JOINT MULTIUSER DETECTION AND OPTIMAL SPECTRUM BALANCING FOR DIGITAL SUBSCRIBER LINES

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ABSTRACT

This paper presents a joint multiuser detection and optimal spectrum balancing algorithm for heavily unbalanced crosstalk channels in digital subscriber line systems. To ensure detection of strong crosstalk only, the set of tones subject to multiuser detection must be chosen carefully. The problem of tone selection is highly coupled with the transmit power spectra and thus the optimal solution requires the two problems to be solved jointly. This paper makes use of the idea of dual decomposition to solve the above problem. By minimizing the Lagrangian dual of the primal problem, the joint tone selection and optimal spectral balancing problem can be solved globally and efficiently. Simulations show considerable bit rate increase when multiuser detection is performed with optimal spectrum balancing.

1. INTRODUCTION

Crosstalk noise is a major limiting factor in wideband digital subscriber line (DSL) systems. Future DSL systems are envisioned to employ dynamic spectrum management (DSM) techniques to mitigate the effect of crosstalk [1]. DSM aims to facilitate cooperation among mutually interfering lines in a DSL binder. Cooperation may be implemented in two different levels. When power-spectral-density (PSD) level cooperation is possible, the optimal set of power-spectral-densities may be computed for each line in the binder so that the effect of the mutual interference is minimized. In this case, multiple transmitters in the binder operates independently, but at mutual accommodating PSD levels. The class of algorithms that are capable of computing the best set of PSDs is called spectrum balancing algorithms (e.g. [2] and [3]). When cooperation is possible, not only at the PSD level, but also at the transmission signal level, the multi-line DSL binder can then be truly designed as a multiple-input multiple-output (MIMO) system where multiuser detection algorithms can be implemented [4]. In this case, each line has the full knowledge of the transmitted signal from neighboring lines, and crosstalk can be completely cancelled.

This paper explores a different form of cooperation that is most applicable to DSL configurations where the crosstalk channels are heavily unbalanced. For example, in a downstream ADSL deployment with a optical network unit (ONU), some remote terminals (RT) served from the central office (CO) can be located much closer to a nearby ONU than to their own CO. In this case, the crosstalk emitted by the ONU can overwhelm the intended transmission from the CO. Likewise, in upstream VDSL systems, some RT's can reside very close to the CO and exerts very strong interference to other users.

Signal-level cooperation is often not possible in some of these cases described above. This is true for ADSL systems where the transmitters and the receivers are not physically co-located. This is also often the case for VDSL systems where multiple transmission lines may be owned by different service providers. In these cases, PSD-level cooperation, although capable of producing a large gain as compared to the current practice of static spectrum management, is still not theoretically the best possible.

One of the main contributions of this paper is that multiuser detection and crosstalk cancellation can further improve the system performance *even when signal-level cooperation is not possible.* Intuitively, crosstalk cancellation is effective only when the crosstalk signal is strong. In DSL systems, multiuser detection must only be carried out at tones with strong interference for optimal performance. Thus, the problem of tone selection for multiuser detection and the optimal multiuser spectrum balancing is strongly coupled. The main novelty of this paper is a method that determines the optimal transmit spectra jointly with the optimal tone selection for multiuser detection. The algorithm is based on the idea of dual optimization, recently introduced in [3] and [5]. As the results of this paper show, multiuser detection can bring further improvement to the overall systems performance beyond that of optimal spectral balancing alone.

The ideas of crosstalk cancellation and power allocation has been considered separately in the past. For example, [6] proposed a maximum likelihood multiuser detector (ML-MUD) that considers all possible combinations of the interference signals and determines the most likely combination given the received signals. Alternatively, in an interference canceling multiuser detector (IC-MUD), interference from adjacent users can be estimated, reconstructed, and subtracted from the received signal. It is shown in [7] that this type of interference canceling scheme can achieve a substantial performance gain for near-end crosstalk cancellation. In terms of power allocation, [8] proposed an efficient method for allocating power in DSL systems with multiuser detection. However, crosstalk is assumed to be strong and crosstalk cancellation is performed in all channels. Hence, none of the previous work considers the joint optimization of bit/power allocation and crosstalk cancellation. The main contribution of this paper is to show that such a joint optimization can be done in a numerically efficient way. Throughout the paper, perfect knowledge of channel state information of the direct and crosstalk channels is assumed. The DSL systems are assumed to operate in a frequency-division duplex mode so that only far-end crosstalk is considered. The multiuser detection scheme used in the algorithm is of the interference canceling type.

