A GENERAL APPROACH TO BLOCK-BASED PHYSICAL MODELING WITH MIXED MODELING STRATEGIES FOR DIGITAL SOUND SYNTHESIS

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ABSTRACT

Block-based physical modeling algorithms for sound synthesis have gained more and more interest in the past few years. They have the advantage, that new models can be created by combining existing blocks and they offer the opportunity to combine different modeling techniques. In this paper we propose a general approach how to create physical models in blocks and how to connect these blocks assuring their correct physical interaction. Furthermore we show how to implement the interconnection of these models with wave digital filter principles. The complete procedure is applied to the example application scenario of the one-dimensional wave equation. Three different modeling techniques are used. One part is implemented with a finite difference time domain approach, one with the functional transformation method, and the boundaries are implemented by wave digital filters.

1. INTRODUCTION

Physical modeling is one of the most promising approach for digital sound synthesis in the last few years. Besides producing good sounds it pays regard to a natural and intuitive interaction between human and sound model, unlike the wide-spread sampling method. In focusing on the sound production mechanism rather than on the sound itself, physical modeling techniques provide more musical expression freedom to the user.

In this scope block-based algorithms become more and more interesting. Quite a number of physical modeling methods are available nowadays (see e.g. [1]). Furthermore, automatic synthesis strategies exist for building virtual musical instruments from a selection of blocks, each based on the physical modeling paradigm [2, 3]. Using these strategies via an intuitive user interface allows the musician to interact with virtual physical entities (strings, membranes, bells, wind instruments, etc.) rather than with technical components (VCO, filters, etc.).

2. BLOCK MODELS AND THEIR PHYSICAL INTERACTION

As a first step towards block-based physical modeling, a partial differential equation description for physical models is considered. The correct physical interaction between blocks is specified in terms of boundary conditions.

2.1. Physical modeling with partial differential equations

Physical models are based on physical laws which themselves are represented by differential equations. At least for linear systems, this set of differential equations can be compiled into a single partial differential equation (PDE) for the scalar output $y(x,t)$ with a certain order of the derivatives in space x and time t.

Alternatively one can also represent the model by a vector PDE, where the space and time-derivatives are only of order one (see [1] for instance). This vector PDE can generally be written as

$$\mathbf{A}\mathbf{y}(x,t) + \mathbf{B}\nabla\mathbf{y}(x,t) + \mathbf{C}\frac{\partial}{\partial t}\mathbf{y}(x,t) = \mathbf{f}_{\mathrm{e}}(x,t)\ . \qquad (1)$$

All elements in (1) are matrices of size $n \times n$, respectively vectors of size $n \times 1$. Higher order systems result in a larger number of dimensions n. $\mathbf{y}(x,t)$ denotes the vector of dependent variables and $\mathbf{f}_{\mathrm{e}}$ is the vector of excitation signals. Depending on the type of problem, $\mathbf{C}$ is a mass or capacitance matrix. The first order spatial derivative ∇ is defined according to the spatial dimensions of $\mathbf{y}(x,t)$. The notation according to (1) will be used in the sequel.

2.2. Interaction by boundary conditions

However, besides the PDE (1), initial conditions (ICs) for the temporal derivatives and boundary conditions (BC) for the spatial derivatives are needed for a unique system description. Especially the BCs are of interest in this scope, as they define the behavior of the model at the intersection to the other block models. Fig. 1 depicts a schematic diagram of two interconnected string models as an illustration.

Both outputs $y_1(x_1,t)$ and $y_2(x_2,t)$ are defined in the regions V_1 resp. V_2 in Fig. 1. The PDEs governing each region may be realized with different models. Assuming a seamless transition from one model to the other, there has to be a function $y(x,t)$ that solves a global PDE on the combined region $V = V_1 \cup V_2 \cup \partial V$.

