ANC ALGORITHMS THAT DO NOT REQUIRE IDENTIFYING THE SECONDARY PATH

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ABSTRACT

Most available control algorithms for active noise control (ANC) require the identification of the secondary path. This estimation will not only increase the control system complexity, but it can add to the residual noise power and even cause the adaptive system to diverge when the identification is not sufficiently close to the real system. In this paper, we use a geometrical analysis of the filtered-x LMS algorithm to introduce a new ANC algorithm for a single frequency and narrow-band noise where no identification of the secondary path is required. Then, we extend our new ANC algorithm without the secondary path identification to the active control of broadband noise through the use of a sub-band ANC implementation. When compared to other available control algorithms requiring no secondary path identification, our method possesses a simple structure, good performance, and a reasonable convergence rate. Simulation results confirm the effectiveness of our proposed algorithms.

1. INTRODUCTION

Active noise control (ANC) and active vibration control (AVC) methods have received considerable attention in recent research due to many industrial applications. The filtered-x LMS algorithm is the most common algorithm that is applied in both feed-forward and feed-back systems due to its simplicity [1, 2]. However, most available adaptive control algorithms, including the filtered-x LMS algorithm, require the identification of the secondary path. The requirement to identify the secondary path causes several problems: 1) increased complexity; 2) potential divergence due to identification errors; and 3) increased residual noise due to the fact that the online identification requires an auxiliary input.

A control algorithm where no secondary path identification is required could potentially solve these problems. Currently, several such methods exist [3-7]. However, the methods introduced by Feintuch *et al* [3] and Bjarnason *et al* [4] do require some advance knowledge regarding the secondary path. Furthermore, their methods only work for certain narrowband noises and systems. The algorithm introduced in [6] uses the simultaneous equation method, and so requires implementing an auxiliary filter. Although this technique can converge quickly, it is not convenient to implement and has considerable computation complexity. The method introduced in [7] requires three adaptive filters to simultaneously minimize two artificial errors – clearly this method is complex and computationally burdensome. In [5], random search algorithms based on a simple parameter perturbation optimization method are employed to find the coefficients of the adaptive control filter. Although simple in structure, this method converges very slowly when compared to efficient adaptive (gradient based) algorithms like the filtered-x LMS. Additionally, the added perturbation will increase the residual noise power.

In this paper, we introduce new adaptive control algorithms to cancel single frequency noise, narrow-band noise, and broadband noise. Our proposed methods do not require any secondary path identification. They do enjoy a simple structure, yield good performance and converge quickly.

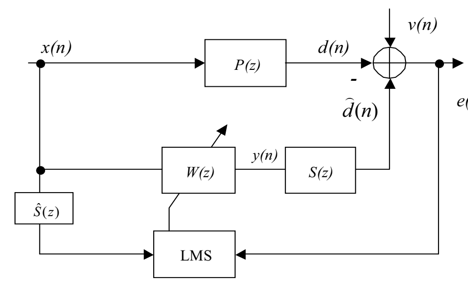

Figure 1. Block diagram of the filtered-x LMS algorithm

2. A GEOMETRICAL ANALYSIS OF FILTERED-X LMS

The filtered-x LMS block diagram is shown in Fig. 1, where *P(z)*, *S(z)* and $\hat{S}(z)$ represent the main path, secondary path and the estimated secondary path respectively; *W(z)* is the adaptive filter; *x(n)* is the reference signal and *v(n)* is additive zero-mean noise that is uncorrelated with *x(n)*. Now, define the reference signal vector $X(n)$ as [*x(n) x(n-1)* ... *x(n-M)*], where *M* is the order of the adaptive filter *W(n)* so that we have the update

$$W(n) = W(n-1) + \mu e(n) X_f^*(n) \tag{1}$$

where $X_f(n)$ is the reference signal vector $X(n)$ filtered by $\hat{S}(z)$. The positive, real number μ is the step-size that controls the convergence speed and stability.

