# COSINE MODULATED MULTITONE FOR VERY HIGH-SPEED DIGITAL SUBSCRIBER LINES

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## ABSTRACT

In this paper, the use of cosine modulated filter banks (CMFB) for multi-carrier modulation in the application of very high-speed digital subscriber lines (VDSL) is studied. We refer to this modulation technique as cosine modulated multitone (CMT). CMT is fundamentally the same as the discrete wavelet multitone (DWMT) except the receiver structure. With the modified receiver structure, CMT uses only two taps per sub-carrier for equalization. This is an order of magnitude less complex than DWMT where each sub-carrier requires more than 20 taps for equalization. We compare CMT with zipper discrete multitone (z-DMT) and filtered multitone (FMT), the two modulation techniques that have been included in the VDSL draft standard.

## 1. INTRODUCTION

Multi-carrier modulation (MCM) has attracted considerable attention in recent years as a practical and viable technology for high-speed data transmission over spectrally shaped noisy channels. The most popular MCM technique uses the properties of the discrete Fourier transform (DFT) in an elegant way so as to achieve a computationally efficient realization. While the terminology discrete multitone (DMT) is used in the DSL literature to refer to this MCM technique, in wireless applications the terminology orthogonal frequency division multiplexing (OFDM) has been adopted. Asynchronous zipper DMT (z-DMT) [1] is the latest version of DMT that has been proposed as an effective frequency division duplexing (FDD) method for VDSL application, and has been adopted in the VDSL draft standard [8].

Radio frequency interference (RFI) is a major challenge that any VDSL modem has to deal with. The poor side-lobe behavior of DFT filters and also the very high level of RFI result in significant interference from RFI which degrades the performance of z-DMT significantly. To deal with RFI, there is a current trend in the industry to adopt filter bankbased MCM techniques.

Filtered multitone (FMT) is a filter bank solution that has been proposed by IBM and has been widely studied recently [5, 6, 7]. Another filter bank solution that has been widely addressed in literature is discrete wavelet multitone (DWMT). This is based on cosine modulated filter banks (CMFB). In DWMT, it is proposed that channel equalization in each sub-carrier be performed by combining the signals from the desired band and its adjacent bands. These equalizers that have been referred to as post-combiner equalizers impose significant load to the computational complexity of the receiver. This complexity and lack of in-depth theoretical understanding of DWMT have kept industry lukewarm about it in the past. A revisit to CMFB-MCM/DWMT has been made recently [9, 10]. In [10], in particular, it was noted that by properly restructuring the receiver, each post-combiner equalizer could be replaced by a two tap filter.

In this paper, we extend the application of this modified CMFB-MCM to VDSL channels. In order to distinguish between the proposed technique and DWMT, we refer to it as cosine modulated multitone (CMT). By performing extensive computer simulations, we contrast CMT against z-DMT and FMT and make an attempt to highlight the relative advantages that each of these three techniques offer.

## 2. COSINE MODULATED MULTITONE

In transceiver proposed in [10], the synthesis CMFB follows the conventional implementation reported in the literature [3]. However, for the analysis, a non-simplified structure of CMFB is used. Fig. 1 presents a block diagram of this nonsimplified structure for an *M*-band analysis CMFB; see [3] for development of this structure.<sup>1</sup>  $G_k(z), 0 \le k \le 2M-1$ , are polyphase components of the filter bank prototype filter  $P_0(z)$ . The coefficients  $d_0, d_1, \dots, d_{2M-1}$  are chosen in order to equalize the group delay of the filter bank sub-channels. This gives  $d_k = e^{j\theta_k}W_{2M}^{(k+0.5)N/2}$  for k = $0, 1, \dots, M-1$ , and  $d_k = d_{2M-1-k}^*$ , for k = M, M + $1, \dots, 2M - 1$ , where  $\theta_k = (-1)^k \frac{\pi}{4}, W_{2M} = e^{-j2\pi/2M}$ , \* denotes conjugate, and *N* is the order of  $P_0(z)$ .

Let  $Q_0^{\rm a}(z), Q_1^{\rm a}(z), \cdots, Q_{2M-1}^{\rm a}(z)$  denote the transfer functions between the input x(n) and the analyzed outputs

<sup>&</sup>lt;sup>1</sup>In this paper, we have used slightly different notations from [3] to simplify the development of the results.



**Fig. 1**. The analysis CMFB structure that is proposed for CMT.

 $u_0^o(n), u_1^o(n), \cdots, u_{2M-1}^o(n)$ , respectively. We recall from the theory of CMFB that

$$Q_k^{\rm a}(z) = d_k P_0(z W_{2M}^{k+0.5}), \quad k = 0, 1, \cdots, 2M - 1.$$
 (1)

Using (1), one finds that

$$Q_{2M-1-k}^{a}(e^{-j\omega}) = [Q_{k}^{a}(e^{j\omega})]^{*}.$$
 (2)

This shows that  $Q_k^{a}(z)$  and  $Q_{2M-1-k}^{a}(z)$  have frequency responses which are magnitude symmetric and phase anti symmetric with respect to origin,  $\omega = 0$ .