2. OPTIMAL SPECTRUM BALANCING

This section sets the stage for future developments by first providing an overview of the optimal spectrum balancing problem without multiuser detection. In a K-user DSL system, an overall system performance measure is the weighted sum rate of all participating users. The design constraints are individual power constraints for each user. The optimization problem can be written as follows:

$$\max_{\{S_1^n,\ldots,S_K^n\}_{n=1}^N} \sum_{k=1}^{K} w_k R_k \text{ s.t. } P_k \le \mathbf{P_k} \ \forall k \tag{1}$$

where $\mathbf{P_k}$ is the kth user's power constraint, S_k^n is the transmit power for user k in tone n, and R_k is the total rate achieved by user k. The weights $w_k \ge 0$ are chosen so that $\sum_{k=1}^K w_k = 1$. The total power used by user k is computed as $P_k = \Delta f \sum_{n=1}^N S_k^n$, where Δf is the frequency width of the DMT tones and N is the total number of frequency tones. DMT modulation allows independent data transmission on each tone, i.e. R_k in (1) can be calculated by $R_k = \frac{1}{T}\sum_{n=1}^N b_k^n$, where T is the symbol period and b_k^n denotes the achievable bit rate for user k in tone n given by

$$b_k^n = \left\lfloor \log_2\left(1 + \frac{1}{\Gamma}\cdot\frac{h_k^n S_k^n}{\sigma_k^n + \sum_{i\neq k}\alpha_{i,k}^n S_i^n}\right)\right\rfloor \tag{2}$$

Here, Γ is the SNR gap, σ_k^n is the channel noise for user k in tone n, h_k^n is the direct channel response of user k in tone n, and $\alpha_{i,k}^n$ is the crosstalk transfer function from the ith user to the kth user in tone n. This model tacitly assumes that multiuser detection is not possible, hence every user sees the crosstalk from each other as noise on a tone-by-tone basis.

The optimal spectrum balancing (OSB) algorithm proposed in [3] provides an efficient method for solving the optimal transmit power spectrum problem (1) in spite of the apparent non-convexity of (2). The main idea is to form the dual of the original problem and to decompose the dual problem in a tone-by-tone basis. In doing so, the optimal spectrum can be determined with a much lower complexity as compared to an exhaustive search over all possible bit allocations and over all frequency tones. More specifically, the dual objective is as follows:

$$\begin{aligned} & g(\lambda_1,\cdots,\lambda_K) \qquad (3)\\ &= \max_{\{S_1^n,\ldots,S_K^n\}_{n=1}^N} \sum_{k=1}^K w_k R_k - \sum_{k=1}^K \lambda_k (P_k - \mathbf{P_k}) \\ &= \left(\sum_{n=1}^N \max_{S_1^n,\ldots,S_K^n} \sum_{k=1}^K (w_k b_k^n - \lambda_k S_k^n)\right) + \sum_{k=1}^K \lambda_k \mathbf{P_k}. \end{aligned}$$

Note that the evaluation of $g(\lambda_1,\cdots,\lambda_K)$ is decoupled into N per-tone maximization problems. As a result, if B is the maximum bitloading for each tone and discrete bitloading is used, a complexity reduction from $O(B^{NK})$ for an exhaustive search over all tones to $O(NB^K)$ can be achieved.

It turns out that for all DMT systems, the primal and the dual problems always have the same solutions regardless of the convexity of the original problem. This is shown in [5] under a very general time-sharing condition that is always satisfied for DMT-based systems. This means that

$$\min_{\lambda_1,\cdots,\lambda_K} g(\lambda_1,\cdots,\lambda_K) = \max_{S_1^n,\ldots,S_K^n} \sum_{k=1}^K w_k R_k. \tag{4}$$

As the dual problem is always convex, it can be efficiently solved by using a subgradient search method. The complexity of the subgradient search is polynomial in K. Thus, as long as the optimization within each tone can be done with manageable complexity, the entire problem may be solved efficiently.

3. JOINT MULTIUSER DETECTION AND OPTIMAL SPECTRUM BALANCING

In the spectrum balancing algorithm as described in the previous section, crosstalk is always regarded as noise. This is optimal when the crosstalk channel gains, $\alpha_{i,k}^n$, are small for $i \neq k$. However, practical configurations can often be such that an interfering transmitter is very close to the receiver of a neighboring user, for example, as in Fig. 1. In this case, crosstalk cancellation at the nearby receiver may potentially bring significant performance gains.

