AN INSTRUMENTAL VARIABLE METHOD FOR ADAPTIVE FEEDBACK CANCELLATION IN HEARING AIDS

Ann Spriet[1,2], Ian Proudler[3], Marc Moonen[1], Jan Wouters[2]

[1]K.U. Leuven, ESAT/SCD-SISTA, Kasteelpark Arenberg 10, 3001 Leuven, Belgium
[2]K.U. Leuven, Lab. Exp. ORL, Kapucijnenvoer 33, 3000 Leuven, Belgium
[3]QinetiQ Ltd., Malvern Techn. Centre, St Andrews Road, Malvern, Worcestershire, WR14 3PS, UK

ABSTRACT

In this paper, we propose an instrumental variable method for adaptive feedback cancellation (IV-AFC) in hearing aids that is based on the auto-regressive modelling of the desired signal. The IV-AFC offers better feedback suppression for spectrally colored signals than the standard continuous adaptation feedback cancellers. In contrast to a previously proposed prediction error method based feedback canceller, the IV-AFC does not suffer from stability problems when the adaptive feedback canceller is highly time-varying.

1. INTRODUCTION

Acoustic feedback limits the maximum gain that can be used in a hearing aid without making it unstable. A promising solution for acoustic feedback is the use of an adaptive feedback canceller. However, because of the presence of a closed signal loop, i.e., the so-called forward path $G(q)$, the standard continuous adaptation feedback cancellers (CAF) fail to provide a reliable feedback path estimate if the desired signal $x[k]$ is spectrally colored [1, 2].

In [2, 3], an adaptive feedback canceller based on the direct method of closed-loop identification and a *fixed* estimate of the desired signal model has been proposed. It has been shown that an unbiased feedback path estimate can be obtained by means of a filtered-X algorithm if the desired signal $x[k]$ can be modelled as $H(q)w[k]$, with $w[k]$ white noise, and the desired signal model $H(q)$ is known. In practice, $H(q)$ is unknown and highly time-varying so that it is desirable to also estimate this model adaptively. In [4], we have derived a prediction error method based adaptive feedback canceller (PEM-AFC) that identifies both the desired signal model $H(q)$ and the feedback path $F(q)$. For highly time-varying signals such as speech, the PEM-AFC has a clear benefit over the filtered-X algorithm of [2, 3]. In the filtered-X algorithm of [2, 3] and the PEM-AFC [4], the feedback compensated signal $e[k]$ is filtered with the estimate of $H^{-1}(q)$ before using it to update the adaptive feedback canceller $\hat{F}(q)$. If the estimate of $H^{-1}(q)$ contains a group delay, the correction term in the adaptation of $\hat{F}(q)$ is delayed. As a result, instability may occur when the adaptive filter $\hat{F}(q)$ is fast time-varying, e.g., in highly time-varying environments [5, 6].

This research was carried out at ESAT and Lab. Exp. ORL of K.U. Leuven, in the frame of IUAP P5/22, the Concerted Research Action GOA-MEFISTO-666, FWO Project nr. G.0233.01, *Signal Processing and automative patient fitting of auditory prostheses*, IWT project 020540, *Innovative speech processing algorithms for improved performance in cochlear implants* and was partially sponsored by QinetiQ. The scientific responsibility is assumed by its authors.

In this paper, we propose an instrumental variable (IV) method for adaptive feedback cancellation (IV-AFC). We show that the IV method produces an unbiased feedback path estimate if the input-output data are pre-filtered with $H^{-1}(q)$ and the pre-filtered input data are used as instrumental variables. As such, the IV-AFC corresponds to a modified version of the PEM-AFC: the PEM-AFC rather uses pre-filtered versions (with $H^{-1}(q)$) of the input and the error signal. Simulations demonstrate that the IV-AFC, as the PEM-AFC, outperforms the standard CAF. Moreover, in contrast to the PEM-AFC, the IV-AFC does not suffer from stability problems when the adaptive filter $\hat{F}(q)$ is highly time-varying, while the computational complexity is only slightly increased.

