AN SEMI-BLIND EIGEN APPROACH TO TIME-DOMAIN EQUALIZER DESIGN FOR VDSL SYSTEMS

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ABSTRACT

In this paper we propose a semi-blind time domain equalizer (TEQ) design method for VDSL system. In the VDSL system frequency division duplex is used to separate upstream and downstream signals. In downstream or upstream transmission, only half the tones carry data and the other half are null tones. The proposed semi-blind method exploits the null tones of VDSL training symbols. It does not require the channel impulse response nor the channel noise statistics, like all eigen approaches proposed earlier. Example will be given to demonstrate that proposed TEQ design method can effectively shorten the channel impulse responses for VDSL application.

1. INTRODUCTION

The time domain equalizers play an important role in the application of DMT (discrete multitone) to DSL (digital subscriber loop) transmission [1, 2]. For a DMT system with cyclic prefix length L, there will be no IBI (inter-block interference) if the channel order is no larger than L. In DSL applications, the channel impulse response can spread to a duration much larger than cyclic prefix. At the receiver, there is usually a TEQ that shortens the equivalent channel to reduce IBI.

In the literature, many TEQ designs have been proposed. In many of the existing methods, optimal TEQ in a certain sense can be computed once the channel impulse response and channel noise statistics are given, [3]-[8]. In [3]-[7], the TEQ design problem is formulated as a eigen problem. The optimal TEQ is the eigen vector corresponding to the largest eigen value of an appropriately defined channel dependent matrix. These methods requires the channel to be known beforehand. Therefore the receiver needs an estimate of the channel using training symbols before TEQ optimization can take place. In [7], zero padding is applied in the frequency domain to impose extra null symbols. Using a quadratic objective function based on the null symbols, blind adaptive equalization that does not require the channel impulse response can be achieved. The equalizer designed in [7] is different from the usual TEQ for DMT systems in the sense that the goal is to have ideal equalization so that the equivalent channel has only one tap. In [8], a blind adaptive TEQ (called MERRY) that exploits cyclic prefix is proposed. MERRY is shown in [8] to be a globally convergent algorithm.

In this paper we propose a semi-blind TEQ design method for VDSL system using an eigen filter approach. In the VDSL system frequency division duplex is used to separate upstream and downstream signals. In downstream application, the upstream tones are not used and are referred to as the null tones in this paper. Similarly, in upstream transmission, the downstream tones are the null tones. The proposed TEQ design method will exploit the null tones to shorten the channel. It is semi-blind in the sense that the transmitted signal are VDSL training symbols, but there is no need to estimate the channel impulse response first as in all earlier eigen-based approaches. Training time can be saved and furthermore the results does not depend on the accuracy of channel estimation. Example will be given to demonstrate that TEQ with good shortening effect can be obtained using the proposed method.

2. VDSL SYSTEM TRAINING SYMBOLS

In the VDSL system frequency division duplex is used to separate upstream and downstream signals. In downstream application, only half the tones are used and these tones are referred to as the data tones; in the tones reserved for upstream transmission zeros are used and these tones are called the null tones in this paper. Similarly, in upstream application, the tones used for transmission are the data tones and the downstream tones are the null tones. In an VDSL training symbol, some of the data tones are set aside for pilots and the others used for transmitting Special Opera-

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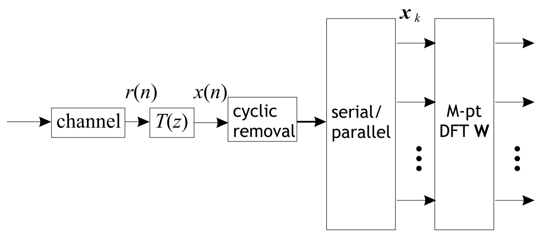

Figure 1: The block diagram of the VDSL receiver.

tion Channel (SOC) message. In the training stage, tones with indices that are even or multiples of 10 plus 9 are reserved for pilots. Constellation point of 00 is used on pilot tones. The selected constellation points are rotated by $0, \pi/2, \pi, 3\pi/2$ in pseudo random manner by multiplying with a rotation sequence. The rotation sequence is reused for every VDSL symbol. As a result, the actual symbols sent on pilot tones are the same for each training symbol.

