A MODAL APPROACH TO SOUNDFIELD REPRODUCTION IN REVERBERANT ROOMS

Terence Betlehem and Thushara D. Abhayapala

National ICT Australia
Research School of Information Sciences and Engineering
The Australian National University
ACT 0200, Australia
Email: [Terence.Betlehem, Thushara.Abhayapala]@anu.edu.au

ABSTRACT

In this paper, we present a novel method of soundfield reproduction (SFR) for reverberant acoustic environments. Using an efficient parametrization of the acoustic transfer function (ATF) over a region of space, we devise a method for accurate SFR over the whole of the reproduction region. This method is based on a practical method of determining the ATF between each loudspeaker and the reproduction region.

1. INTRODUCTION

A problem relevant to emerging surround sound technology is the reproduction of a soundfield. Using a set of loudspeakers, it is possible for listeners to fully experience what it is like to be in the original sound environment. SFR has been discussed since the 1960s. However much of the work so far does not address SFR in reverberant environments. In this paper, using an efficient parametrization of the room transfer function we extend SFR to reverberant enclosures.

A major portion of recent SFR work hinges on the Kirchoff-Helmholtz equation. Here, SFR inside a control region is achieved by controlling the pressure and its normal derivative over the region boundary [1]. In similar work, pressure is controlled on the boundary and points inside the control region [1]. However a simpler design can be achieved with a spherical harmonic approach [2].

The reverberant case is made difficult by the rapid variation of the ATFs over the room [3]. The standard approach is to equalize the ATFs over multiple points [4]. However equalization is poor away from the design points.

In this paper, we present a method of performing SFR in a reverberant room. This method is based on an efficient parametrization of the ATF in which the ATF is written as weighted sum of the *modes*[1] of the control region. Using this parametrization, we reconstruct a soundfield accurately over the whole control region.

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[1]The term *mode* shall refer to an orthogonal basis function satisfying the wave equation.

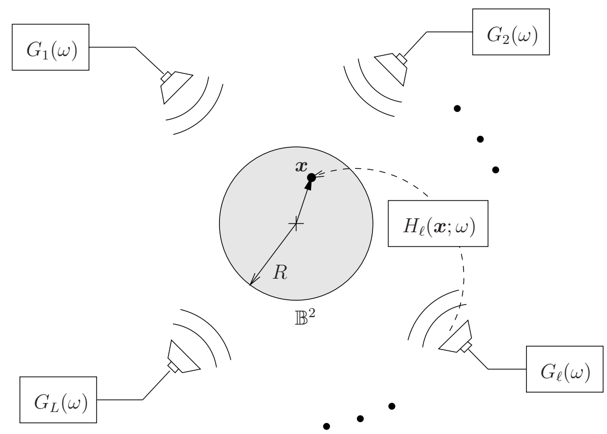

Fig. 1. Use of L loudspeakers to reproduce a desired field in a control region $\mathbb{B}^2$ with loudspeaker signals $G_\ell(\omega)$ and ATFs $H_\ell(\boldsymbol{x}, \omega)$ from the ℓth loudspeaker to point $\boldsymbol{x} \in \mathbb{B}^2$.

2. SOUND FIELD REPRODUCTION

In this section, we devise a method of performing 2-D SFR within a reverberant enclosure, that ensures good reproduction in the plane of the loudspeakers. The objective is to determine, for each frequency of interest, the loudspeaker filter weights required to reproduce a desired soundfield.

2.1. Problem Definition

We aim to reproduce the pressure $P_d(\boldsymbol{x};\omega)$ of a desired soundfield at each point $\boldsymbol{x}$ in the source-free region of interest $\mathbb{B}^2 = \{\boldsymbol{x} \in \mathbb{R}^2 : \|\boldsymbol{x}\| \leq R\}$ using an array of L loudspeakers.

As shown in Fig. 1, each loudspeaker ℓ transmits an output signal $G_\ell(\omega)$. This signal encapsulates both the input signal applied to loudspeaker ℓ as well as any filtering of it.