Notation

The symbol q^{-1} denotes the discrete-time delay operator, i.e., $q^{-1}u[k] = u[k-1]$. A discrete-time filter with coefficient vector $\mathbf{f} = \left[\begin{array}{cccc} f_0 & f_1 & \cdots & f_{L_F-1} \end{array}\right]^T$ and filter length L_F is represented as a polynomial $F(q)$ in q, i.e.,

$$F(q) \quad = \quad f_0 + f_1 q^{-1} + \ldots + f_{L_F-1} q^{-L_F+1}. \qquad (1)$$

Filtering $u[k]$ with $F(q)$ is denoted as $F(q)u[k]$ or $\mathbf{f}^T\mathbf{u}[k]$ with $\mathbf{u}[k] = \left[\begin{array}{cccc} u[k] & u[k-1] & \cdots & u[k-L_F+1] \end{array}\right]^T$. The filter $F(q,k)$ refers to a time-varying filter with coefficient vector $\mathbf{f}[k]$.

2. CLOSED-LOOP SYSTEM SET-UP

Figure 1 depicts the closed-loop system set-up of a hearing aid. The *open-loop system to be identified* is described by

$$y[k] = F(q)u[k] + x[k], \qquad (2)$$

where $y[k]$ is the microphone signal and $u[k]$ the loudspeaker signal. In general, the desired signal $x[k]$ is an audio signal (e.g., a speech signal). Many audio signals can be closely approximated by a low-order autoregressive (AR) random process

$$x[k] = H(q)w[k] = \frac{1}{1+q^{-1}P(q)} w[k], \qquad (3)$$

with $w[k]$ white noise[1] and $P(q)$ an FIR filter.

The output signal $y[k]$ is fed-back to the input $u[k]$ according to

$$u[k] = G(q)\left(y[k] - \hat{F}_0(q)u[k]\right). \qquad (4)$$

Using (2) and (4), the input $u[k]$ can be written as

[1]Note that the white noise assumption is not satisfied for periodic signals such as voiced speech segments. E.g., the excitation $w[k]$ of a voiced speech segment is an impulse train [7].

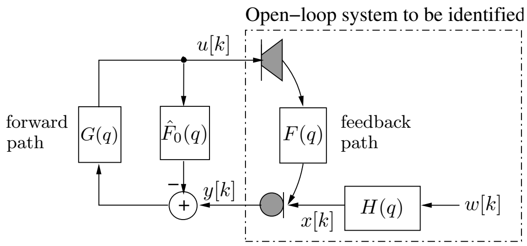

Fig. 1. Closed-loop system set-up of a hearing aid.

$$u[k] = \frac{G(q)}{1 - G(q)\left(F(q) - \hat{F}_0(q)\right)} x[k] = C(q)x[k], \quad (5)$$

The filter $\hat{F}_0(q)$ in the feedback cancellation path is an initial estimate of the feedback path $F(q)$ chosen such that the closed-loop system $C(q)$ is stable. During adaptation, it is typically replaced with the feedback path estimate $\hat{F}(q)$. In the sequel, we assume that the forward path $G(q)$ contains a delay $d_f \geq 1$ sample.

3. INSTRUMENTAL VARIABLE BASED AFC

3.1. Instrumental variable method [8, 9]

Let $A(q)$ be a pre-filter and let

$$x_p[k] = A(q)x[k]; \;\; u_p[k] = A(q)u[k]; \;\; y_p[k] = A(q)y[k]. \quad (6)$$

The idea behind the Instrumental Variable (IV) method is to use a generic regression vector $\boldsymbol{\xi}[k] \in \mathbb{R}^{L_{\hat{F}} \times 1}$, called the IV vector, that is uncorrelated with $x_p[k]$ but correlated with the pre-filtered signal $u_p[k]$, and to compose the feedback path estimate $\hat{\mathbf{f}}[k]$ as

$$\hat{\mathbf{f}} = \left(\boldsymbol{\Xi}^T[k]\mathbf{U}_p[k]\right)^{-1} \boldsymbol{\Xi}^T[k]\mathbf{y}_p[k], \quad (7)$$

where

$$\boldsymbol{\Xi}[k] = \begin{bmatrix} \lambda^0 \boldsymbol{\xi}^T[k] \\ \lambda^{\frac{1}{2}} \boldsymbol{\xi}^T[k-1] \\ \vdots \\ \lambda^{\frac{k}{2}} \boldsymbol{\xi}^T[0] \end{bmatrix}, \quad \mathbf{U}_p[k] = \begin{bmatrix} \lambda^0 \mathbf{u}_p^T[k] \\ \lambda^{\frac{1}{2}} \mathbf{u}_p^T[k-1] \\ \vdots \\ \lambda^{\frac{k}{2}} \mathbf{u}_p^T[0] \end{bmatrix}, \quad (8)$$

$$\mathbf{y}_p[k] = \begin{bmatrix} \lambda^0 y_p[k] & \lambda^{\frac{1}{2}} y_p[k-1] & \cdots & \lambda^{\frac{k}{2}} y_p[0] \end{bmatrix}^T, \quad (9)$$

$$\mathbf{u}_p[k] = \begin{bmatrix} u_p[k] & u_p[k-1] & \cdots & u_p[k - L_{\hat{F}} + 1] \end{bmatrix}^T. \quad (10)$$