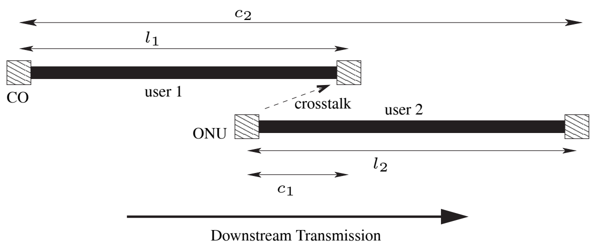

Fig. 1. Loop Topology for 2-User ADSL Downstream

The main idea proposed in this paper is that multiuser detectors (MUD) can be applied in conjuction with spectrum optimization in situations such as that in Fig. 1. A multiuser detector at the receiver of user 1 works by first detecting and subtracting the signal from user 2 in the received signal, then detecting the signal from user 1. Implementation of this scheme requires error-free decoding of user 2 at user 1. Thus, the bit rate of user 2 is restricted by the quality of the crosstalk channel. Therefore,

$$\begin{cases} \tilde{b}_1^n = \left\lfloor \log_2 \left(1 + \frac{1}{\Gamma} \frac{h_1^n S_1^n}{\sigma_1^n} \right) \right\rfloor \\ \tilde{b}_2^n = \min \left(\left\lfloor \log_2 \left(1 + \frac{1}{\Gamma} \frac{\alpha_{2,1}^n S_2^n}{\sigma_1^n + h_1^n S_1^n} \right) \right\rfloor , \right. \\ \qquad\qquad \left. \left\lfloor \log_2 \left(1 + \frac{1}{\Gamma} \frac{h_2^n S_2^n}{\sigma_2^n + \alpha_{1,2}^n S_1^n} \right) \right\rfloor \right) \end{cases} \tag{5}$$

is an achievable rate pair.

Since channel gains are frequency selective, multiuser detection must only be applied to a selective number of tones, primarily the high frequencies where the crosstalk coupling between lines are strong. Thus, the multiuser detection scheme is effective when it is applied only to high frequency tones. Making such a tone selection for multiuser detection is not trivial but important for achieving the optimal weighted sum rate.

The main point of this paper is to show that the optimal set of MUD tones can be determined along with the optimal power spectra in a computationally tractable manner using the dual algorithm described in the previous section. With the option of achieving $\tilde{b}_k^n$ with MUD, (3) can be modified to

$$\begin{aligned} & g(\lambda_1, \cdots, \lambda_K) \\ & = \left(\sum_{n=1}^{N} \max_{S_1^n, \ldots, S_K^n} \left(\max \left(\sum_{k=1}^{K} w_k b_k^n, \sum_{k=1}^{K} w_k \tilde{b}_k^n \right) \right. \right. \\ & \qquad \left. \left. - \sum_{k=1}^{K} \lambda_k S_k^n \right) \right) + \sum_{k=1}^{K} \lambda_k \mathbf{P_k}. \end{aligned} \tag{6}$$

The S_k^n sequence that minimizes $g(\lambda_1, \cdots, \lambda_K)$ is the optimal power spectra and the choice of b_k^n or $\tilde{b}_k^n$ in the inner maximization determines the MUD mode for tone n. Global optimality is guaranteed for the same reason as without MUD, which is proven in [5]. Although an extra maximization computation is required, the order of complexity remains at $O(NB^K)$.

For a general 2-user interference channel, an MUD can be installed at both/either/neither receivers, resulting in a total of four options. However, the placement of MUD can often be easily determined for practical channels given the channel lengths. Referring to Fig. 1, simulation experience shows that an MUD at user i is effective only if $\frac{c_i}{l_i} < 1$, where c_i and l_i are length parameters as shown in Fig. 1. Clearly, it is not possible that both $c_1 < l_1$ and $c_2 < l_2$. Hence, the possibility of using two MUD's can be eliminated, and the MUD should only be placed at user i with a smaller $\frac{c_i}{l_i}$. The decision of whether an MUD should be used at all depends on the extra receiver complexity required and the performance gain obtained. The simulations in the later section illustrates the benefit of multiuser detection as a function of the length of the crosstalk channel.

The above method for finding the optimal power spectrum with MUD at the receivers can be extended to more than two users. However, the algorithm does become more complex. With two users as in previous example, there are only two modes for the MUD: either cancel or ignore the crosstalk. If instead there are S users connecting to CO and T users connecting to ONU in Fig. 1, there are ST cancelable strong crosstalk channels, giving a total of 2^{ST} MUD modes. However, in most situations, only the strongest crosstalk needs to be cancelled. Thus, low complexity versions of the above algorithm are possible.