To characterize the acoustic properties of the enclosure, we define the ATF $H_\ell(\boldsymbol{x};\omega)$ as the frequency response between loudspeaker ℓ and point $\boldsymbol{x}$. The sound pressure at any point $\boldsymbol{x}$ due to loudspeaker ℓ is equal to:

$$P_\ell(\boldsymbol{x};\omega) = G_\ell(\omega) H_\ell(\boldsymbol{x};\omega). \tag{1}$$

From Fig. 1, the sound pressure in the reproduced field resulting from the L loudspeakers is then equal to

$$P(\boldsymbol{x};\omega) = \sum_{\ell=1}^{L} P_\ell(\boldsymbol{x};\omega) = \sum_{\ell=1}^{L} G_\ell(\omega) H_\ell(\boldsymbol{x};\omega). \tag{2}$$

The design task of SFR is to choose filter weights $G_\ell(\omega)$ to minimize the reproduction error $\mathcal{J}$ over $\mathbb{B}^2$,

$$\mathcal{J} = \int_{\mathbb{B}^2} |P(\boldsymbol{x};\omega) - P_d(\boldsymbol{x};\omega)|^2 da(\boldsymbol{x}), \tag{3}$$

where $da(\boldsymbol{x}) = x\,dx\,d\phi_x$ is the differential area element at $\boldsymbol{x}$, $x = \|\boldsymbol{x}\|$ and ϕ_x is the polar angle of $\boldsymbol{x}$.

One approach to solving this problem is to write the least squares solution for a set of uniformly-spaced points over $\mathbb{B}^2$ [4]. Below we outline a *modal space* approach, which allows design over the whole region.

2.2. Modal Space Approach

In the modal space approach, we express the sound pressure variables $P_d(\boldsymbol{x};\omega)$, $P(\boldsymbol{x};\omega)$ and the ATFs $H_\ell(\boldsymbol{x};\omega)$ in terms of the modes of the soundfield. Provided all sound sources lie outside of $\mathbb{B}^2$, at any point inside $\mathbb{B}^2$ the pressure variables can be written in modal form as [5]:

$$P_d(\boldsymbol{x};\omega) = \sum_{n=-\infty}^{\infty} \beta_n^{(d)}(\omega) J_n(kx) e^{in\phi_x}, \tag{4}$$

$$P(\boldsymbol{x};\omega) = \sum_{n=-\infty}^{\infty} \beta_n(\omega) J_n(kx) e^{in\phi_x}, \tag{5}$$

where $\beta_n^{(d)}(\omega)$ and $\beta_n(\omega)$ are the nth order modal coefficient of the desired and reproduced soundfields, $J_n(\cdot)$ is the Bessel function of order n, $k = \omega/c$ is the acoustic wave number and c is speed of sound in air. We call the functions $J_n(kx)e^{in\phi_x}$ the *modes* of the soundfield. Reproduction of the sound pressure $P_d(\boldsymbol{x};\omega)$ over $\mathbb{B}^2$ with $P(\boldsymbol{x};\omega)$ is equivalent to reproduction of the modal coefficients $\{\beta_n^{(d)}(\omega)\}$ with $\{\beta_n(\omega)\}$.

Since, from (1), $H_\ell(\boldsymbol{x};\omega)$ describes the soundfield of a source when a loudspeaker is excited by a unit impulse, we can also write it in modal form as:

$$H_\ell(\boldsymbol{x};\omega) = \sum_{n=-\infty}^{\infty} \alpha_n(\ell,\omega) J_n(kx) e^{in\phi_x}, \tag{6}$$

where $\alpha_n(\ell,\omega)$ are the modal coefficients of the ATFs for loudspeaker ℓ. These modal coefficients completely characterize the reverberant soundfield generated by each loudspeaker within $\mathbb{B}^2$:

Observation 1 *When the modal coefficients $\alpha_n(\ell,\omega)$ for each loudspeaker are known for a given room, the ATF $H_\ell(\boldsymbol{x};\omega)$ between each loudspeaker and any position $\boldsymbol{x}$ inside $\mathbb{B}^2$ is also known, and is given by (6).*

Substituting (5) and (6) into (2), the modal coefficients of the reproduced soundfield are related to $\alpha_n(\ell,\omega)$ through

$$\beta_n(\omega) = \sum_{\ell=1}^{L} G_\ell(\omega) \alpha_n(\ell,\omega). \tag{7}$$

A benefit of the modal approach is that key variables are expressed in terms of orthogonal functions. Using the orthogonality property of exponential functions,

$$\int_0^{2\pi} e^{-in\phi} e^{im\phi} d\phi = 2\pi\delta_{nm}, \tag{8}$$

we derive an expression for the error $\mathcal{J}$ of the reproduced soundfield over $\mathbb{B}^2$ as a function of the modal coefficients. Substituting (4) and (5) into (3):

$$\mathcal{J} = \int_{\mathbb{B}^2} \left| \sum_{n=-\infty}^{\infty} [\beta_n(\omega) - \beta_n^{(\mathrm{d})}(\omega)] J_n(kx) e^{in\phi_x} \right|^2 da(\boldsymbol{x}).$$