If we assume the input is a pure tone with frequency ω, then the filters *P(z)*, *W(z)*, $S(z)$ and $\hat{S}(z)$ reduce to the complex values P_ω, $W_\omega(n)$, S_ω and $\hat{S}_\omega$ respectively. We can write (1) as

$$\begin{aligned} W_\omega(n) &= W_\omega(n-1) + \mu x_\omega^*(n)\hat{S}_\omega^*[x_\omega(n)P_\omega - x_\omega(n)W_\omega(n-1)S_\omega] \\ &= W_\omega(n-1) + \mu P_x(\omega)S_\omega\hat{S}_\omega^*[P_\omega / S_\omega - W_\omega(n-1)] \end{aligned} \tag{2}$$

$P_x(\omega)$ represents the power of the reference signal at the frequency ω. When the adaptive filter converges, $W_\omega(n) = W_\omega(n-1)$; so we have $W_\omega(\infty) = P_\omega / S_\omega$.

If the estimated secondary path $\hat{S}(z)$ is error free, i.e. if $\hat{S}(z)=S(z)$, then Eq. (2) can be written as

$$W'_{\omega}(n)=W_{\omega}(n-1)+\mu P_x(\omega)\left|S_{\omega}\right|^2\left[\frac{P_{\omega}}{S_{\omega}}-W_{\omega}(n-1)\right] \quad (3)$$

The physical meaning of (3) is that $W'_{\omega}(n)$ goes in the point-to-point direction from $W_{\omega}(n-1)$ towards P_{ω}/S_{ω}, with a length of $\mu P_x(\omega)|S_{\omega}|^2\left|P_{\omega}/S_{\omega}-W_{\omega}(n-1)\right|$ as shown in Fig. 2. However, in practice, there is always some estimation error. At the frequency ω, $\hat{S}(z)$ can be expressed as:

$$\hat{S}_{\omega}=c_{\omega}S_{\omega}e^{j\theta_{\omega}} \quad (4)$$

where c_{ω} is a real constant representing the amplitude estimation error and θ_{ω} represents the phase estimation error. Combining Eqs. (4) and (2), we have

$$W_{\omega}(n)=W_{\omega}(n-1)+\mu P_x(\omega)\left|S_{\omega}\right|^2 c_{\omega}\left[\frac{P_{\omega}}{S_{\omega}}-W_{\omega}(n-1)\right]e^{-j\theta_{\omega}} \quad (5)$$

Clearly, $W_{\omega}(n)$ doesn't go in the point-to-point direction from $W_{\omega}(n-1)$ toward P_{ω}/S_{ω} but rather instead there is the angle difference θ_{ω} that is indicated in Fig. 2. If $\left|\theta_{\omega}\right|<90°$, and $\mu c_{\omega}P_x(\omega)|S_{\omega}|^2<2\cos(\theta_{\omega})$, then the distance from $W_{\omega}(n)$ to P_{ω}/S_{ω} will be less than that from $W_{\omega}(n-1)$ to P_{ω}/S_{ω}. Consequently, the adaptive filter will slowly converge. On the other hand, if $\left|\theta_{\omega}\right|\geq 90°$, the adaptive filter will never converge.

Using the orthogonality of signals, we can extend this result to broadband input signals. In this case, the step size μ should be the smallest over all frequencies, i.e.

$$\mu<\min_{\omega}\frac{2\cos(\theta_{\omega})}{c_{\omega}P_x(\omega)\left|S_{\omega}\right|^2} \quad (6)$$

This analysis simply shows the $\pm 90°$ stability bound of the filtered-x LMS algorithm. The amplitude estimation error of $\hat{S}(z)$ should only affect the bound of step size μ – it will not cause divergence if μ is chosen small enough. These results have been observed in [3, 8, 9], but our analysis uses geometry to obtain an intuitive explanation that will yield a new broadband algorithm.

3. ANC for Single Frequency Noise Without Requring Identification of the Secondary Path

If we ignore the secondary path effect, then the LMS update is

$$W(n)=W(n-1)+\mu e(n)X^*(n) \quad (7)$$

Using our intuition from Section 2, for a single frequency input

$$\begin{aligned}W_{\omega}(n)&=W_{\omega}(n-1)+\mu P_x(\omega)S_{\omega}\left[\frac{P_{\omega}}{S_{\omega}}-W_{\omega}(n-1)\right]\\&=W_{\omega}(n-1)+\mu P_x(\omega)\left|S_{\omega}\right|\left[\frac{P_{\omega}}{S_{\omega}}-W_{\omega}(n-1)\right]e^{j\angle S_{\omega}}\end{aligned} \quad (8)$$

