A NEW GRADIENT AND REGION BASED GEOMETRIC SNAKE

Yongsheng Pan, J. Douglas Birdwell, Seddik M. Djouadi

Laboratory of Information Technologies
Department of ECE University of Tennessee
Knoxville, TN, 37996
{ypan, birdwell}@lit.net djouadi@ece.utk.edu

ABSTRACT

In this paper, a new geometric snake model is proposed, based upon techniques of curve evolution and the utilization of gradient information and region information simultaneously. This model successfully solves the boundary leakage problem. With the help of a hierarchical approach, it can handle complicated cases, such as triple junctions. Furthermore, it supports vector-valued images and can be easily extended to handle color and textured images. Experimental results demonstrate the model's power in image segmentation.

1. INTRODUCTION

Active contour models are widely used in image segmentation problems. They evolve one or more curves in an image, subject to constraints derived from the image and the evolving curve(s), to detect objects in the image by stopping the curve(s) at boundaries between objects. These models are usually implemented using variational methods, with partial differential equation solution techniques and level set methods.

The geometric model of active contours, proposed by Caselles *et al.* [1] and Malladi *et al.* [2], evolves curves in a Eulerian formulation and is implemented via level set algorithms [3]. This model, compared to the classic parametric model put forward by Kass *et al.* [4], has several advantages, mainly the capacity to automatically handle topological changes and detect multiple objects simutaneously.

In these models, gradient information is usually used to stop a curve at boundaries of objects [5]. Although this works in many cases, the method allows boundary leakage when image gradients are not very high. Siddiqi *et al.* [6], provides mathematical derivations of length and areas minimizing flows and combines gradient and area information for shape segmentation. Their methods have good performance but still need improvement, becasue boundary leakage still exists. Yezzi *et al.* [7] propose a fully global gradient flow for image segmentation using coupled curve evolution equations. This model provides very good performance for bi-modal and tri-model images. However, this model requires *a priori* knowledge of the number of regions in the image. Furthermore, when multiple objects (more than 3) exist in the image, the implementation of this method becomes very complicated.

In the papers of Chan and Vese [8,9], region information is utilized for image segmentation. In their methods, the segmentation problem is formulated as an energy minimization of a Mumford-Shah based minimal partition problem and implemented using level set methods. This model provides very good performance. Tsai *et al.* [10] implemented this model using curve evolution methods and extended it to image noise reduction, interpolation and magnification, besides segmentation. However, in this model, gradient information is not utilized.

Xie *et al.* [11] proposed a region-aided geometric snake model, which also makes use of the combination of gradient and region information. The method resolves the boundary leakage problem by using the region information. In the first step of its implementation, a region growing method is applied on the image. The capture range of the results is then extended by using a gradient vector diffusion method. In the last step, the image is segmented using an improved curve evolution model. However, it seems difficult to deal with textured images by applying this method together with the tensor measure.

In this paper, a new geometric snake model is proposed, which uses both the gradient and region information for image segmentation. This model is able to segment objects where weak edges exist with no boundary leakage problem. A hierarchical scheme for multi-modal images is also proposed, which helps to segment multi-modal images without prior knowledge of the number of regions. It is easily exended to color and textured images. Complicated cases, such as images with triple juctions, are solved using this model.

The paper is organized as follows. In section 2, the weighted gradient flows for both length and area are introduced. The method in [6] is discussed and the proposed model is explained in section 3. In section 4, experimental results are shown and analyzed. This is followed by section 5, in which conclusions and possible directions for future work are discussed.

2. LENGTH AND AREA MINIMIZING FLOWS

In this section, a concise introduction of typical length and area minimizing flows is given. For more details of mathematical derivation, refer to [6].

Let $C = C(p,t)$ be a family of smooth closed planar curves, where t represents the time and p the curve parameter. The parameter p takes values in $[0,1]$. Assume $C(0,t) = C(1,t)$ and $C_p(0,t) = C_p(1,t)$, restricting the family to closed curves. Let $\mathcal{N}$ represent the inward unit normal, κ the Euclidean curvature and s the Euclidean arc length, which satisfies $ds = \|\partial C/\partial p\| dp$. By selecting different functionals, several minimizing flows are defined.

2.1. Weighted Length Minimizing Flow

When the standard Euclidean metric is replaced by a conformal metric $ds_\phi^2 = \phi^2(dx^2 + dy^2)$, and the weighted length functional is defined using this metric as

$$L_\phi(t) = \int_0^1 \| \frac{\partial C}{\partial p} \| \phi dp \qquad (1)$$

then the minimization of this functional leads to the following flow

$$C_t = \{\phi\kappa - \bigtriangledown\phi \cdot \mathcal{N}\}\mathcal{N} \qquad (2)$$

The doublet in the right part of the equation, together with the weight function ϕ, attracts the curve to features of interest. When ϕ equals 1, the flow shrinks any simple closed curves into a point.