When the channel order is smaller than the length of the cyclic prefix, we know there is no IBI after removing guard samples (cyclic removal). In the absence of channel noise, the outputs of the $M \times M$ DFT matrix at the receiving end are the scaled versions of the transmitter inputs. The scalars are the M-point DFT of the channel impulse response. The null tones will be nothing but channel noise if transmission in the opposite direction is idle, which occurs in some parts of the training phase. However, if the channel order is larger than the length of cyclic prefix, there will be IBI even after removing guard samples. The output of the null tones now has not only channel noise but also interference from the data tones of the previous block due to IBI (assuming channel order is smaller than the length of one block $N = M + L$, where L the length of cyclic prefix). Our proposed method will exploit the fact that the symbols sent on pilot tones and the null tones are fixed in each VDSL training symbol to formulate a quadratic objective function for TEQ optimization.

3. PROPOSED TEQ DESIGN

In this paper, we propose to a semi-blind TEQ design method for VDSL systems by minimizing the ISI present in the null tones. The design does not require the channel impulse response. To be more specific, suppose the number of pilot tones is M_p and the number of null tones is M_n. The numbers M_p, M_n are determined by the spectral plan. Considering the i-th output block, we collect the outputs of the pilot tones and the outputs of the null tones respectively in vectors $\mathbf{p}_i$ and $\mathbf{n}_i$, for $i = 1, 2, \cdots, B$, where B denotes the number of received output block available for equalizer design (Fig. 1). The dimensions of $\mathbf{p}_i$ and $\mathbf{n}_i$ are respectively M_p and M_n. We compute the averaged vectors,

$$\overline{\mathbf{p}} = \frac{1}{B}\sum_{i=1}^{B}\mathbf{p}_i, \quad \overline{\mathbf{n}} = \frac{1}{B}\sum_{i=1}^{B}\mathbf{n}_i.$$

We note that the average pilot tone vector $\overline{\mathbf{p}}$ will be pilot symbols plus interference from other data tones as averaging remove most noise. Similarly, averaged null tone vector $\overline{\mathbf{n}}$ is mostly interference from data tones. (If the channel is shorter than cyclic prefix length than there is no IBI and null tones will be zero.) Moreover the interference comes mainly from pilot tones. This is because the symbols in message-bearing tones are different from block to block and the average is approximately zero whereas the pilot symbols are the same in each block. Therefore we will minimize the null tone energy $\overline{\mathbf{n}}^\dagger\overline{\mathbf{n}}$. On the other hand, the performance of the TEQ is greatly affected by the signal power in the tones used for transmission. So we propose to maximize the pilot tone energy, which is a good measure of the signal power in the tones used for transmission. An objective function that reflects the pilot tone energy over the null tone interference is

$$\phi = \frac{\overline{\mathbf{p}}^\dagger\overline{\mathbf{p}}}{\overline{\mathbf{n}}^\dagger\overline{\mathbf{n}}}. \tag{1}$$

Although $\overline{\mathbf{p}}^\dagger\overline{\mathbf{p}}$ contains pilot energy as well as terms due interference, we can still use it to reflect pilot energy as interference is usually much smaller than pilot symbols. We will optimize the TEQ coefficients to maximize ϕ.

Remarks. Notice that our method is semi-blind. Namely, the receiver knows the training symbol contains pilots tones but it knows neither the channel impulse response nor the spectrum of channel noise. Although our method exploits null tones like [7], there are several differences. First our method attempts to shorten the channel rather than equalize the channel completely like [7]. Also the close form equalizer solution requires the channel impulse response and the second order statistics of channel noise in [7] while ours do not.

In what follows, we will see that the numerator and denominator of the objective function in (1) can be formulated as quadratic terms of the TEQ coefficients and the problem can be solved elegantly by computing eigen vectors of an appropriately defined positive definite matrix. Suppose the TEQ has order T,

$$T(z) = \sum_{i=0}^{T} t(i)z^{-i}.$$

The output of the TEQ can be written as

$$x(n) = \sum_{\ell=0}^{T} t(\ell)r(n-\ell),$$

where $r(n)$ is the channel output as indicated in Fig. 1. Let the i-th input vector of the DFT matrix at the receiver be

$$\mathbf{x}_i = \begin{pmatrix} x(iN+\Delta) \\ x(iN+\Delta+1) \\ \vdots \\ x(iN+\Delta+M-1) \end{pmatrix},$$

where Δ is a parameter determined from symbol synchronization. Then $\mathbf{x}_k$ can be written in terms of TEQ coefficients as