The forgetting factor $\lambda \in (0, 1]$ has been included to allow for tracking in time-varying scenarios.

In the sequel, we give the IV vector $\boldsymbol{\xi}[k]$ and the pre-filter $A(q)$ for which an unbiased feedback path estimate $\hat{\mathbf{f}}$ is obtained.

3.2. Unbiased feedback path estimate

Assume that $L_{\hat{F}} = L_F$ and that $F(q)$ is time-invariant. Then, the feedback path estimate (7) equals

$$\hat{\mathbf{f}} = \mathbf{f} + \underbrace{\left(\boldsymbol{\Xi}^T[k]\mathbf{U}_p[k]\right)^{-1} \boldsymbol{\Xi}^T[k]\mathbf{x}_p[k]}_{\text{bias}}, \quad (11)$$

where

$$\mathbf{x}_p[k] = \begin{bmatrix} \lambda^0 x_p[k] & \lambda^{\frac{1}{2}} x_p[k-1] & \cdots & \lambda^{\frac{k}{2}} x_p[0] \end{bmatrix}^T.$$

If the IV vector $\boldsymbol{\xi}[k]$ and $x_p[k]$ are uncorrelated (i.e., $\boldsymbol{\Xi}^T[k]\mathbf{x}_p[k] = \mathbf{0}$) and $\boldsymbol{\Xi}^T[k]\mathbf{U}_p[k]$ is non-singular, the bias term in (11) goes to zero for $k \to \infty$ and $\lambda = 1$.

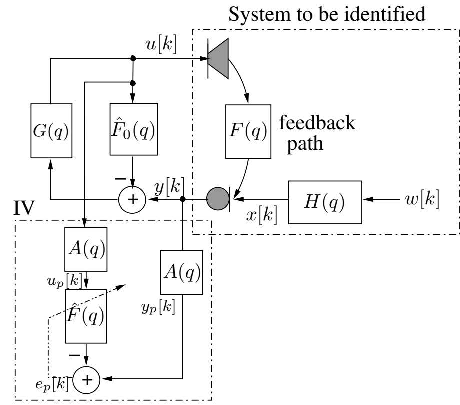

Fig. 2. Instrumental variable method.

Depending on the choice of the IV vector $\boldsymbol{\xi}[k]$ and the pre-filter $A(q)$, different IV methods are possible. Here we develop one based on the idea that if $x[k] = H(q)w[k]$ with $w[k]$ white noise, the choice

$$A(q) = H^{-1}(q), \quad \boldsymbol{\xi}[k] = H^{-1}(q)\mathbf{u}[k] = \mathbf{u}_p[k] \quad (12)$$

offers the best accuracy among all choices for the instruments $\boldsymbol{\xi}[k]$ and the pre-filter $A(q)$, provided that $u_p[k]$ and $x_p[k]$ are uncorrelated and $\boldsymbol{\Xi}^T[k]\mathbf{U}_p[k]{=}\mathbf{U}_p^T[k]\mathbf{U}_p[k]$ is non-singular [8, 9]. The feedback path estimate $\hat{\mathbf{f}}$ then becomes

$$\begin{aligned} \hat{\mathbf{f}} &= \left(\mathbf{U}_p^T[k]\mathbf{U}_p[k]\right)^{-1} \mathbf{U}_p^T[k]\mathbf{y}_p[k] \\ &= \mathbf{f} + \left(\mathbf{U}_p^T[k]\mathbf{U}_p[k]\right)^{-1} \mathbf{U}_p^T[k]\mathbf{x}_p[k]. \end{aligned} \quad (13)$$

For $A(q) = H^{-1}(q)$, $x_p[k]$ equals $w[k]$ and $u_p[k]$ equals $C(q)w[k]$ with $C(q)$ defined in (5). Hence, if $w[k]$ is white and $G(q)$ contains a delay $d_f \geq 1$, the IV vector $\boldsymbol{\xi}[k] = \mathbf{u}_p[k]$ and $x_p[k]$ are uncorrelated, so that the feedback path estimate (13) is unbiased for $k \to \infty$.

