# Coding Assisted Iterative Channel Estimation for Impulse Radio Ultra-Wide Band Communication Systems


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***Abstract*— In this paper, we consider a coded ultra-wide band (UWB) impulse radio system. At the receiver, to estimate multipath delays and amplitudes, the soft information output from a soft-input soft-output (SISO) decoder is used to assist the estimation. The SISO decoder and the channel estimator work in an iterative way and no training sequence is used in the estimation. Compared with the non data-aided channel estimation without using the soft information output from the SISO decoder, the performance of this iterative channel estimation is better. We also explain this iterative method by using the expectation maximization (EM) algorithm, which can be used to prove the convergence of this iterative method.**


## I. INTRODUCTION

With the recent approval of unlicensed UWB devices operating in the $3.1 - 10.6$GHz frequency range by the FCC [1], there has been a growing interest in the research of UWB wireless systems. The impulse radio UWB technology [2][3] has some attractive advantages. Firstly, it is a baseband modulation and demodulation technology and therefore its transceiver is simple. Secondly, because the pulse duration is very short, the impulse radio system has very high multi-path resolution, which leads to the reduced fading and thus improved communication quality. Thirdly, the repetition period of the pulse is very large compared with the pulse duration, which has two advantages, one is that using time hoping multiple access technology, the impulse radio system can accommodate many users; the other is that the power spectral density is very low, so the impulse radio system has very little impact on other narrow band systems operating in the same frequency range.

Besides of these attractive advantages, there are still many challenges ahead [4]. One of these challenges is the timing acquisition. At the receiver, the correlator should know the starting of the pulse to start the correlation, otherwise, it can not detect the data successfully. If the timing error is larger than the pulse duration, dramatic performance loss will be encountered [5][6]. Also, to exploit the advantage of the impulse radio system, Rake receiver should be used to collect the energy of each multi-path. In the Rake receiver, each finger should precisely know the delay and amplitude of its corresponding multipath to perform optimal combining (maximum ratio combining). The objective of this paper is to estimate the delays and amplitudes of multipaths in the environment of impulse radio.

In [7], maximum likelihood (ML) method is used to estimate the delay and amplitude of each multipath, where the data-aided (DA) and non data-aided (NDA) methods are proposed. It is shown that the NDA method will encounter performance loss when the number of users is large. In a practical system, to improve the reliability, error correction coding (ECC) is usually used. In this paper, a convolutional encoder is added at the transmitter of an impulse radio system. At the receiver, we use soft-input soft-output (SISO) decoder, which can output soft information of each transmitted bit, and this soft information is used to enhance the estimation of the delays and amplitudes of multipaths. The channel estimator and the decoder can work together in an iterative way, i.e., the enhanced estimation is also used by the detector and the soft output of the detector is again used by the channel decoder to improve the quality of the soft information of the transmitted bits. We show in this paper that this iterative scheme can improve the performance of the NDA channel estimation. In fact, this iterative method can be related to the expectation maximization (EM) algorithm [12] and thus its convergence can be explained. The EM algorithm is also used in [8] to do turbo synchronization for conventional narrow band communication systems and recently used in [13] to do multiuser synchronization for DS-CDMA systems.

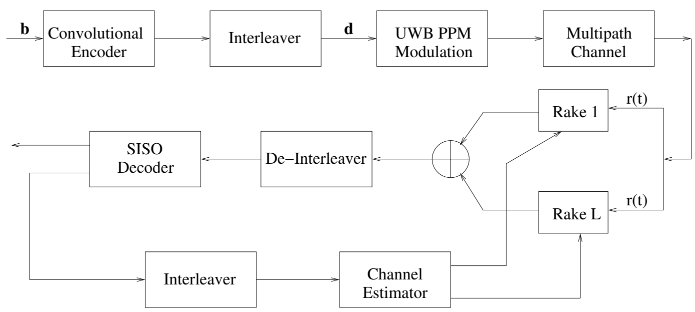


Fig. 1. System Model

The paper is organized as follows. In Section II, the system model of an impulse radio is introduced and the problem of interest is formulated. In Section III, the non data-aided estimation is derived in a way that it can be easily extended to the iterative method. In Section IV, the iterative non data-aided method is introduced and also explained by using the EM algorithm. In Section V, we show some simulation results of this iterative method.