This implies for the vector model (1) that $\mathbf{y}(x,t)$ is differentiable for $x \in V$, including the boundary $x_{\mathrm{b}} \in \partial V$, i.e. we have to assure that the boundary values at both sides of x_{b} are identical

$$\lim_{x_1 \to x_{\mathrm{b}}} \mathbf{y}_1(x_1,t) = \lim_{x_2 \to x_{\mathrm{b}}} \mathbf{y}_2(x_2,t)$$
$$\Downarrow$$
$$\mathbf{y}_1(x_{\mathrm{b}},t) = \mathbf{y}_2(x_{\mathrm{b}},t)\ . \qquad (2)$$

To fulfill equation (2) one simply has to arrange all elements of the output $\mathbf{y}_1(x_{\mathrm{b}},t)$ resp. $\mathbf{y}_2(x_{\mathrm{b}},t)$ to pairs of port variables. These port variables constitute ports, that only have to be connected to

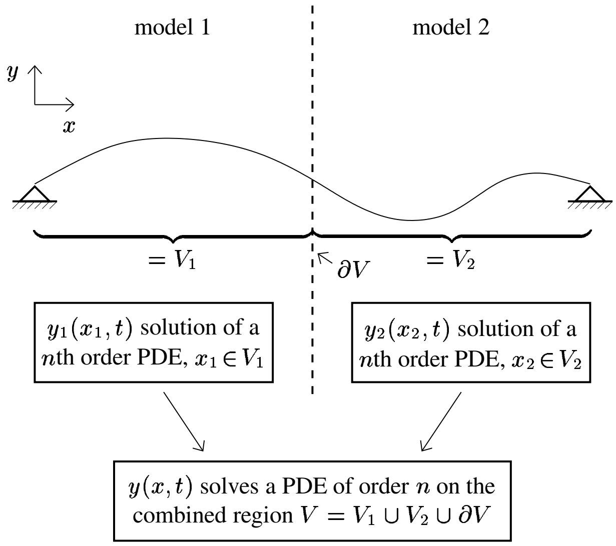

Fig. 1. Two connected string models. $y_1(x_1, t)$ and $y_2(x_2, t)$ are the solutions of nth order PDEs.

the appropriate ports (the ports that share the same boundary region ∂V) of the other block elements. The result is a simple network of multi-port block elements, equivalent to electrical networks.

3. INTERACTION TOPOLOGY

For a discrete solution of this network of block elements, a classical method from network theory is applied in this section, the realization with wave digital filters.

3.1. Discrete realization with wave digital filters

Wave Digital Filters (WDFs) provide an elegant and efficient method to solve continuous networks in the discrete time domain. A detailed description of WDFs can be found in [4]. Their main advantage in this context is the discretization process [5]. The discretization is not performed as a whole, but separately for each network element by the bilinear transformation . Potential computational problems, e.g. delay-free-loops, are avoided by the definition of the so called wave-variables

$$\begin{bmatrix} \mathbf{a}[k] \\ \mathbf{b}[k] \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{R} \\ \mathbf{I} & -\mathbf{R} \end{bmatrix} \begin{bmatrix} \mathbf{y}[k] \\ \mathbf{v}[k] \end{bmatrix} . \tag{3}$$

The vectors $\mathbf{a}[k]$ and $\mathbf{b}[k]$ are called the incident and the reflected wave, respectively. This transition from the "*Kirchhoff*" variables $\mathbf{y}[k]$ and $\mathbf{v}[k]$ (K-variables in the sequel) to the wave variables (W-variables in the sequel) is the key step to a computable system. By a proper choice of the matrix of port resistances $\mathbf{R}$ in (3), one can achieve a delayed response for most common network elements.

WDFs offer the opportunity to separate the design of the block elements from the definition of their interaction by appropriate adaptor elements (see [4]). The implementation of the interaction topology can even be automatized and realized during execution as described in [2], what makes them the method of choice for block-based physical modeling.

3.2. Transition to wave digital filter models

However, a wide range of physical modeling techniques are not based on W-variables. In fact, two of the three physical modeling techniques employed in the example application scenario of section 4 are based on K-variables, incompatible with WDF so far.

To facilitate the interaction of blocks from different modeling paradigms, the adaption to W-variables has been solved for any physical modeling strategy by a method proposed in [6]. It is based on the the general description of discrete systems by state space structures (SSSs). Details on this procedure are not given here, it only has to be mentioned that any discrete system can be represented by a SSS. With slight changes in the structure and with a proper choice of the port resistance matrix, a W-variable output can be added to the SSS. The result is a W-variable compatible model, regardless what physical variables the original model has used.

4. SIMULATING THE WAVE EQUATION WITH MIXED MODELING STRATEGIES

As a classical and widely-used example application, the simulation of the one-dimensional wave-equation is drawn on. Alltogether three quite different modeling techniques are used, that all fit in the proposed approach to block-based physical modeling.