It follows that:

$$\mathcal{J} = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} [\beta_m(\omega) - \beta_m^{(\mathrm{d})}(\omega)]^* [\beta_n(\omega) - \beta_n^{(\mathrm{d})}(\omega)] \times \int_0^{2\pi} e^{-im\phi_x} e^{in\phi_x} d\phi_x \int_0^R J_m(kx) J_n(kx) x\,dx.$$

Applying orthogonality property (8), the error reduces to:

$$\mathcal{J} = K \sum_{n=-\infty}^{\infty} w_n(kR) |\beta_n^{(\mathrm{d})}(\omega) - \beta_n(\omega)|^2. \tag{9}$$

where $K \triangleq 2\pi/k^2$ and

$$w_n(kR) \triangleq k^2 \int_0^R [J_n(kx)]^2 x\,dx = \int_0^{kR} [J_n(x)]^2 x dx.$$

In the next section, we show that for finite radius R, the above parametrizations in (4), (5) and (6) can accurately be truncated to a finite number of terms.

2.3. Active Modes

Because of the high-pass character of Bessel functions, not all modes make a significant contribution to the soundfield inside $\mathbb{B}^2$. In (9), the sequences of modal coefficients

$\{\beta_n^{(d)}(\omega)\}$ and $\{\beta_n(\omega)\}$ for a source-free region are known to be bounded [6]. As a result the energy contribution of each term to reproduction error is controlled by $w_n(kR)$. Since $w_n(kR)$ drops rapidly to zero past $n = N$, only the modes of index up to $N = \lceil kR \rceil$ contribute significant energy to the soundfield inside $\mathbb{B}^2$ [2, 6]. The $2N+1$ modes, $J_{-N}(kx)e^{-iN\phi_x} \dots J_N(kx)e^{iN\phi_x}$ are referred to as the *active* modes of $\mathbb{B}^2$.

Accurate SFR requires reproduction of these active modes. Also, the ATFs mentioned in Observation 1, are accurately known just by measuring modal coefficients $\{\alpha_n(\ell,\omega)\}_{n=-N}^{N}$.

2.4. Least Squares Solution

We now derive the least squares solution for the speaker filter weights that minimizes the reproduction error in (9).

Because the modes become inactive for $n > N$, reproduction error in (9) can be truncated to N_T for $N_T \geq N$:

$$\mathcal{J}_{N_T} = K \sum_{n=-N_T}^{N_T} w_n(kR) |\beta_n(\omega) - \beta_n^{(d)}(\omega)|^2. \quad (10)$$

This truncated reproduction error $\mathcal{J}_{N_T}$ is written in matrix form, as follows. Defining the vector of loudspeaker filter weights, $\boldsymbol{g} = [G_1(\omega), G_2(\omega), \dots, G_L(\omega)]^T$ where $(\cdot)^T$ is the transpose operator, the vector of the modal coefficients of the reproduced soundfield, $\boldsymbol{\beta} = [\beta_{-N_T}(\omega), \beta_{-N_T+1}(\omega), \dots, \beta_{N_T}(\omega)]^T$ and matrix of the modal coefficients of the room responses of all loudspeakers,

$$\boldsymbol{A} = \begin{bmatrix} \alpha_{-N_T}(1,\omega) & \dots & \alpha_{-N_T}(L,\omega) \\ \alpha_{-N_T+1}(1,\omega) & \dots & \alpha_{-N_T+1}(L,\omega) \\ \vdots & \ddots & \vdots \\ \alpha_{N_T}(1,\omega) & \dots & \alpha_{N_T}(L,\omega) \end{bmatrix},$$