## II. SYSTEM MODEL

In our system, we assume that the information bits of the desired user are $\mathbf{b} = [b_1, b_2, \cdots b_K]$, these bits are encoded to be the bit sequence $\mathbf{d} = [d_1, d_2, \cdots, d_N]$. So, the rate of the channel code is $R_c = K/N$. We use time-hopping multiple access impulse radio to send these coded bits of the desired user. The system model is shown in Fig. 1. Two kinds of modulation schemes are usually used in current UWB impulse radio systems, one is pulse position modulation (PPM), the other is pulse amplitude modulation (PAM). We focus on

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PPM in this paper, while for PAM, the derivation procedure is similar. In PPM the signal format is:

$$s(t)=\sum_{j=-\infty}^{\infty} w_t(t-jT_f-c_jT_c-\Delta d_{\lfloor j/N_s\rfloor}), \tag{1}$$

where $w_t(t)$ is the transmitted pulse called *monocycle*. We assume that the duration of the pulse $w_t(t)$ is $T_p$, which is usually in the order of fractional nanoseconds. $T_f$ in (1) is the pulse repetition period, it is usually hundreds or thousands times of $T_p$, and $c_j$ is the time-hopping sequence for the desired user. In the time-hopping multiple access impulse radio system, there is a distinctive time shift $c_jT_c$ for each user in each pulse period $T_f$. In (1), $\lfloor\cdot\rfloor$ is the floor function, $N_s$ is the number of pulses per bit. For PPM, the value of $d_i=0$ or $d_i=1$ is modulated on the time shift of the pulse, and the difference in the time shift is $\Delta$. In this model, the binary symbol rate is $R_s=1/T_fN_s$ bits/sec. We assume that the channel has $L$ multipaths, and the impulse response of the channel is:

$$h(t)=\sum_{l=1}^{L}\gamma_l\delta(t-\tau_l), \tag{2}$$

where $\gamma_l$ and $\tau_l$ are the amplitude and delay of the $l$th path respectively. Since the pulse duration is very small, we usually assume that there is no overlap between different paths, i.e., $\tau_l-\tau_{l'}>T_p$ for $l\neq l'$. Thus, the received signal waveform is:

$$r(t) \;=\; \sum_{l=1}^{L}\gamma_l s(t-\tau_l)+n_i(t)+n_g(t). \tag{3}$$

In this model, we use $n_i(t)$ to denote the effect of multi-user interference, and $n_g(t)$ to denote the AWGN noise at the receiver front-end. We approximate that the total effect of $n_i(t)$ and $n_g(t)$, i.e., $n(t)=n_i(t)+n_g(t)$ as Gaussian distributed. We assume that the received pulse is $w_r(t)$, and this pulse is known by the receiver. Notice that in (3) we have also assumed that the channel keeps constant during the transmission of the coded block. At the receiver, the received waveform is correlated with the template $v(t)=w_r(t)-w_r(t-\Delta)$.

We assume that there are $L$ fingers at the receiver and the output of each finger for the $i$th coded bit $d_i$ is

$$r_i^l=\begin{cases} +\gamma^2E_r+n_i^l, & for\;\; d_i=0,\\ -\gamma^2E_r+n_i^l, & for\;\; d_i=1,\end{cases}$$

where $E_r$ is the energy of each symbol bit output from the correlator and

$$E_r=\sum_{j=0}^{N_s-1}\int_{jT_f+c_jT_c}^{(j+1)T_f+c_jT_c} w_r^2(t-jT_f-c_jT_c)dt, \tag{4}$$

and $n_i^l$ is the Gaussian noise with zero mean and variance $\sigma^2$. The decision statistic of the $i$th coded bit is:

$$r_i=\sum_{l=1}^{L} r_i^l.$$

In our system, we will assume that the number $L$ of paths is known, and the objective of our paper is to estimate the delay and amplitude of each path, i.e., $\tau\triangleq[\tau_1,\tau_2,\cdots,\tau_L]$ and $\gamma\triangleq[\gamma_1,\gamma_2,\cdots,\gamma_L]$.

## III. Non Data-Aided Channel Estimation

In this section, we discuss the non data-aided channel estimation which is similar to [7], but is revised so that it can be easily extended to the iterative case. The iterative method will be discussed in the next section.