The feedback path estimate (13) minimizes the least-squares cost function

$$\left\| \mathbf{y}_p[k] - \mathbf{U}_p[k]\hat{\mathbf{f}} \right\|_2 \quad (14)$$

Hence, the coefficient vector $\hat{\mathbf{f}}[k]$ can be adapted by applying the standard adaptive filtering techniques (e.g., RLS or LMS) to the pre-filtered data $y_p[k]$ and $u_p[k]$, as depicted in Figure 2. For RLS, this results in the update equation:

$$\begin{aligned} \hat{\mathbf{f}}[k] &= \hat{\mathbf{f}}[k-1] + \mathbf{R}^{-1}[k]\mathbf{u}_p[k]\left(y_p[k] - \hat{\mathbf{f}}^T[k-1]\mathbf{u}_p[k]\right) \\ \mathbf{R}[k] &= \lambda\mathbf{R}[k-1] + \mathbf{u}_p[k]\mathbf{u}_p^T[k] \end{aligned} \quad (15)$$

The matrix $\mathbf{R}^{-1}[k]$ in (15) may be updated using the matrix inversion lemma (cf. Algorithm 2).

3.3. Estimation of the desired signal model

In practice, the desired signal model $H(q)$ is unknown and time-varying so that $H^{-1}(q)$ has to be estimated adaptively. In general though, the feedback path $F(q)$ and the desired signal model $H(q)$ are not both identifiable in the closed-loop system at hand [2]. In [4], we demonstrated that not only inserting non-linearities or a probe signal $r[k]$, but also adding a delay in the forward path or the cancellation path can render the system identifiable. If the total delay $d = d_f + d_c \geq L_{H^{-1}}$ with d_c the common delay in the feedback path $F(q)$ and the cancellation path $\hat{F}(q)$, the desired signal $H(q)$ and the feedback path $F(q)$ can be both identified in closed-loop. Assuming that $L_A \geq L_{H^{-1}}$, we set $d_f \geq L_A$.

If $F(q)$ was known, we could compute $x[k]$ as

$$x[k] = y[k] - F(q)u[k]. \tag{16}$$

Assuming an AR model for $x[k]$ (cf. (3)), an estimate $A(q) = 1 + q^{-1}\bar{A}(q)$ of the inverse desired signal model $H^{-1}(q)$ could then be computed by solving the linear prediction problem

$$\bar{A}(q,k)=\arg\min_{\bar{A}(q,k)} \sum_{\bar{k}=0}^{k} \lambda_{\bar{a}}^{k-\bar{k}} \left(x[k] + \bar{A}(q,k)x[k-1]\right)^2. \tag{17}$$

where $\lambda_{\bar{a}}$ is the forgetting factor. Since $F(q)$ is unknown, we use the feedback compensated signal

$$\epsilon[k] = y[k] - \hat{F}(q,k-1)u[k] \tag{18}$$

instead of $x[k]$ to compute $\bar{A}(q,k)$ in (17) [8, 10]. To update the AR coefficients $\bar{\mathbf{a}}[k]$, we use the Burg lattice algorithm [11], described in *Algorithm 1*.

The pre-filtered data $y_p[k]$, $u_p[k]$ in (15) are then computed as

$$y_p[k] = A(q,k-1)y[k], \quad u_p[k] = A(q,k-1)u[k]. \tag{19}$$

The complete algorithm, which is a special case of the IV approximate maximum likelihood algorithm of [10], is summarized in the tabulated *Algorithm 2*. We refer to this algorithm as IV based adaptive feedback canceller (IV-AFC). Note that the IV-AFC can be seen as a modified version of the PEM-AFC where the input-output data, rather than the input and the error signal are pre-filtered with $H^{-1}(q)$.