where $\angle S_{\omega}$ is the angle of S_{ω} and $\left|S_{\omega}\right|$ is its amplitude. From our previous discussion, we can see that if

$$\mu<\frac{2\cos\left(\angle S_{\omega}\right)}{P_x(\omega)\left|S_{\omega}\right|} \quad (9)$$

and $\angle S_{\omega}$ is within $\pm 90°$, then the update of $W_{\omega}(n)$ will still converge to its ideal value even without identifying the secondary path. However, if $\angle S_{\omega}$ is more than $\pm 90°$ out, then $W_{\omega}(n)$ will diverge. In this case, if we use

$$W(n)=W(n-1)-\mu e(n)X^*(n) \quad (10)$$

Which is equivalent to changing the sign of μ, then for a single frequency input $X_{\omega}(n)$, we have

$$\begin{aligned}W_{\omega}(n)&=W_{\omega}(n-1)-\mu P_x(\omega)\left|S_{\omega}\right|\left[\frac{P_{\omega}}{S_{\omega}}-W_{\omega}(n-1)\right]e^{j\angle S_{\omega}}\\&=W_{\omega}(n-1)+\mu P_x(\omega)\left|S_{\omega}\right|\left[\frac{P_{\omega}}{S_{\omega}}-W_{\omega}(n-1)\right]e^{j(\angle S_{\omega}-\pi)}\end{aligned} \quad (11)$$

By changing the sign of μ, we actually move the angle difference from out of range to within range as shown in Fig. 3. However, without identifying the secondary path, we wouldn't know if $\angle S_{\omega}$ is within the $\pm 90°$ range or outside of it, so we couldn't determine which direction to move. Our proposed solution to this problem is based on the following assumption:

Assumption 1. The additive noise *v(n)* in Fig. 1. is wide sense stationary or varying slowly with known power range $P_{max}/P_{min}=c$, where P_{max} represents the maximum power of *v(n)* and P_{min} represents the minimum power *v(n)*.

Using this assumption, we develop our new algorithm for active control of a single frequency noise without identifying the secondary path (see Alg. 1). In short, this algorithm must be initialization, and the iteration must determine the search direction, update the filter, and monitor the performance (see Fig. 4). The main idea behind our algorithm is of course the choice of sign for μ. This is done by monitoring the excess noise power. For initialization, *N* is the length of data used to determine the search direction of the adaptive filter. This length is related to the frequency of the reference signal and the variance of the additive noise *v(n)*. Also

$$c'=\max\{c,1.2\} \quad (12)$$

where c is defined in Assumption 1. We use the minimum value of c' as 1.2 (used step 7 in Alg. 1) to tolerate some variation in our adaptive algorithm. Since we don't have the secondary path information, we cannot use Eq. (9) to obtain μ. However, it could be estimated using prior information about the secondary path or from trial and error.

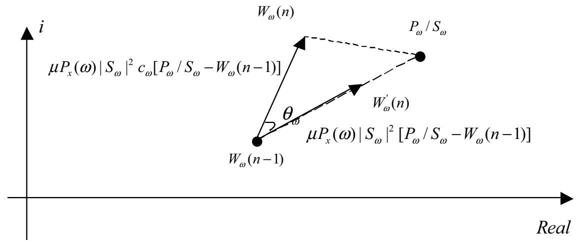

Figure 2. The complex plane expression of Eqs. (3) and (5)

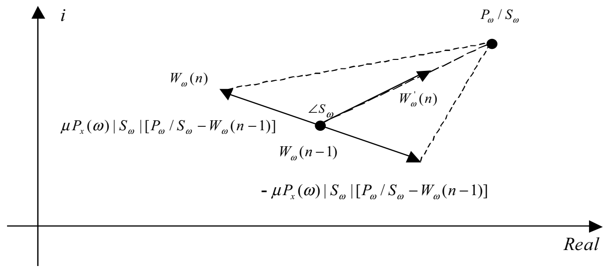

Figure 3. The expression of Eq. (10)

Algorithm 1. Single tone noise and certain narrowband noise ANC without the secondary path identification.