If the decided coded bit sequence is $\mathbf{d}=[d_1,d_2,\cdots,d_N]$, and the estimated delay and amplitude of the $L$ paths are $\gamma=[\gamma_1,\gamma_2,\cdots,\gamma_L]$ and $\tau=[\tau_1,\tau_2,\cdots,\tau_L]$, respectively, then the likelihood function of the three unknown parameters $\mathbf{d}$, $\gamma$ and $\tau$ is $\Upsilon(\mathbf{d},\gamma,\tau)=$

$$\exp\left\{\frac{1}{N_0}\left[2\sum_{l=1}^{L}\gamma_l\sum_{i=1}^{N}r_i(d_i,\tau_l)-NE_r\sum_{l=1}^{L}\gamma_l^2\right]\right\} \tag{5}$$

where $N_0=2\sigma^2$. In the derivation of (5), we assumed that there was no overlap between the pulses from different multipaths. In (5), $E_r$ is equal to (4) and

$$r_i(d_i,\tau_l)=\sum_{j=(i-1)N_s}^{iN_s-1}\int_{jT_f+t_j\tau_l}^{(j+1)T_f+t_j+\tau_l} r(t)w_r(t-jT_f-t_j-\tau_l)dt,$$

where $t_j=c_jT_c+\Delta d_i$. Since in non data-aided estimation, the data $d_i$ for $i=1,2,\cdots,N$ are assumed unknown, and it is further assumed that $d_i=0$ and $d_i=1$ are with equal probability. Under this assumption, we can average out the effect of $d_i$ in $\Upsilon(\mathbf{d},\tau)$ by calculating:

$$\Upsilon(\gamma,\tau)=\int\Upsilon(\mathbf{d},\gamma,\tau)p(\mathbf{d})d\mathbf{d}, \tag{6}$$

where $p(\mathbf{d})$ is

$$p(\mathbf{d})=\prod_{i=1}^{N}\left[\frac{1}{2}\delta(d_i)+\frac{1}{2}\delta(d_i-1)\right].$$

Substituting $\Upsilon(\mathbf{d},\gamma,\tau)$ into (6), and take logarithm then we have $\log\Upsilon(\gamma,\tau)$, which is shown at the top of the next page. Using the approximation $\log\left[e^{x_1}+e^{x_2}\right]=\max(x_1,x_2)$, and assuming that the maximum of the two items in the upper equation is achieved for $d_i=k_i$, where either $k_i=0$ or $k_i=1$, the upper equation can be approximated as $\log\Upsilon(\gamma,\tau)\approx$

$$-\frac{NE_r}{N_0}\sum_{l=1}^{L}\gamma_l^2+\sum_{i=1}^{N}\left[\log(1/2)\frac{2}{N_0}\sum_{l=1}^{L}\gamma_l r_i(k_i,\tau_l)\right]. \tag{8}$$

We calculate the optimal values of $\gamma$ and $\tau$ in two steps. In the first step, we assume that $\tau$ is known, and we calculate the optimal value of $\gamma$ by taking the derivative of (8) with respect to $\gamma_l$ and setting it is zero. Thus, the optimal value of $\gamma_l$ is:

$$\gamma_l^{opt}=\log(1/2)\frac{1}{NE_r}\sum_{i=1}^{N}r_i(k_i,\tau_l), \tag{9}$$

for $l=1,2,\cdots,L$. Substituting the upper equation into (8) and maximize it over $\tau$, we have:

$$\tau_l^{opt}=\arg\max_{\tau_l}\left(\sum_{i=1}^{N}r_i(k_i,\tau_l)\right)^2, \tag{10}$$

for $l=1,2,\cdots,L$.