4.1. Model Overview

The one-dimensional wave equation is well known in acoustics and may serve as a simple model for air-columns in pipes and tubes. For a concise notation the velocity potential Φ is introduced, that is related to the pressure p and the sound particle velocity v by

$$\frac{\partial \Phi(x,t)}{\partial x} = \Phi'(x,t) = v(x,t) \ , \tag{4}$$

$$\frac{\partial \Phi(x,t)}{\partial t} = \dot{\Phi}(x,t) = -\frac{1}{\varrho_0}\, p(x,t) \ . \tag{5}$$

Using this quantity, the wave-equation can be written as

$$\Phi'' - \frac{1}{c^2}\ddot{\Phi} = f_{\mathrm{e}}(x,t) \ , \tag{6}$$

where c denotes the speed of sound in the media and $f_{\mathrm{e}}(x,t) = \gamma_0(x - x_{\mathrm{e}}) \cdot f_{\mathrm{e}}(t)$ is an excitation that acts only in the point x_{e}.

Three different implementations are bonded together in the discrete-time domain with the proposed approach as described in section 3. An overview of the complete implementation is given in Fig. 2.

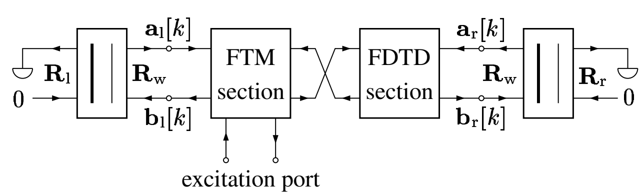

Fig. 2. Connection of two physical identical *string segments* modeled with different methods. Due to the identical port resistances, the adaptor between the string segments simplifies to an exchange of the W-variables. The string implemented with the FTM is excited by an external excitation at x_{e}.

As described in section 2.2 and can be seen in Fig. 2, the input and output of each block is needed in terms of W-variables in order to connect the different blocks. For this purpose, it is necessary to convert the K-variables into W-variables and to define a port resistance R (see [4] for details). The port resistance has the physical meaning of the relation between the port variables. For mechanical waves, mechanical impedance is defined as ratio of force and particle velocity. Here the output variable $\dot{\Phi}(x,t)$ (sound pressure units) and the input variable $\Phi'(x,t)$ (particle velocity units) are used, both related in terms of *responsiveness* [7].

Therefore equation (6) is solved for $\dot{\Phi}(0,t)$ resp. $\dot{\Phi}(l,t)$ and the boundary conditions are defined as

$$\Phi'(0,t) = \Phi_0(t), \qquad \Phi'(l,t) = \Phi_l(t)\ .$$

Here l denotes the length of the air column. Furthermore homogeneous initial conditions are assumed.

4.2. The finite difference time domain block

The Finite Difference Time Domain (FDTD) method is based on a central finite difference scheme of both time and space derivatives of the wave equation. Second order derivatives for the one dimensional case can be expressed as follows [8]

$$\Phi'' \approx \frac{\Phi(k+1,n) - 2\Phi(k,n) + \Phi(k-1,n)}{\Delta x^2}, \tag{7}$$

$$\ddot{\Phi} \approx \frac{\Phi(k,n+1) - 2\Phi(k,n) + \Phi(k,n-1)}{\Delta t^2}\ . \tag{8}$$

In discrete form, time and space positions are denoted as $k = x/\Delta x$ and $n = t/\Delta t$, where Δx and Δt are the temporal and spatial sampling intervals. These sampling intervals must be related by $\Delta x = c\Delta t$ to assure numerical stability and to avoid truncation errors and numerical dispersion [8].