Initialization:

1. Initialize the adaptive filter coefficient *W(n)* with zeros, length of samples *N*, step size μ and factor c'.

Direction Search:

2. Don't change the adaptive filter coefficients, and measure the mean noise power $Ee1 = \sum_{i=0}^{N-1} e^2(i)$ and reference noise power $Ex1 = \sum_{i=0}^{N-1} x^2(i)$ for *N* samples.
3. Update the adaptive filter using Eq. (7) and measure the mean noise power $Ee2$ and mean reference noise power $Ex2$ as in step 1 for another *N* samples.
4. If $Ee2/Ex2 > Ee1/Ex1$, change the sign of μ.

Update:

5. Update the adaptive filter using Eq. (7).

Performance Monitoring stage:

(For a system with a time-varying secondary path)

- Initialize $Ex(0) = Ex1$ and $Ee(0) = Ee1$

6. Calculate the mean noise power $Ee(n)$ and mean reference signal power $Ex(n)$ iteratively:

$$Ee(n) = \frac{n+N-1}{n+N} Ee(n-1) + \frac{1}{n+N} e^2(n)$$

$$Ex(n) = \frac{n+N-1}{n+N} Ex(n-1) + \frac{1}{n+N} x^2(n)$$

7. If $Ee(n)/Ex(n) > c'Ee(n-N)/Ex(n-N)$, or $Ee(n)/Ex(n) > c'Ee1/Ex1$ go to step 2 and redo the direction search; otherwise, go to step 5 and update.

If the secondary path is stationary, then the performance monitoring stage is not needed. Also, the measurement of the reference signal mean power could be eliminated if the reference noise is wide sense stationary. This method can also be applied to narrow-band noise, or even broad-band noise, if at that band the secondary path phase response satisfies

$$-\frac{\pi}{2} + k\pi < \angle S_{\omega} < \frac{\pi}{2} + k\pi \quad k \text{ an arbitrary integer} \qquad (13)$$

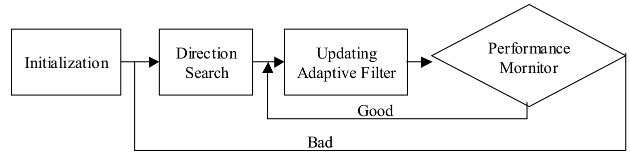

Figure. 4. Diagram of proposed algorithm for single frequency noise

4. BROADBAND ANC WITHOUT SECONDARY PATH IDENTIFICATION

The algorithm for single frequency noise without the secondary path identification has some application in practice. However, when the noise to be cancelled is broad-band noise, or when the secondary path phase response doesn't satisfy Eq. (13), then the above proposed method can't be applied. In this case, we can employ the method introduced in [10] or [11] to divide the broad-band signal into a narrow-band signal, and try to make each sub-band signal meet the condition in (13). Then we can apply the method discussed above in Section 3 to each sub-band.

The derived sub-band implementation of ANC for broadband noise without identifying the secondary path follows:

1. Sub-band analysis of the reference and error signals as in [10, 11].
2. Determine the appropriate update direction. To avoid sub-band interference, we only determine one sub-band at a time. Thus, we only update the coefficients in one sub-band when using Morgan's configuration [10], or we update the adaptive filter coefficients based on one sub-band reference and error signal in DeBrunner's configuration [11].
3. Update the adaptive filter while monitoring system performance exactly as described in Section 3. If the performance deteriorated, we would need to redo step 2.

The block diagram of the resulting algorithm that is based on DeBrunner's configuration is shown in Figure 5. The number of sub-bands is the critical factor. If we have prior knowledge of the secondary path phase response, we can choose the filter bank so that the phase response of each sub-band for the secondary path satisfies (13). Otherwise, we can use many sub-bands. However, increasing the number of sub-bands increases the decimation rate in Morgan's configuration with a corresponding convergence lag, while the increase means an additional computational burden in DeBrunner's configuration. Furthermore, because our algorithm needs to find the adaptive filter direction for each sub-band, we will have a corresponding increase in the time spent in the direction searching stage.