## IV. Iterative Non Data-Aided Channel Estimation

In the preceding section, it is assumed that there is no *a priori* information of the transmitted bit, and thus we use that the values of $d_i=0$ and $d_i=1$ have equal probability for $i=1,2,\cdots,N$. But in the coded system, if the outer channel decoder can calculate the probability of the value of $d_i$, then the channel estimator can take these probabilities as *a priori* information of the transmitted bits to enhance the channel estimation. Based on the enhanced channel

$$\log \Upsilon(\gamma,\tau) = -\frac{NE_r}{N_0}\sum_{l=1}^{L}\gamma_l^2 + \sum_{i=1}^{N}\log\left\{\exp\left[\log(1/2)\frac{2}{N_0}\sum_{l=1}^{L}\gamma_l r_i(0,\tau_l)\right] + \exp\left[\log(1/2)\frac{2}{N_0}\sum_{l=1}^{L}\gamma_l r_i(1,\tau_l)\right]\right\}. \quad (7)$$

estimation, the detector can also update its soft outputs, and these soft outputs can be used again by the SISO decoder to calculate the probability of the value of $d_i$. This process can work in an iterative way until no changes in the estimated value occurs. In this section, we will discuss this iterative non data-aided channel estimation.

Assume that the log likelihood ratio (LLR) of the coded bit $d_i$ output from the SISO decoder is $L(d_i) \triangleq \frac{p(d_i=1)}{p(d_i=0)}$. Then, we have $p_i(1) \triangleq p(d_i = 1) = \frac{\exp(L(d_i))}{1+\exp(L(d_i))}$ and $p_i(0) \triangleq p(d_i = 0) = \frac{1}{1+\exp(L(d_i))}$. Now, $p(\mathbf{d})$ is

$$p(\mathbf{d}) = \prod_{i=1}^{N}\left[p_i(0)\delta(d_i) + p_i(1)\delta(d_i-1)\right]. \quad (11)$$

Substituting this equation into (6) and processing similarly as before, we have the log likelihood function as in equation (7) except now two $\log(1/2)$s are replaced by $\log(p_i(0))$ and $\log(p_i(1))$ respectively. Similarly, this equation can be approximated, and we can take the derivative of the approximated equation with respect to $\gamma_l$ and set it to zero to calculate the optimal value of $\gamma_l$. Accordingly, $\gamma_l^{opt}$ is

$$\gamma_l^{opt} = \frac{1}{NE_r}\sum_{i=1}^{N}\log(p_i(k_i))r_i(k_i,\tau_l), \quad (12)$$

for $l = 1, 2, \cdots, L$, and $\tau_l^{opt}$ is

$$\tau_l^{opt} = \arg\max_{\tau_l}\left(\sum_{i=1}^{N}\log(p_i(k_i))r_i(k_i,\tau_l)\right)^2, \quad (13)$$

for $l = 1, 2, \cdots, L$. Having the estimated channel amplitude and delay, the LLR output from the detector can be calculated by:

$$L'(d_i) = \frac{2}{N_0}\sum_{l=1}^{L}\gamma_l^{opt}\left[r_i(1,\tau_l^{opt}) - r_i(0,\tau_l^{opt})\right]. \quad (14)$$

These updated LLRs can be input to the outer SISO decoder.

If we take the transmitted bits $\mathbf{d}$ as the hidden data, the correlator output $\mathbf{r} \triangleq [r_1^1, r_2^1, \cdots, r_N^1, \cdots, r_1^L, r_2^L, \cdots, r_N^L]$ as the incomplete data, $\mathbf{y} \triangleq [\mathbf{r}, \mathbf{d}]$ as the complete data, and the parameters to estimate as $\xi \triangleq [\tau, \gamma]$, then the iterative method described before can be related to the expectation maximization (EM) algorithm. The EM algorithm is characterized by two steps in each iteration, i.e., the expectation step:

$$\mathcal{Q}(\xi, \widehat{\xi}^{(n-1)}) = \int_{\mathbf{y}} p(\mathbf{y}|\mathbf{r}, \widehat{\xi}^{(n-1)})\log p(\mathbf{y}|\xi)d\mathbf{y} \quad (15)$$

and the maximization step:

$$\widehat{\xi}^{(n)} = \arg\max_{\xi}\{\mathcal{Q}(\xi, \widehat{\xi}^{(n-1)})\} \quad (16)$$