$$\mathbf{x}_i = \mathbf{R}_i \mathbf{t}, \qquad (2)$$

where $\mathbf{R}_i$ is an $M \times (T+1)$ matrix given by

$$\begin{pmatrix} r(iN+\Delta) & \cdots & r(iN+\Delta-T) \\ r(iN+\Delta+1) & \cdots & r(iN+\Delta+1-T) \\ \vdots & \ddots & \vdots \\ r(iN+\Delta+M-1) & \cdots & r(iN+\Delta+M-1-T) \end{pmatrix},$$

and $\mathbf{t}$ is a column vector of size $(T+1)$. The i-th pilot tone vector $\mathbf{p}_i$ can be expressed as

$$\mathbf{p}_i = \mathbf{W}_1 \mathbf{x}_i,$$

where $\mathbf{W}_1$ is an $M_p \times M$ submatrix of the $M \times M$ DFT matrix $\mathbf{W}$, obtained by keeping only the rows that correspond to pilot tones used in the design. Similarly, we can express the null tone vector $\mathbf{n}_i$ as

$$\mathbf{n}_i = \mathbf{W}_2 \mathbf{x}_i,$$

where $\mathbf{W}_2$ is an $M_n \times M$ submatrix of the $M \times M$ DFT matrix $\mathbf{W}$, obtained by keeping only the rows that correspond to the null tones used in the design. Using (2), we can write $\mathbf{p}_i$ and $\mathbf{n}_i$ respectively as

$$\mathbf{p}_i = \mathbf{W}_1 \mathbf{R}_i \mathbf{t}, \quad \mathbf{n}_i = \mathbf{W}_2 \mathbf{R}_i \mathbf{t}.$$

Using these expressions, we have

$$\overline{\mathbf{p}} = \mathbf{W}_1 \overline{\mathbf{R}} \mathbf{t}, \quad \mathbf{n}_i = \mathbf{W}_2 \overline{\mathbf{R}} \mathbf{t},$$

where

$$\overline{\mathbf{R}} = \frac{1}{B} \sum_{i=1}^{B} \mathbf{R}_i.$$

Therefore, we have

$$\overline{\mathbf{p}}^\dagger \overline{\mathbf{p}} = \mathbf{t}^\dagger \underbrace{\overline{\mathbf{R}} \mathbf{W}_1^\dagger \mathbf{W}_1 \overline{\mathbf{R}}}_{\mathbf{A}} \mathbf{t} = \mathbf{t}^\dagger \mathbf{A} \mathbf{t},$$

$$\overline{\mathbf{n}}^\dagger \overline{\mathbf{n}} = \mathbf{t}^\dagger \underbrace{\overline{\mathbf{R}} \mathbf{W}_2^\dagger \mathbf{W}_2 \overline{\mathbf{R}}}_{\mathbf{B}} \mathbf{t} = \mathbf{t}^\dagger \mathbf{B} \mathbf{t},$$

where $\mathbf{A}$ and $\mathbf{B}$ are square matrices of size $(T+1)$. Also, both matrices are positive definite. The objective function given in (1) become

$$\phi = \frac{\mathbf{t}^\dagger \mathbf{A} \mathbf{t}}{\mathbf{t}^\dagger \mathbf{B} \mathbf{t}}.$$

Now the objective function is written as quadratic forms of the TEQ coefficients. The energy constraint $\sum_{i=0} |t(i)|^2 = 1$ becomes $\mathbf{t}^\dagger \mathbf{t} = 1$.

Optimal solutions. We now find the optimal $\mathbf{t}$ that maximize ϕ subject to the constraint $\mathbf{t}^\dagger \mathbf{t} = 1$. As $\mathbf{B}$ is positive definite, we can decompose $\mathbf{B}$ as $\mathbf{B} = \mathbf{C}^{-\dagger}\mathbf{C}^{-1}$. Then ϕ can be written as the ratio $\phi = \frac{\mathbf{t}^\dagger \mathbf{A} \mathbf{t}}{\mathbf{t}^\dagger \mathbf{C}^\dagger \mathbf{C} \mathbf{t}}$. Let $\mathbf{u} = \mathbf{C}^{-1}\mathbf{t}$, then $\mathbf{t} = \mathbf{C}\mathbf{u}$ and

$$\phi = \frac{\mathbf{u}^\dagger \mathbf{C}^\dagger \mathbf{A} \mathbf{C} \mathbf{u}}{\mathbf{u}^\dagger \mathbf{u}}.$$

Using Rayleigh's principle, ϕ can be maximized by choosing $\mathbf{u}$ to be the eigen vector corresponding to the largest eigen value of $\mathbf{C}^\dagger \mathbf{A} \mathbf{C}$. The optimal TEQ is given by $\mathbf{t} = \mathbf{C}\mathbf{u}$.