In a conventional CMFB, analysis filters are generated by adding the pairs of  $Q_k^a(z)$  and  $Q_{2M-1-k}^a(z)$ , for  $k = 0, 1, \dots, M-1$ . In the CMT, however, we resort to using the complex coefficient analysis filters  $Q_k^a(z)$ , for  $k = 0, 1, \dots, M-1$ . In the absence of channel, the perfect reconstruction property of CMFB implies that [3]

$$u_k(n) = \frac{1}{2} [s_k(n) + jr_k(n)]$$
(3)

where  $s_k(n)$  is the transmitted symbol and  $r_k(n)$  arises because of ISI from the *k*th sub-channel and ICI from other sub-channels.

We assume that the number of sub-bands is sufficiently large such that the channel frequency response, H(z), can be approximated by a complex constant gain  $h_k$  over the kth sub-channel. Then, in the presence of channel,

$$u_k(n) \approx \frac{1}{2} [s_k(n) + jr_k(n)] \times h_k + \nu_k(n)$$
(4)

where  $\nu_k(n)$  is the channel additive noise after filtering. Obviously, the accuracy of the approximation in (4) improves as the number of sub-carriers, M, increases.

Considering (4), an estimate of  $s_k(n)$  can be obtained as follows:

$$\hat{s}_{k}(n) = \Re\{w_{k}^{*}u_{k}(n)\}$$

$$= w_{k,\mathrm{R}}u_{k,\mathrm{R}}(n) + w_{k,\mathrm{I}}u_{k,\mathrm{I}}(n)$$
(5)

where the subscripts R and I denote the real and imaginary parts of. By choose the optimum value of  $w_k^*$  as  $w_{k,opt}^* = \frac{2}{h_k}$ , we completely removes the ISI plus ICI term  $r_k(n)$ .

## 3. EFFICIENT REALIZATION OF ANALYSIS CMFB

Efficient implementation of synthesis CMFB using discrete cosine transform (DCT) can be found in [3]. This will be used at the transmitter side of a CMT transceiver. At the receiver, as discussed above, we use a modified structure of analysis CMFB. Thus, efficient implementations that are available for the conventional analysis CMFB [3] are of little use here. In this section, we develop a computationally efficient realization of the analysis CMFB by modifying the structure of Fig. 1.

At the receiver, we need to implement filters  $Q_0^{a}(z)$ ,  $Q_1^{a}(z), \dots, Q_{M-1}^{a}(z)$ . Recalling (2) and noting that x(n) is real-valued, we argue that these filters can equivalently be implemented by realizing  $Q_k^{a}(z)$  for  $k = 0, 2, 4, \dots, 2M - 2$ , *i.e.*, for even values of k only;  $Q_1^{a}(z)$ , for instance, is realized by taking the conjugate of the output of  $Q_{2M-2}^{a}(z)$ .

We note from Fig. 1 that

$$Q_{2k}^{a}(z) = d_{2k} \sum_{l=0}^{2M-1} \left(z^{-1} W_{2M}^{-1/2}\right)^{l} G_{l}(-z^{2M}) W_{2M}^{-2kl}$$
  
$$= d_{2k} \sum_{l=0}^{M-1} \left[z^{-l} \left(G_{l}(-z^{2M})\right) + j z^{-M} G_{l+M}(-z^{2M})\right) W_{2M}^{-l/2} W_{M}^{-kl}$$
(6)

Using (6) to modify Fig. 1 and using the noble identities, [3], to move the decimators to the position before the polyphase component filters, we obtain the efficient implementation of Fig. 2. This implementation has a computational complexity that is approximately one half of that of the original structure in Fig. 1 - the 2M-point IDFT is replaced by an M-point IDFT. In Fig. 2, the block C is to reorder and conjugate the output samples of IDFT, wherever needed.



Fig. 2. Efficient implementation of the analysis CMFB.

#### 4. COMPUTATIONAL COMPLEXITY

Computational complexity is an important issue of concern in any system implementation. In this section, we compare the computational complexity of CMT transceiver against z-DMT and FMT. The number of operations given for each scheme are based on some of the best available algorithms in the literature. In particular, we have considered using the split radix FFT algorithm [4] for implementation of DFT. We have counted each complex multiplication as three real multiplications and three real additions [4].

Per sample complexity of z-DMT, CMT and FMT are obtained as

$$C_{\rm DMT} = 4\log_2 M - 1 \tag{7}$$

$$C_{\rm CMT} = 6\log_2 M + 8m + 2 \tag{8}$$

$$C_{\rm FMT} = 4\log_2 M + 4\gamma + 4(N_{\rm f} + N_{\rm b}) - 7 \quad (9)$$

where M indicates the number of sub-carriers. In CMT, the length of prototype filter is 2mM. In FMT, the parameter  $\gamma$ denotes the number of coefficients in each of the polyphase filter components, and  $N_{\rm f}$  and  $N_{\rm b}$  denote the number of taps in the feed-forward and feedback sections of DFE, respectively - FMT uses a decision feedback equalizer (DFT) for each subcarrier to equalize for distortion introduced by the band edges of the filter banks [5, 6, 7].