In order to achieve solvable equations, slightly changed PDE and BCs are used

$$\begin{aligned} \text{PDE} \quad &: \quad \rho A\,\dddot{\Phi} - \frac{1}{c^2}\,\dot{\Phi}'' = \dot{f}_\mathrm{e}, \qquad (9) \\ \text{BC} \quad &: \quad \dot{\Phi}'(x,t)\,|_{x=0} = \dot{\Phi}_0'(t), \quad \dot{\Phi}'(x,t)\,|_{x=l} = \dot{\Phi}_l'(t)\ . \end{aligned}$$

In this case, the discrete solution of the one-dimensional wave equation yields

$$\begin{aligned} \dot{\Phi}(k,n+1) = \dot{\Phi}(k-1,n) + \dot{\Phi}(k+1,n) - \dot{\Phi}(k,n-1) \\ + c^2\Delta t(f_e(k,n) - f_e(k,n-1))\ . \end{aligned} \tag{10}$$

The boundary conditions in (9) require the use of first order derivatives of variables. These derivatives are approximated using backward differences

$$\dot{\Phi}' \approx \frac{\dot{\Phi}(k,n) - \dot{\Phi}(k-1,n)}{\Delta x}, \tag{11}$$

$$\ddot{\Phi} \approx \frac{\dot{\Phi}(k,n) - \dot{\Phi}(k,n-1)}{\Delta t}, \tag{12}$$

resulting in the discrete equations

$$\dot{\Phi}(0,n) = \dot{\Phi}(1,n) - \frac{\Delta x}{\Delta t}(\dot{\Phi}_0'(n) - \dot{\Phi}_0'(n-1)) \tag{13}$$

$$\dot{\Phi}(L,n) = \dot{\Phi}(L-1,n) + \frac{\Delta x}{\Delta t}(\dot{\Phi}_l'(n) - \dot{\Phi}_l'(n-1)) \tag{14}$$

where $L = l/\Delta x$.

4.3. The functional transformation method block

The Functional Transformation Method (FTM) solves the PDE with help of suitable time-frequency and space-frequency transformations. Details on the procedure are given in [1], here only the results are presented. A similar application and extensions to the theory can also be found in [9].

The FTM starts directly from the vector PDE (1). The particular equation is

$$\nabla \begin{bmatrix} \dot{\Phi} \\ \Phi' \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ -\frac{1}{c^2} & 0 \end{bmatrix} \frac{\partial}{\partial t} \begin{bmatrix} \dot{\Phi} \\ \Phi' \end{bmatrix} = \begin{bmatrix} 0 \\ -f_\mathrm{e} \end{bmatrix}\ . \tag{15}$$

This equation is solved analytically via a transfer function model yielding the relation

$$\begin{aligned} \dot{\Phi}(x,t) = \frac{1}{l} \sum_{\mu=-\infty}^{\infty} \cos\left(\frac{\mu\pi}{l}x\right) \cdot e^{j\omega_\mu t} * \\ * \left[\cos\left(\frac{\mu\pi}{l}x_\mathrm{e}\right) f_\mathrm{e}(t) + \Phi_0'(t) + (-1)^\mu \Phi_l'(t)\right]\ , \end{aligned} \tag{16}$$

where $\omega_\mu = \mu\frac{c}{l}$ is the frequency of the μ-th harmonic.

A discrete implementation of (16) can be achieved by a set of second order recursive systems, each simulating one specific harmonic of the system, see [1] for details.

4.4. The terminating wave resistances

The simplest parts of the model are the terminating wave resistances. In this example memoryless port resistances are used, that simplify to a non-reflecting WDF, as it is already depicted in Fig. 2.

By adjusting the values of the port resistances it is possible to simulate perfect reflection, negative perfect reflection, zero reflection, and all values in between. The zero reflection scenario can be used to model free field conditions.

4.5. Results

All discrete implementations are adopted to W-variables according to [6], except for the port-resistances which are already in their "native environment". The resulting WDF are plugged together by so called parallel adaptors (see [4]). The adaptor between the FTM and the FDTD segment simplifies to a wave-bridging, as the port resistances of the models have the same value, the speed of sound c.

The speed of sound is set to $300\,\frac{\mathrm{m}}{\mathrm{s}}$. The length of the FTM part is 1m and the FDTD part is 0.5m . The sampling rate is 150,000 Sa/s, resulting in 500 harmonics below the Nyquist frequency for the FTM part and 250 discrete points for the FDTD simulation.

Fig. 3 depicts the simulation of perfect reflection at the right side and simulated free-field conditions at the left side. The lattice indicates the parting line between FTM section (left side) and FDTD section (right side). Both, free-field simulation and model crossover look promising.

To analyze errors, a slightly changed scenario is procured. Both ends of the model are terminated to simulate free-field conditions. Again the simulation looks good (see Fig. 4), however a more in depth examination shows simulation inaccuracies. After hitting the ends of the model (at about t =4.1ms), the impulse ideally should vanish completely, and the model should loose all its energy. However, plotting the output in a logarithmic scale (reference value is the initial amplitude of the impulse) shows non-zero reflections.