In (15) and (16), $\widehat{\xi}^{(n)}$ is the estimated $\gamma$ and $\tau$ in $n$th iteration. Because of the independence between the channel parameter $\xi$ and the transmitted symbol bits $\mathbf{d}$, $p(\mathbf{y}|\xi)$ in (15) can be simplified to $p(\mathbf{y}|\xi) = p(\mathbf{r}|\mathbf{d},\xi)p(\mathbf{d}|\mathbf{r})$, and (15) becomes

$$\begin{aligned}\mathcal{Q}(\xi, \widehat{\xi}^{(n-1)}) &= \int_{\mathbf{d}} p(\mathbf{d}|\mathbf{r}, \widehat{\xi}^{(n-1)})\log p(\mathbf{r}|\mathbf{d},\xi)d\mathbf{d} \\ &+ \int_{\mathbf{d}} p(\mathbf{d}|\mathbf{r}, \widehat{\xi}^{(n-1)})\log p(\mathbf{d}|\mathbf{r})d\mathbf{d}\end{aligned}$$

In the right hand side of upper equation, the second term is independent of the the channel parameters $\xi$, and thus in the maximization step, this term can be deleted. In the first term, $p(\mathbf{d}|\mathbf{r}, \widehat{\xi}^{(n-1)})$ can be calculated by using (11), which is the probability of the transmitted bits conditioned on the estimation of the current iteration and the output of the correlator. Although in each iteration, $p(\mathbf{d})$, which is calculated by the SISO decoder, is not the true *a posteriori* probability $p(\mathbf{d}|\mathbf{r}, \widehat{\xi}^{(n-1)})$, for high SNR we can approximately do so. The $p(\mathbf{r}|\mathbf{d},\xi)$ in (17) corresponds to (5), which is the probability of the correlator's outputs conditioned on the transmitted bits and the channel parameters. Accordingly, previous derivation in Section III and Section IV is to simplify $\mathcal{Q}(\xi, \widehat{\xi}^{(n-1)})$. The second step of the EM algorithm, i.e., the maximization step, corresponds to the optimization process in (9) and (10) in Section III and (12) and (13) in Section IV. Since the convergence of the EM algorithm holds, the convergence of our iterative method holds too.

## V. SIMULATIONS

The outer channel encoder in our simulations is chosen as a rate $R = 1/2$ recursive convolutional code with generating matrix $[1 \quad 1+D+D^2/1+D^2]$, the information block length $K = 2045$ after adding 3 tail bits, the coded block length $N = 4096$. At the receiver, the SISO decoder is based on the BCJR algorithm.

In the simulations, the pulse that the receiver received is the second derivative of the Gaussian function

$$w_r(t) = \left[1 - 16\pi\left(\frac{t - T_p/2}{T_p}\right)^2\right]\exp\left[-8\pi\left(\frac{t - T_p/2}{T_p}\right)^2\right].$$

In our simulations, we assume that $T_f = 40T_p$, $T_c = 2T_p$, and $N_s = 2$. So, in these parameters, the maximum number of users that can accommodate is $N_u = 20$. We also assume that PPM modulation scheme is used and $\Delta = T_p$. For the time-hopping multiple access, $c_j$ is chosen randomly between 0 and 19 for each user. To estimate the delay of each path, the output of the integrator in Fig. 1 should be over sampled, the sampling interval length in our simulations is $T_s = 0.05T_p$.

In our simulations, we assume that the number of users is $N_u$, and the number of multi-paths is $L$. For the desired user, we assume that the path gain keeps constant, while we assume that the path gains for other users are independent random variables with Rayleigh distribution [14]. The delay profile for both desired user and other users are modelled as exponentially distributed, i.e., $E(\gamma_l^2) = e^{-l/4}$ for $l = 1, 2, \cdots, L$. The delay $\tau_l$ for each user are modelled as $\tau_l = 5lT_p$. Fig. 2 shows the variance of the estimated $\tau$ for each iteration. In this figure, we assume that $N_u = 20$, which is the maximum number of users the system can accommodate, and $L = 1$. The choice of $L = 1$ is to keep the simulations in a tolerable time, since for more multipaths the estimation process is essentially the same. From this figure we can see, when the signal-to-noise ratio (SNR) is high, the quality of the estimation improves with the number of iterations, but it is not the case for low SNR. This can be explained as follows. Because of the low SNR, the probability of the transmitted symbol bits provided by the SISO decoder $p(\mathbf{d})$ is not a good approximation of the *a posteriori* probability $p(\mathbf{d}|\mathbf{r}, \xi^{(n-1)})$ and this may exacerbate the estimation of the channel parameters. That is, in this case, the iteration is no longer an EM algorithm, thus it may not converge.