In comparing z-DMT, CMT and FMT, we find that fair comparisons are made when for z-DMT we follow [1] and choose M to be 2048, the length of cyclic prefix to be 100, and the length of the pulse-shaping and windowing samples to be 140 and 70, respectively. The length of cyclic suffix is determined according to the channel group delay. For CMT we choose M = 512 and m = 3 (which we experimentally found to be good choices). For FMT we follow [7] and choose M = 128,  $\gamma = 10$ ,  $N_f=26$ ,  $N_b=9$  and  $\alpha = 0.125$ , where  $\alpha$  is the excess bandwidth. With these typical numbers, we obtain  $C_{\text{DMT}} = 43$ ,  $C_{\text{CMT}} = 80$ ,  $C_{\text{FMT}} = 201$ operations per sample. Here, we should note that the complexity of z-DMT given here does not include the RFI canceller which can exhibit a significant computational peak load, whenever a new RFI is detected

### 5. SIMULATION RESULTS

To make comparisons with the previous works possible, we follow simulation parameters of [1]. We use a transmission bandwidth of 300 kHz to 11 MHz. The noise sources include a mix of ETSI'A' [11], 25 NEXT (near-end cross-talk) and 25 FEXT (far-end cross-talk) disturbers. Transmit band allocation is also performed according to [1]. The parameters of the three schemes are set as in Section 4.

We compare CMT with z-DMT and FMT for the following cases: (i) Crosstalk dominated channels, where RFI is absent. (ii) Transmission under RFI ingress noise.

### 5.1. Crosstalk Dominated Channels

In our simulations, NEXT and FEXT noise are generated according to the coupling equations for a 50-pair binder ca-

ble as

$$PSD_{NEXT} = K_{NEXT}S_{d}(f)(N_{d}/49)^{0.6}f^{1.5}$$
  

$$PSD_{FEXT} = K_{FEXT}S_{d}(f)|H(f)|^{2}d(N_{d}/49)^{0.6}f^{2} (10)$$

where  $K_{\text{NEXT}} = 8.818 \times 10^{-14}$  and  $K_{\text{FEXT}} = 7.999 \times 10^{-20}$ ,  $S_{\rm d}(f)$  is the PSD of a disturber,  $N_{\rm d}$  is the number of disturbers, H(f) is the channel frequency response, and d is the channel length in meters.

Fig. 3 compares the bit-rates of z-DMT, CMT and FMT on TP1 lines of different lengths. CMT achieves a higher transmission rate because of higher bandwidth efficiency no cyclic extensions or guard bands. Also shown in this figure is the upper bound of the transmission rate, which is obtained from an ideal system where a bank of ideal filters with zero transition bands and a channel with flat gain over each sub-band are assumed. An observation in Fig. 3 that requires some comments is that the performance of FMT is worse than that of FMT obtained in [7], especially when the length of the line is larger than 1000 m. This we believe is because we use a different noise model than [7].



**Fig. 3**. Comparison of bit-rates of z-DMT, CMT and FMT on TP1 lines of different lengths.

#### 5.2. Effect of RFI Ingress Noise

In z-DMT, suppression of RFI has to be made at the demodulator output [2]. In CMT and FMT, the sharp roll-off and the high stop-band attenuation of the analysis filters allow cancellation of the RFI without resorting to any additional post demodulator RFI canceller.

Figs. 4 and 5 present a set of results that compare the performance of z-DMT, CMT and FMT in the presence of RFI. The RFI is chosen to be a 4 kHz narrowband signal. In Fig. 4, the center frequency of the RFI is at 1.9 MHz. This is near the center of the first HAM band in the VDSL band. We observe that in this case the RFI canceller clears RFI almost perfectly. There is only slight degradation in SNRs

near the band edges. However, the RFI canceller fails when the RFI center frequency moves to a point near one of the VDSL signal band edges. This is shown in Fig. 5 where the center frequency of the RFI is shifted to 1.82 MHz which is only 10 kHz away from the VDSL signal band.



**Fig. 4**. RFI performance of z-DMT, CMT and FMT when an RFI with bandwidth of 4 kHz and center frequency of 1.9 MHz is present.



**Fig. 5**. RFI performance of z-DMT, CMT and FMT when an RFI with bandwidth of 4 kHz and center frequency of 1.82 MHz is present.

## 6. CONCLUSIONS

In this paper, we presented a thorough study of CMT, a novel filter bank-based multi-carrier modulation that uses cosine modulated filter banks. We compared CMT against z-DMT and FMT over VDSL with respect to computational complexity, achievable bit-rates, and resistance to cross-talks and RFI. Except computational complexity, where CMT was found to be more complex than z-DMT, CMT showed superior performance with all other respects. Compared to FMT, CMT was found to be superior with respect to computational complexity and achievable bit-rate. CMT and FMT show similar resistance to cross-talks and RFI. 

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