4. SIMULATION EXAMPLES

In the simulations, the DFT size is 8192, cyclic prefix length is 640, sampling rate is 35.328 MHz and 1000 blocks of VDSL symbols are used. We consider downstream transmission so the upstream tones are null tones. The noise is comprised of FEXT, NEXT and AWGN as described in the testing environment of [2]. The VDSL training symbol is as given in [2]. The channel used is loop 7 from [2], the longest test loop in [2]. Fig. 2 shows the impulse response of the channel. The SIR of the channel is 35.7dB.

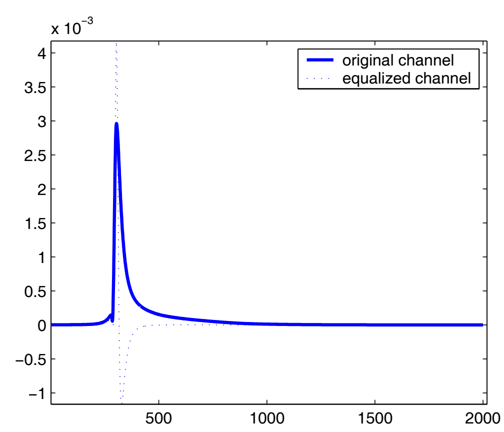

Figure 2: The impulse responses of the original VDSL loop 7 and the equalized channel.

We compute the average received vector $\bar{\mathbf{r}}$ and matrices $\mathbf{A}, \mathbf{B}$ as described in section 3 and the optimal TEQ of 20 taps is computed. The resulting TEQ is as shown in Fig. 3 and the corresponding magnitude response of the TEQ is shown in Fig. 4. Convolving the optimized TEQ with the original channel, we get the equalized channel as shown in Fig. 2. The SIR of the equalized channel is 67.1dB. We can see from the magnitude response Fig. 4 that the zeros of the TEQ are mostly placed in the upstream bands as the underlined application is downstream transmission. The bands marked with 'u' and 'd' correspond respectively to upstream and downstream bands.

Using the proposed TEQ design method, we list in Table 4 the SIR of the equalized channels for the test loops listed in [2]. The test loops VDSL 1-4 used are long loops of length 4,500 ft. For comparison we also list the SIRs of equalized channels using MSSNR (maximum shortening signal-to-noise ratio)[3], in which perfect channel knowledge is available. In both cases, the TEQ is of 20 taps. We can see that the proposed TEQ design can achieve SIR over 59 dB for all test loops.

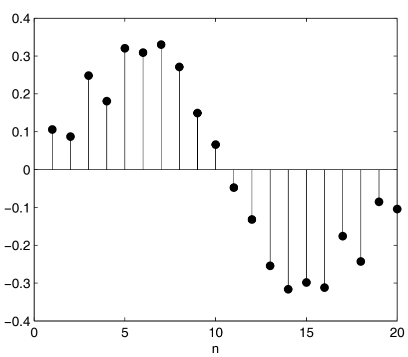

Figure 3: The impulse response of the TEQ optimized using the proposed semi-blind eigen approach.

VDSL loop	proposed	MSSNR
VDSL-1L	69.5	96.5
VDSL-2L	59.4	84.8
VDSL-3L	69.1	86.8
VDSL-4L	61.7	73.6
VDSL-5	90.8	98.8
VDSL-6	84.2	92.1
VDSL-7	67.1	79.8

Table 1: Comparison of SIR (dB) performances on VDSL loops.

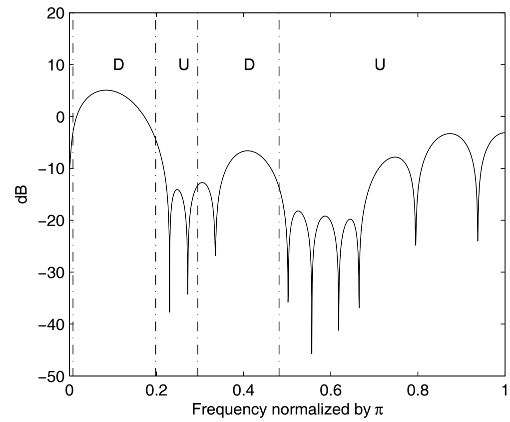

Figure 4: The magnitude response of the TEQ in Fig. 3.

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