To alleviate this concern, we propose an adaptive sub-band selection method. At the sub-band analysis stage, we first guess the number of sub-bands required to do the analysis. Then we determine the appropriate search direction for each sub-band. However if we change the sign of the step size μ, we need to make sure this step size μ reduces the residual noise power. If not, it means the phase response of the secondary path in this sub-band does not satisfy (13), and consequently we need more sub-bands. Thus, we add more sub-bands and search the directions for each newly created sub-band. The flowchart of this proposed algorithm combined with the adaptive sub-band selection is shown in Figure 6.

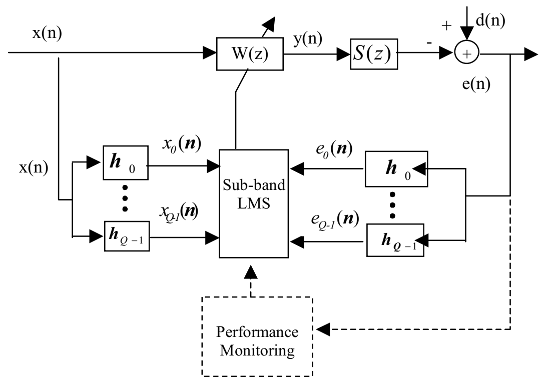

Figure 5. Sub-band implementation of ANC without identifying the secondary path based on DeBrunner's method (The dashed line is the optional performance monitoring stage).

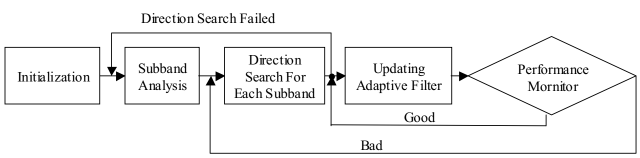

Figure 6. Broadband ANC without secondary path identification with adaptive sub-band selection

5. Simulation Results

We have space for only one simulation example of broadband ANC without identifying the secondary path as in Fig. 5 -- a four sub-band example with the same coefficients as in [12]. The ANC system is sampled at 100 Hz, the main path is modeled by an FIR filter with impulse response [0 0 0 1 -2.7083 4.1861 -3.0451 0.73071]; the secondary path is initiallymodeled by an IIR filter with numerator [0 1 0.96 0.4923] and denominator [1 1.06 0.3352]. At 120s the secondary path changes to an FIR filter model with impulse response [1 0.7 0.3352 -0.2 0.02].

The direction search for each sub-band takes 2 seconds, i.e. 200 samples. The learning curve for an average of 50 runs is shown in Fig. 7. From this figure, we see that our algorithm is robust to the sudden change of the secondary path. The filtered-x LMS algorithm requires online secondary path identification for this situation. However, as the simulations show in [12], most ANC with online secondary path identification will diverge.

6. Conclusion and Discussion

In this paper, we analyzed the filtered-x LMS algorithm and point out the ±90° bound property from a geometrical point of view. With this new insight, we first propose a new ANC algorithm without the secondary path identification for active control of a single frequency noise and certain narrow-band noise. We extend this algorithm to control broadband noise by employing a sub-band implementation of the ANC algorithm. Our proposed algorithms outperform other available algorithms from either convergence rate, or implementation cost, or both.

Compared to the conventional filtered-x LMS, our proposed algorithms require significantly fewer computations. However, as can be seen in Figs. 2 and 3, without the secondary path identification, the adaptive filter converges more slowly. Currently, we are working on a new method to increase the convergence rate of these proposed methods.

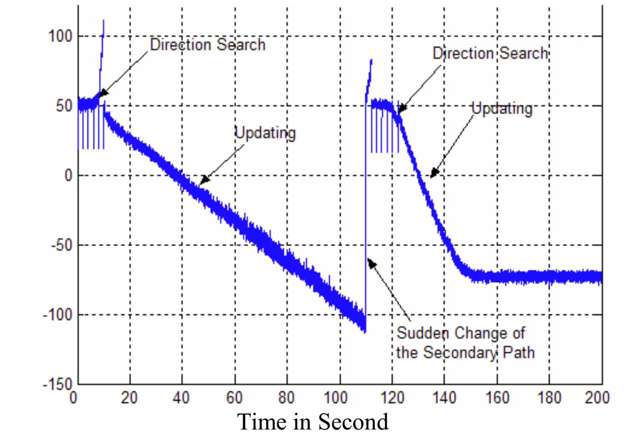

Fig 7. Learning curve for a sudden change of the secodnary path

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