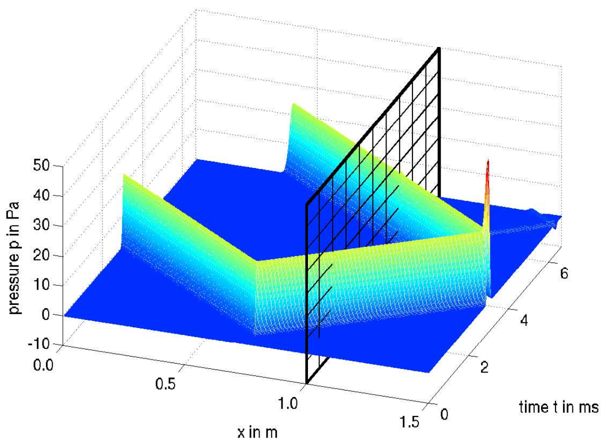

Fig. 3. Simulation of the one-dimensional wave-equation with mixed modeling techniques. The left part is modeled with the FTM, the right part with the FDTD method, and the boundaries with WDFs. The lattice denotes the border. The model was excited with a band-limited impulse.

As it can be seen in Fig. 5, there are large differences in performance between the methods. The FTM-part damps the incident waves by over 60 dB, while the FDTD part only scores 27 dB. One reason is certainly the different size of the simulated region, but there are also principal differences in accuracy. Nevertheless, improvements can always be achieved by a higher number of harmonics resp. simulated points.

5. CONCLUSIONS

This paper has considered the physically correct interaction in a block-based physical modeling environment for digital sound synthesis. The realization of such an environment should not burden the user with interfaces, impedance maching, conversion of variables, and alike. Instead, a high level modeling approach is required, which handles the connection of blocks from different modeling paradigms in a physical meaningful and thus reliable fashion.

It has been shown that a successful connection of different physical models can be achieved if the block outputs comply with the wave digital filter paradigm. It is the key feature of the presented approach that a wave-variable port can be attached also to blocks realized with finite difference methods or functional transformation models.

6. REFERENCES

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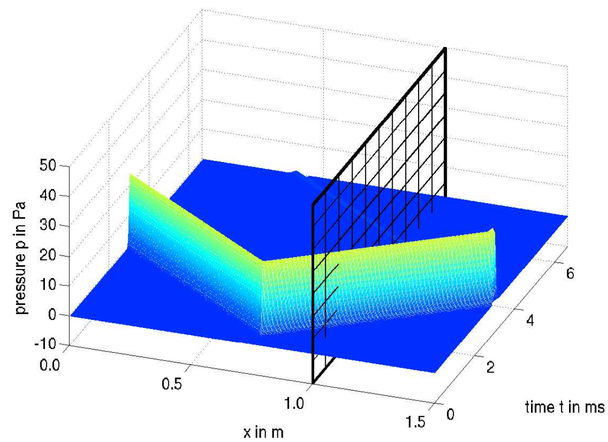

Fig. 4. Same simulation as depicted in Fig. 3 but with a different port resistance at the right boundary to simulate free-field conditions.

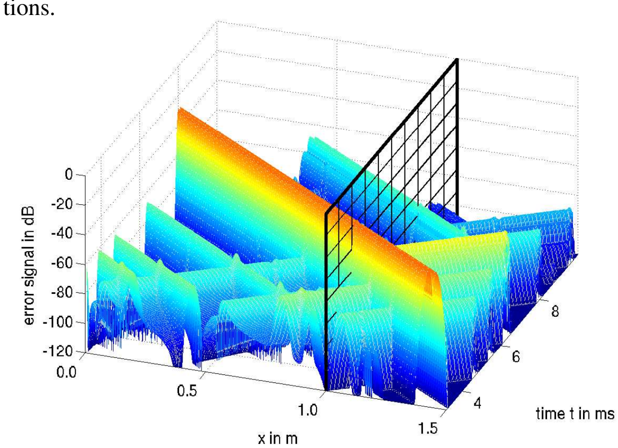

Fig. 5. Error signal after the wanted impulse has left the modeled region (see Fig. 4) plotted in dB. Reference value is the previous amplitude of the impulse.

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