4. SIMULATIONS

This section examines the improvement in bit rate when the multiuser detection is applied with optimal spectrum balancing. For all simulations, a target error probability of 10^{-7} with a 3 dB coding gain and 6 dB noise margin are used. Discrete bit-loading is assumed for all cases.

A 2-user ADSL downstream scenario as shown in Fig. 1 with $l_1 = l_2 = 12$kft and $c_1 = 1$kft is simulated. The crosstalk from transmitter 2 to receiver 1 is large due to the close distance between them. Fig. 2 shows the achievable rate increase offered by the joint multiuser detection algorithm. In this configuration, a 14% increase for one user or 7% for both users can be observed. Fig. 3 illustrates the computed power allocation for both users. Without multiuser detection, frequency division multiplexing is enforced at high frequency. With the proposed joint multiuser detection and spectrum balancing algorithm, both users transmit data even when crosstalk is severe at high frequency. The extra transmitted bits contribute to the overall bit rate increase. Simulations of 2-user VDSL systems also show similar performance gain. With more than 2 users, however, the benefit of multiuser detection turns out to be smaller.

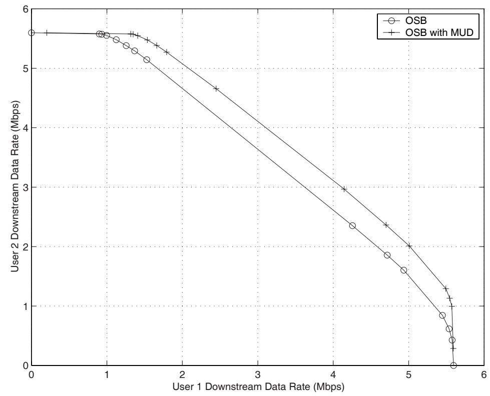

Fig. 2. Achievable rate region for 2-user ADSL downstream using optimal spectrum balancing alone, and joint spectrum balancing and multiuser detection

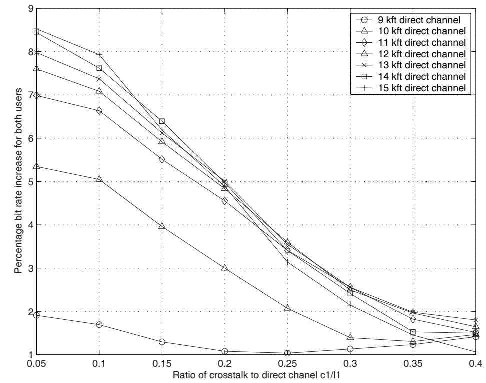

Fig. 4. Percentage bit rate increase as a function of the direct and crosstalk channel lengths in 2-user ADSL downstream

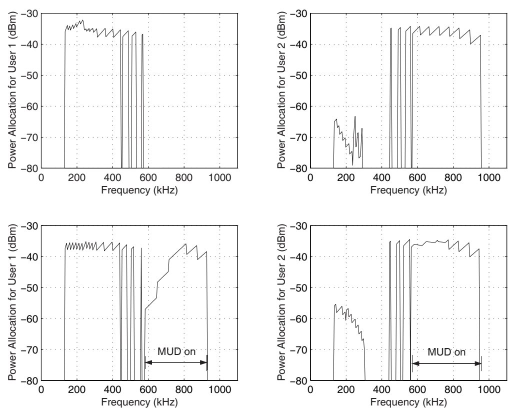

Fig. 3. Power allocations for 2-user ADSL downstream with optimal spectrum balancing alone (top left and right) and with joint spectrum balancing and multiuser detection (bottom left and right). The total rates are $(R_1, R_2) = (4.2560\,\text{Mbps}, 2.3520\,\text{Mbps})$ for without multiuser detection and $(R_1, R_2) = (4.1440\,\text{Mbps}, 2.9680\,\text{Mbps})$ for with multiuser detection.

Fig. 4 investigates the relationship between the length of the crosstalk channel and the bit rate increase with multiuser detection. The same scenario as in Fig. 1 is examined, but with a range of common ADSL line lengths. Both direct channel lengths l_1 and l_2 are assumed to be constant in all cases. In general, the performance gain decreases when the ratio $\frac{c_1}{l_1}$ increases. The maximum gain also increases with the length of the direct channel so that an 8.5% increase for both users or 17% increase for one user is possible.

5. CONCLUSIONS

This paper presented an efficient algorithm for jointly determining the optimal transmit spectrum and the optimal tone selection for multiuser detection in digital subscriber lines. The multiuser detection scheme provides considerable bit rate increase when the cable configuration generates strong crosstalk channels even in the absence of signal-level cooperation.

6. REFERENCES

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