filter perfectly compensates for the time delay.

4. DESIGN FOR VDSL APPLICATIONS

The proposed method is used to design the interpolation filter and the FEQ for VDSL applications. Its performance is then compared with that of using the fixed cubic Lagrange interpolator [6] and the FEQ designed for minimizing J. In VDSL systems [5], the working frequencies are divided into four subbands; two of them are used for downstream and the other two for upstream transmissions. We simulate for the downstream connection with $N = 8192$, baud-rate sampling $q = 1$, and $M = 3, P = 3$ for the interpolation filter. The tone indexes in the two downstream bands are $k \in [33, 880]$ and $k \in [1207, 1970]$. The frequencies of the amateur radio bands and AM radios are not employed; thus the total number of used tones is 1412. The simulation channels use the VDSL1 loop, a 24-gauge twisted pair, of short (1500ft), medium (3000ft), or long-range (4500ft) values. The noise considers the white, the near-end and far-end cross-talk noises, while the signal power of each tone, following the cabinet deployment scenario defined in [5], is all identical. The weighting factor is uniform; that is $C(k) = 1/1412$ for the employed tone k, otherwise $C(k) = 0$. The algorithm starts with an initial cubic Lagrange interpolation filter and stops when $|J^{(l)} - J^{(l-1)}|/J^{(l-1)}$ is less than 10^{-4} for short and medium channels, and 10^{-5} for long channel where $J^{(l)}$ is the cost J evaluated after the l-th iteration.

The cost $J^{(l)}$ in dB versus iteration for short channel is shown in Fig. 2. As shown, $J^{(l)}$ is, as expected, decreasing with each iteration. To quantify the improvement of transmission rate, the noise power of each tone is evaluated as $\mathrm{E}[|X_k - \hat{X}_k|^2]$ and the signal-to-noise ratio of the k-th tone (SNR_k) as $S_{x_k}/\mathrm{E}[|X_k - \hat{X}_k|^2]$, then the allowed bit number of each tone for transmission can be approximately obtained by $b_k = \log_2(1 + \mathrm{SNR}_k/\Gamma)/2$ where $\Gamma = 9.55$

for the bit-error-rate less than 10^{-7}. Hence, the sum of b_k multiplied by the transmission rate of 4 kbps of each tone is the total transmission rate. Fig. 3 shows the obtained SNR_k for the short channel of the two design examples. The figure reflects the property that the cubic Lagrange interpolation filter compensates for the timing offset of the signal nearly perfect in the low-frequency band but poor in the high-frequency band; the proposed joint design improves the performance in the high-frequency band at the cost of slight performance degradation in the low-frequency band. The transmission rate and its increment for the three channels are listed in Table 1 in which the proposed method gains an increase of 8 Mbps for short channel but nearly without any gain for medium and long channels. The reason is that the channel responses in the high-frequency band attenuate more quickly than in the low-frequency band as the channel length increases; this quick attenuation renders the improvement of the designed interpolator in the high-frequency band ineffective. This shortage may be improved by the use of the weighting factor and the topic of its empirical design is in our future investigation.

5. CONCLUSION

This paper presents a joint design of the interpolation filter and FEQ in DMT systems for xDSL applications. The design algorithm is simple to realize and ensures convergence. Simulations for VDSL downstream communications show that for short channel a substantial improvement in transmission rate can be achieved compared with the conventional design.

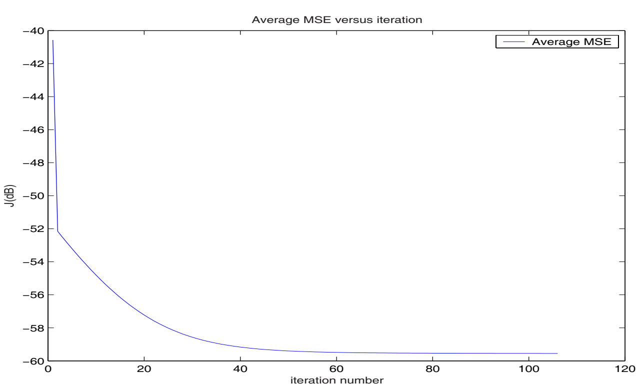

Fig. 2. The MSE J versus the iteration

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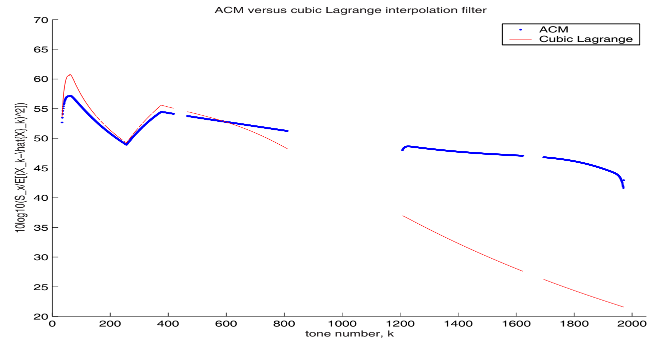

Fig. 3. SNR_k in dB versus tone number k

Table 1. Transmission rate in Mbps

channel	cubic	proposed	improvement
1500ft	28.1	36.1	8.0
3000ft	26.5	27.0	0.5
4500ft	15.3	15.3	0.0

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