2.2. Weighted Area Minimizing Flow

Similarly, consider area in the conformal metric and minimize the modified area functional

$$A_\phi(t) = -\frac{1}{2}\int_0^{L(t)} \phi\langle C, \mathcal{N}\rangle ds \qquad (3)$$

We get the weighted area minimizing flow

$$C_t = \{\phi + \frac{1}{2}\langle C, \bigtriangledown\phi\rangle\}\mathcal{N} \qquad (4)$$

In this case, the weight function and the doublet term attract the curve into boundaries by minizing the area inside the curve. When the weight function is set to 1, the flow changes to a simple area minimizing flow.

3. GEOMETRIC SNAKE COMBINING GRADIENT AND REGION INFORMATION

In this section, a geometric snake model is given, which improves upon the method in [6]. This is first described for bi-model images, and then extended to multi-modal images using a hierarchical approach. A final extension to vector-valued images, i.e. color and textured images, is then developed.

3.1. Model Description

The geometric snake model proposed in [6] makes use of the gradient and area information by using a combination of weighted length and area minimizing flow. It has the following form:

$$C_t = \alpha\{\phi\kappa - \bigtriangledown\phi \cdot \mathcal{N}\}\mathcal{N} + \{\phi + \frac{1}{2}\langle C, \bigtriangledown\phi\rangle\}\mathcal{N} \qquad (5)$$

In this model, the doublets and the weight function are designed to stop the evolving curve at the boundaries of interest. The weight function ϕ is usually selected as $\phi = 1/\{1 + \| \bigtriangledown G_\sigma * I\|^n\}$. Note that this is a function of gradient and is independent of the evolving curve.

However, as pointed out in [8], the minimal length of a closed curve and the minimal area inside it are correlated by the isoperimetric inequality, i.e.

$$Area(\textit{inside}((C))) \leq c \cdot (Length((C)))^{N/(N-1)}$$

where $N > 1$ is an arbitrary dimension, N = 2 for planar images, and c is a constant depending only on N.

This correlation shows that the minimization of the curve length and that of the area inside the curve are somewhat equivalent. Region information is not fully utilized in this way. This may explain why the method in [6], although it improves upon prior models, still exhibits boundary leakage.

To make use of region information in our proposed model, different weight functions are chosen for the length minimizing flow and area minimizing flow. The weight function ϕ_l for length minimization is selected as before to make use of gradient information. The weight function ϕ_r for region minimization, similar to [8], is chosen to minimize the following region functional:

$$\begin{aligned} R_{\phi_r}(t) &= \iint \phi_r dxdy \\ &= \int_{inside(C)} |I(x,y) - c_1|^2 dxdy \\ &+ \int_{outside(C)} |I(x,y) - c_2|^2 dxdy \end{aligned}$$

Here, ϕ_r is selected as a function of the evolving curve to utilize the region information. The level set function can be directly acquired using the Mumford-Shah functional (see [8] for details). Thus the complete level set equation is obtained:

$$\psi_t = \alpha\phi_l(v+\epsilon\kappa)|\bigtriangledown\psi|+(1-\alpha)\delta_\beta(\psi)[(I-c_2)^2-(I-c_1)^2] \quad (6)$$

The first term in the right part of (6) is a combination of the constant motion and curvature motion. In this term, v is the inflationary term, which attracts the curve in one direction: either expanding or shrinking; κ represents the curvature, which keeps the curve smooth when evolving. The doublets are not used in this model. The second term in the right part of (6) makes use of the region information, which helps to stop the curve at the boundaries of interest. In this term, $\delta_\beta(x) = \beta/(\pi(x^2+\beta^2))$ acts like a delta function. α, β and ϵ are positive constant coefficients. α lies between 0 and 1 and determines the weights of the gradient and the region information. When α is set to 1, the model changes to the length and area minimizing flow, as in [6]. When α lies between 0 and 1 and v is set to 0, it becomes similar to the model in [8]. α is usually set to 0.5. For those images where weak edges exist, α is set to be less than 0.5 to increase the weight of the region information.

This proposed model combines gradient and region information. It can be seen to be an extension of the method in [6] by utilizing the region information. It can also be seen as the refinement of the method in [8] by introducing the gradient information.

3.2. Hierarchical Approach

The way to utilize the region information shown above is particularly suitable for a bi-model image. To deal with images containing multiple objects, a hierarchical approach is proposed, which is similar to as the one in [10]. In this approach, an image is segmented into two subimages, and then the method is applied on these subimages respectively. The procedure is implemented iteratively until each region contains only one object.