In contrast to the filtered-X algorithm of [2] and the PEM-AFC [4], any change to the adaptive filter $\hat{\mathbf{f}}$ has an immediate effect on the adaptation error $\epsilon_p[k] = y_p[k]-\hat{\mathbf{f}}^T[k-1]\mathbf{u}_p[k]$, irrespective of the group delay in $A(q)$. As a result, the IV method does not suffer from instability problems when there is a group delay associated with $A(q)$ and the adaptive filter $\hat{\mathbf{f}}$ is highly time-varying. Compared to the PEM-AFC, an additional filtering with $\hat{\mathbf{f}}$ is required to compute the feedback compensated signal $\epsilon[k] = y[k]-\hat{F}(q)u[k]$. In hearing aids, the dominant part of the true feedback path $F(q)$ is short, so that a short filter length $L_{\hat{F}}$ for $\hat{\mathbf{f}}$ is typically used. Hence, the increase in computational complexity with respect to the PEM-AFC is limited.

Note: Like the PEM-AFC, the IV-AFC assumes that the desired signal model $H(q)$ is stationary over a time window with the length L_F of the feedback path $\mathbf{f}$. In hearing aids, the dominant part of the feedback path $F(q)$ is short, so that this assumption is justified [4].

Algorithm 1 Burg lattice algorithm.

Initialization:

$f_0[k] = \epsilon[k] = y[k] - \hat{\mathbf{f}}[k-1]\mathbf{u}[k]$;

$b_0[k] = \epsilon[k]$;

For $i = 1, \ldots, L_A - 1$:

$d_i[k] = \lambda_{\bar{\mathbf{a}}} d_i[k-1] + (1-\lambda_{\bar{\mathbf{a}}})\left[f_{i-1}^2[k] + b_{i-1}^2[k-1]\right]$

$n_i[k] = \lambda_{\bar{\mathbf{a}}} n_i[k-1] + (1-\lambda_{\bar{\mathbf{a}}})(-2)f_{i-1}[k]b_{i-1}[k-1]$

$\kappa_i[k] = \frac{n_i[k]}{d_i[k]}$

$f_i[k] = f_{i-1}[k] + \kappa_i[k]b_{i-1}[k-1]$ (forward residuals)

$b_i[k] = \kappa_i[k]f_{i-1}[k] + b_{i-1}[k-1]$ (backward residuals)

Algorithm 2 IV based adaptive feedback canceller (IV-AFC).

Initialization:

$\hat{\mathbf{f}}[0] = \mathbf{0}$; $\quad \mathbf{R}^{-1}[0] = \frac{1}{c}\mathbf{I}_{L_{\hat{F}}}$ with c a small positive constant

$\bar{\mathbf{a}}[0] = \mathbf{0}$ or reflection coefficients $\kappa_i[0] = 0$, $i = 1, \ldots, L_A - 1$

For each time instant $k = 0, \ldots, \infty$**:**

$$u[k] = G(q)\left(y[k] - \hat{F}_0(q,k)u[k]\right)$$

Pre-filter the input-output data $u[k]$ and $y[k]$ with $A(q,k-1)$:

Compute $u_p[k] = A(q,k-1)u[k]$ and $y_p[k] = A(q,k-1)y[k]$:
Filter $u[k]$ and $y[k]$ through the lattice filter $\kappa_i[k-1]$

Update equation for $\bar{\mathbf{a}}$:

Compute $\epsilon[k] = y[k] - \hat{F}(q,k-1)u[k]$
Compute the reflection coefficients $\kappa_i[k]$, $i = 1, \ldots, L_A - 1$
by applying the Burg lattice algorithm (cf. Algorithm 1) to $\epsilon[k]$

Update equation for $\hat{\mathbf{f}}$:

$$\hat{\mathbf{f}}[k] = \hat{\mathbf{f}}[k-1] + \mathbf{R}_{\hat{\mathbf{f}}}^{-1}[k]\mathbf{u}_p[k]\underbrace{\left(y_p[k] - \hat{\mathbf{f}}^T[k-1]\mathbf{u}_p[k]\right)}_{\epsilon_p[k]}$$

$$\mathbf{R}_{\hat{\mathbf{f}}}^{-1}[k]=\frac{1}{\lambda_{\hat{\mathbf{f}}}}\mathbf{R}_{\hat{\mathbf{f}}}^{-1}[k-1]-\frac{1}{\lambda_{\hat{\mathbf{f}}}^2}\frac{\mathbf{R}_{\hat{\mathbf{f}}}^{-1}[k-1]\mathbf{u}_p[k]\mathbf{u}_p^T[k]\mathbf{R}_{\hat{\mathbf{f}}}^{-1}[k-1]}{1+\frac{1}{\lambda_{\hat{\mathbf{f}}}}\mathbf{u}_p^T[k]\mathbf{R}_{\hat{\mathbf{f}}}^{-1}[k-1]\mathbf{u}_p[k]}$$

Update the feedback canceller $\hat{F}_0(q)$: $\hat{F}_0(q,k+1) = \hat{F}(q,k)$

4. SIMULATION RESULTS

In this section, we compare the performance of the IV-AFC with the standard CAF algorithm and the PEM-AFC.