(7) can be rewritten $\boldsymbol{\beta} = \boldsymbol{A}\boldsymbol{g}$. Additionally define the vectors of the modal coefficients of the desired soundfield, $\boldsymbol{\beta}_d = [\beta_{-N_T}^{(d)}(\omega), \beta_{-N_T+1}^{(d)}(\omega), \dots, \beta_{N_T}^{(d)}(\omega)]^T$ and the diagonal weighting matrix $\boldsymbol{W} = \mathrm{Diag}\{[w_{-N_T}(kR), w_{-N_T+1}(kR), \dots, w_{N_T}(kR)]\}$. Writing the summation in (10) in matrix form:

$$\sum_{n=-N_T}^{N_T} w_n(kR)|\beta_n - \beta_n^{(d)}|^2 = (\boldsymbol{\beta} - \boldsymbol{\beta}_d)^H \boldsymbol{W} (\boldsymbol{\beta} - \boldsymbol{\beta}_d),$$

where $(\cdot)^H$ is the Hermitian operator, we see that:

$$\mathcal{J}_{N_T} = K(\boldsymbol{\beta} - \boldsymbol{\beta}_d)^H \boldsymbol{W} (\boldsymbol{\beta} - \boldsymbol{\beta}_d).$$

Since $\boldsymbol{\beta} = \boldsymbol{A}\boldsymbol{g}$, we expand the truncated reproduction error as a quadratic form in the vector of loudspeaker filter weights:

$$\mathcal{J}_{N_T}(\boldsymbol{g}) = K(\boldsymbol{g}^H \boldsymbol{B} \boldsymbol{g} - \boldsymbol{b}^H \boldsymbol{g} - \boldsymbol{g}^H \boldsymbol{b} + d), \quad (11)$$

where $\boldsymbol{B} = \boldsymbol{A}^H \boldsymbol{W} \boldsymbol{A}$, $\boldsymbol{b} = \boldsymbol{A}^H \boldsymbol{W} \boldsymbol{\beta}_d$, $d = \boldsymbol{\beta}_d^H \boldsymbol{W} \boldsymbol{\beta}_d$. From [7], (11) possesses its global minimum at:

$$\hat{\boldsymbol{g}} = \boldsymbol{B}^{-1}\boldsymbol{b}.$$

This modal space approach is superior to the conventional approaches in that it ensures reproduction (i) of any soundfield and (ii) over the whole control region.

3. ESTIMATING SOUNDFIELD COEFFICIENTS

In this section we describe how to fully determine the soundfield inside a control region $\mathbb{B}^2$ by measuring the modal coefficients $\beta_n(\omega)$ of the modal expansion shown in (5). This task is an important requirement for calculating $\alpha_n(\ell,\omega)$ that characterize the reverberant field generated by each loudspeaker.

The modal coefficient estimates can be obtained by sampling pressure over a single circle of radius R:

$$\beta_n(\omega) = \frac{1}{2\pi J_n(kR)} \int_0^{2\pi} P(R, \phi_x; \omega) e^{-in\phi_x} d\phi_x,$$

provided R is not a Dirichlet eigenvalues of $J_n(kR)$ (where $J_n(kR) = 0$). This equation can be derived by noting from (5) that $\beta_n(\omega)J_n(kR)$ are the Fourier series coefficients of $P(R, \phi_x; \omega)$ in variable ϕ_x.

Approximate modal coefficients $\hat{\beta}_n(\omega)$ are obtained by sampling sound pressure at M evenly-spaced points (R, ϕ_m) where $\phi_m = 2\pi m/M$ for $m = 0, 1, \dots, M-1$:

$$\hat{\beta}_n(\omega) = \frac{1}{J_n(kR)} \frac{1}{M} \sum_{m=0}^{M-1} P(R, \phi_m; \omega) e^{-i\frac{2\pi mn}{M}}. \quad (12)$$

Appropriate choice for M can be deduced by noting the soundfield in $\mathbb{B}^2$ is the result of the $2N+1$ active modes. Since one equation is required for each mode, we need at least $M = 2N+1$ pressure samples where $N = \lceil kR \rceil$.

Due to the presence of $1/J_n(kR)$ in (12), if kR is near one of the Dirichlet eigenvalues, coefficient error is amplified. This error amplification can be negated by oversampling the pressure.

4. SIMULATION EXAMPLE

As an example, we illustrate SFR of a plane wave at 1kHz.

The design parameters are as follows. The control region has radius 0.3m. The room is rectangular with dimensions 6.4m $\times$ 5m and wall absorption coefficient of 0.3. The reverberation is generated with a 2-D adaptation of the image-source method, including image-sources of up to fifth order. Though the SFR design technique is applicable for any loudspeaker type and configuration, we position omnidirectional loudspeakers in a circular array of radius 2m concentric with $\mathbb{B}^2$. This setup yields an average direct-to-reverberant ratio from each loudspeaker of -4.4dB at the center of $\mathbb{B}^2$.