The same effect can be seen in Fig. 3, which is the variance of the estimated amplitude $\gamma_l$ in the same situation.

Fig. 4 is the BER performance for this iterative channel estimation. We simulated two cases, $N_u = 20$ and $N_u = 5$. The performance is compared with the case when perfect channel information is known at the receiver. In the simulations we also assumed $L = 1$. In this figure we show the performance for iter$\# = 1, 2$ and 5, separately. From this figure we can see clearly the effect of iterations. The first iteration can be taken as the non-data aided channel estimation without using the soft information output from the SISO decoder, since for the first iteration we assumed equal probability for the two values of the transmitted bit, which is similar to [7]. Simply after 2 iterations, we get more than 0.5dB performance gain.

## VI. Conclusions

In this paper, the non-data aided ML channel estimation is improved by using a convolutional encoder at the transmitter and an SISO decoder at the receiver. The SISO decoder and the channel estimator work iteratively. The derivation of the non-data aided ML channel estimation is extended to this iterative channel estimation. The relationship between this iterative channel estimation and the EM algorithm is shown. Some simulations are provided to show the improved performance of the iterative channel estimation.

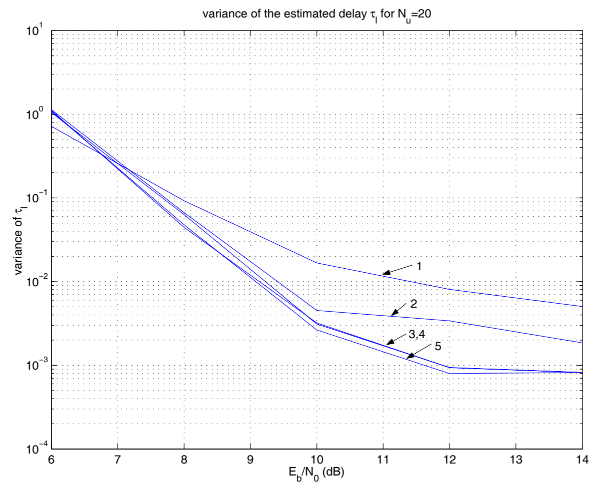


Fig. 2. Variance of the estimated delay

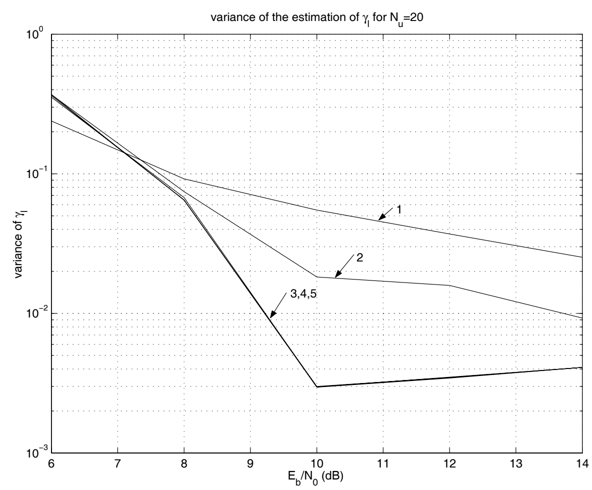


Fig. 3. Variance of the estimated amplitude

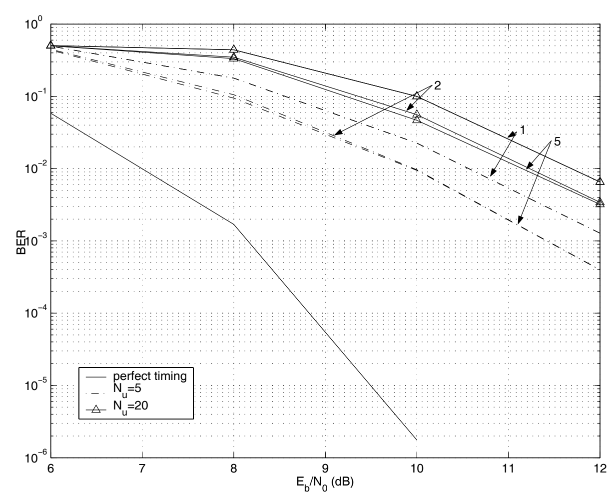


Fig. 4. BER for $N_u = 5$ and $N_u = 20$

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