3.3. Extension to Vector-Valued Images

The proposed method is easy to extend to vector-valued images. For a vector-valued image $I : \mathcal{R}^2 \rightarrow \mathcal{R}^m$, the length weight function is selected as:

$$\phi_l = \frac{1}{m}\sum_{i=1}^{m}\frac{1}{1+\|\bigtriangledown G_\sigma * I_i\|^n}$$

Accordingly, the region weight function is selected so that the level set function takes the following form:

$$\psi_t = \alpha\phi_l(v+\epsilon\kappa)|\bigtriangledown\psi| + (1-\alpha)\delta_\beta(\psi)\frac{1}{m}[\sum_{i=1}^{m}(I_i-c_{2i})^2-(I_i-c_{1i})^2] \quad (7)$$

For color images, three channels of information can be directly used in the above formulae. For textured images, preprocessing techniques, such as wavelet or Gabor transforms, can be performed, and the resulting vector-valued images are applied to this model.

4. EXPERIMENTAL RESULTS

Experimental results from the proposed model are given in this section. The proposed model is implemented using level set methods [3]. In the implementation, the maximal value of the region part is normalized to lie between -1 and 1 so that it is comparable with that of the gradient part.

Fig. 1 illustrates the role of the region term in (6). The parameters v and ϵ are set to expand the curve. With the effects of the region information, however, the left part of the curve shrinks and the right part expands, as can be seen clearly from (b) and (c). The curve finally stops at the correct boundary in (d).

In Fig. 2, comparisons between the proposed method and the ones in [6] and [8] are given when $alpha$ is set to be 0.1. The original image (a) is generated by smoothing an ideal circle using a 13-by-13 Gaussian filter. It can be seen that our method (b) and the method in [8] (c) correctly segment out the object in the image. The results from (b) and (c) show no big differences. In comparison, the method in [6] (d) allows the boundary leakage. This example shows that our model performs very well for weak-edge images.

The results of Fig. 3 shows that our model (with $\alpha = 0.2$) works well for multi-modal images. One point for this case is that our model prefers the image boundary for the initial curve.

In Fig. 4, a color image with a triple junction is processed. Using the hierarchial approach, three parts in the image are segmented sequentially; see (b-d) for details. This example shows that the proposed method can deal with complicated cases successfully.

Fig. 5 shows the segmentation results of a real image (a). Images (c) and (d) show the results of the proposed method and the method in [8], both with the initial contour shown in image (b). It can be clearly seen that our results are perceptually better than (d).

5. CONCLUSIONS AND FUTURE RESEARCH

In this paper, a geometric snake model is proposed, which utilizes both the gradient and the region information for curve evolution. It sucessfully solves the boundary leakage problem. Complicated cases, such as multi-modal images, real color images and images with triple junctions, can be successfully processed. Experimental results shows that the model can be a powerful tool for image segmentation.

6. REFERENCES

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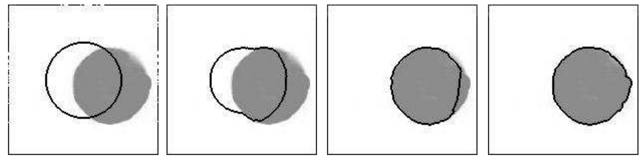
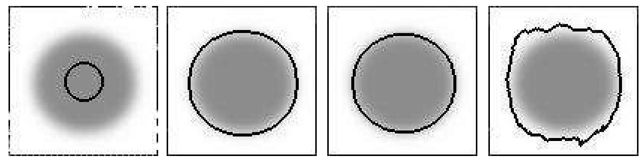


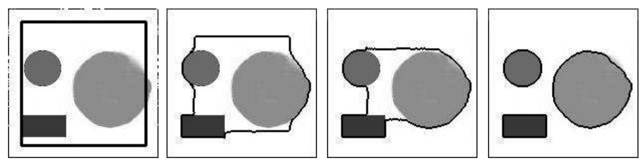
(a) (b) (c) (d)

Fig. 1. Illustration of region information in the model.



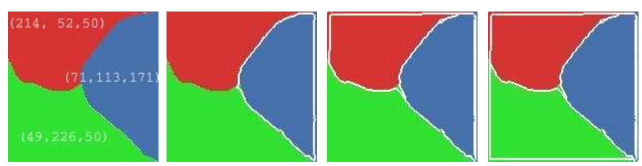
(a) (b) (c) (d)

Fig. 2. The boundary leakage problem



(a) (b) (c) (d)

Fig. 3. Segmentation of multiple objects in an image



(a) (b) (c) (d)

Fig. 4. Segmentation of images with triple junctions

(a) (b) (c) (d)

Fig. 5. Segmentation of real images