4.1. Set-up and performance measure

In the simulation, we have gradually changed the feedback path $F(q)$ from an initial $F_1(q)$ to a final $F_2(q)$ between sample number 60000 and 68000 (i.e., during 0.5 seconds for a sampling frequency $f_s = 16$ kHz) by means of interpolation. Figure 3 depicts the frequency responses of $F_1(q)$ and $F_2(q)$. The filter length L_F of $F_1(q)$ and $F_2(q)$ equals 50. The gain $|G(q)|$ has been set to 4. The delay d_f in $G(q)$ equals 30 samples, i.e., 1.9 msec. For $|G(q)| = 4$, the closed-loop system $\frac{G(q)}{1-G(q)F_2(q)}$ is unstable.

The forgetting factor λ in all algorithms was set to 0.9998. In the IV-AFC and the PEM-AFC, the filter $A(q)$ was updated by means of the Burg lattice algorithm with $\lambda_{\bar{\mathbf{a}}} = 1 - \frac{1}{160}$. The filter length $L_A = 21$, the filter length $L_{\hat{F}} = 50$. During adaptation, the feedback canceller $\hat{F}_0(q)$ was continuously updated by $\hat{F}(q)$.

To assess the performance of the feedback cancellation algorithms we use the misalignment $\zeta(\mathbf{f}[k], \hat{\mathbf{f}}[k])$ between the true and estimated feedback path $\hat{F}(q)$, defined as

$$\zeta(\mathbf{f}[k], \hat{\mathbf{f}}[k]) = \frac{\left\|\mathbf{f}[k] - \hat{\mathbf{f}}[k]\right\|_2^2}{\|\mathbf{f}[k]\|_2^2}. \tag{20}$$

4.2. Simulation results

Figure 4 depicts the misalignment of the CAF, the PEM-AFC and the IV-AFC as a function of time for a stationary speech-weighted

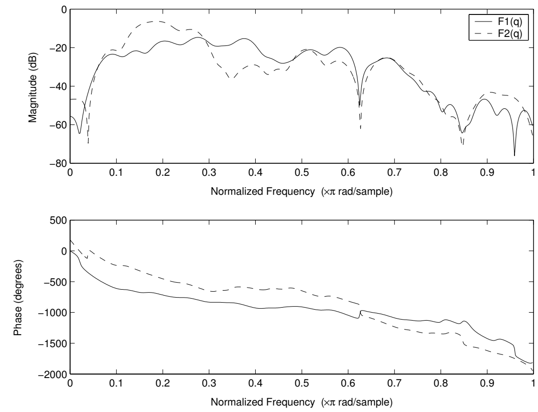

Fig. 3. Frequency response of the feedback path $F_1(q)$ and $F_2(q)$.

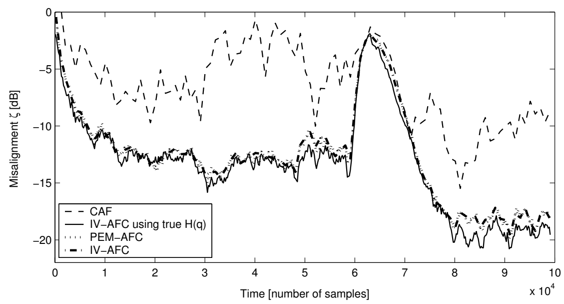

Fig. 4. Misalignment $\zeta(\mathbf{f}[k], \hat{\mathbf{f}}[k])$ of the CAF, the IV-AFC using the true $H(q)$, the PEM-AFC and the IV-AFC as a function of time. Stationary speech-weighted noise signal $x[k]$.

noise signal $x[k]$ created by passing Gaussian white noise through a 20-th order all-pole filter $H(q)$. For comparison, the IV method using the true desired signal model $H(q)$, which we consider in some sense an idealized solution, is also shown. The PEM-AFC and the IV-AFC clearly outperform the CAF and achieve the same performance as the idealized IV method with $A(q) = H^{-1}(q)$.