From the Jacobi Anger expression [5] of the field pressure $P_d(x;\omega) = e^{-ik\boldsymbol{x}\cdot\hat{\boldsymbol{y}}}$ of the plane wave originating from direction $\hat{\boldsymbol{y}}$, modal coefficients are identified as $\beta_n^{(d)}(\omega) = (-i)^n e^{-in\phi_y}$ where ϕ_y is the polar angle of $\hat{\boldsymbol{y}}$. Here $N = \lceil kR \rceil = 6$, prompting need for $2N+1 = 13$ loudspeakers. To measure the ATF coefficients $\{\alpha_n(\ell,\omega)\}_{n=-6}^{6}$, we oversample the pressure around the boundary of $\mathbb{B}^2$ with $M = 20$ sample points.

Fig. 2 shows the reproduction of a plane wave approaching from $\phi_y = \pi/6$. We present the reverberant field design (Fig. 2(c)), a free field design (Fig. 2(a)) and the same free field design in the reverberant room (Fig. 2(b)). The reverberant performance of the free-field design is poor while the reverberant design performs as well in the reverberant room as the free-field design does in an anechoic room.

5. CONCLUSION

We have described a novel method of performing SFR in reverberant enclosures. The key to this method is an efficient parametrization of the acoustical transfer functions. Using a modal parametrization, we have outlined a technique to measure the acoustical transfer functions from a loudspeaker to each point in the SFR region. This approach allows full SFR without prior knowledge of the loudspeaker positions or their transmission characteristics.

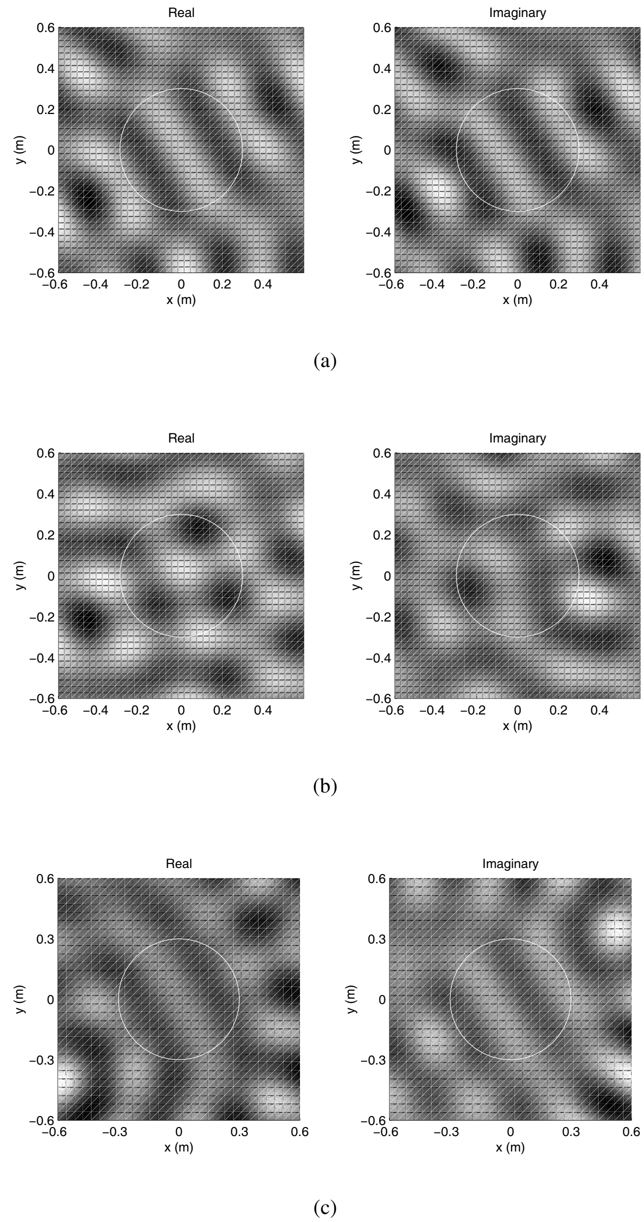

Fig. 2. Reproduction of a plane wave for (a) a free field, (b) the same free field design in the reverberant room, and (c) a reverberant design in the reverberant room. Percentage reproduction errors are 0.87%, 307% and 0.85% respectively.

6. REFERENCES

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