In a second example, $x[k]$ is a real speech signal consisting of sentences spoken by a male speaker. Figure 5 depicts the misalignment of the CAF, the PEM-AFC, the IV-AFC and the IV method that uses an AR model of the long-term average spectrum of the speech signal. Note that the white noise assumption for $w[k]$ is not completely satisfied anymore: the excitation $w[k]$ of voiced speech segments corresponds to an impulse train rather than white noise. As a result, the performance of the PEM-AFC and the IV-AFC is worse than for the stationary signal [4]. The IV-AFC outperforms the CAF and the IV method using the long-term average speech model. The PEM-AFC initially has the same performance as the IV-AFC. However, when the change in the feedback path $F(q)$ occurs (at sample 60000), the PEM-AFC suffers from instability due to a delayed adaptation, while the IV-AFC remains stable.

5. CONCLUSIONS

In this paper, we have derived an IV-AFC that is based on the autoregressive modelling of the desired signal. The IV-AFC offers better feedback suppression than the standard CAF for spectrally colored signals. In contrast to a previously proposed PEM-AFC, the IV-AFC does not suffer from stability problems when the adaptive feedback canceller is fast time-varying.

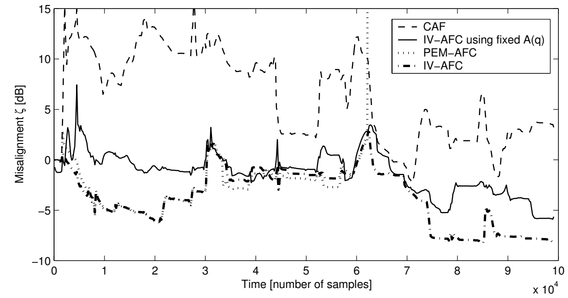

Fig. 5. Misalignment $\zeta(\mathbf{f}[k], \hat{\mathbf{f}}[k])$ of the CAF, the IV-AFC using a fixed $A(q)$, the PEM-AFC and the IV-AFC as a function of time. Real speech signal $x[k]$.

6. REFERENCES

[1] M. G. Siqueira and A. Alwan, "Steady-state analysis of continuous adaptation in acoustic feedback reduction systems for hearing-aids," *IEEE Trans. SAP*, vol. 8, no. 4, pp. 443–453, July 2000.

[2] J. Hellgren and U. Forssell, "Bias of feedback cancellation algorithms in hearing aids based on direct closed loop identification," *IEEE Trans. SAP*, vol. 9, no. 7, pp. 906–913, Nov. 2001.

[3] J. Hellgren, "Analysis of feedback cancellation in hearing aids with filtered-X LMS and the direct method of closed loop identification," *IEEE Trans. SAP*, vol. 10, no. 2, pp. 119–131, Feb. 2002.

[4] A. Spriet, I. Proudler, M. Moonen, and J. Wouters, "Adaptive feedback cancellation in hearing aids with linear prediction of the desired signal," Submitted. Available at ftp://ftp.esat.kuleuven.ac.be/pub/sista/spriet/reports/03-167.pdf

[5] M. Bouchard and S. Quednau, "Multi-channel recursive-least-squares algorithms and fast-transversal-filter algorithms for active noise control and sound reproduction systems," *IEEE Trans. SAP*, vol. 8, no. 5, pp. 606–618, Sept. 2000.

[6] H.-F. Chi, S. X. Goa, S. D. Soli, and A. Alwan, "Band-limited feedback cancellation with a modified filtered-X LMS algorithm for hearing aids," *Speech Communication*, vol. 39, no. 1-2, pp. 147–161, Jan. 2003.

[7] J. R. Deller, J. G. Proakis, and J. H. L. Hansen, *Discrete-Time Processing of Speech Signals*, Macmillan Publishing Company, Englewood Cliffs, New Jersey, 1993.

[8] L. Ljung and T. Söderström, *Theory and Practice of Recursive Identification.*, MIT Press, Cambridge, MA, 1983.

[9] U. Forssell, *Closed-loop identification - methods, theory and applications*, Ph.D. thesis, Linköping Universitet, Linköping, Sweden, 1999.

[10] P. Young, *Recursive estimation and time-series analysis*, Springer-Verlag, 1984.

[11] J. Makhoul, "Stable and efficient lattice methods for linear prediction," *IEEE Trans. ASSP*, vol. 25, no. 5, pp. 423–428, Oct. 1977.