RECURSIVE DECONVOLUTION OF MULTISENSOR IMAGERY USING FINITE MIXTURE DISTRIBUTIONS

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ABSTRACT

In this paper, the case where multiple degraded (blurred and noisy) acquisitions of the same scene are available, is investigated. An efficient iterative deconvolution system is introduced, where the multisensor images are fused, based on the classification of each image in a predetermined number of classes that represent the components of a finite mixture of normal densities (FMN). The EM algorithm is utilized for the learning of the FMN model. The recursive employment of the classification and fusion processes, followed by an optimized adaptive filtering, converges to a global enhanced version of the original scene in only a few iterations. Experimental results establish the efficiency of the proposed scheme.

1. INTRODUCTION

Processing of data from different sensors has been efficiently used in satellite remote sensing, in computer vision and medical applications, for a better understanding of the given situation. Particularly in the case where multiple acquisitions of the same scene are available, e.g. in magnetic resonance imaging (MRI) where multiple fast scans of the same organ may be obtained, restoration is frequently performed on the degraded data that exhibits motion-induced blurring or defocusing effects [1, 2].

In this paper, an efficient iterative deconvolution technique for restoration of such multi-source images is proposed. Specifically, M filtered outputs of the initially distorted images are, first, modelled using a *finite mixture of normal* (FMN) densities [3]. The proposed algorithm attempts to derive for each image, optimal groupings of the observations (image pixels) into K classes, by assuming a K-component normal mixture model for the overall distribution of the data (for any pixel x):

$$f_m(x|\mathbf{\Psi_m}) = \sum_{k=1}^{K} \pi_{m,k} f_{m,k}(x|\theta_{\mathbf{m,k}}), \quad m = 1, \ldots, M \qquad (1)$$

where $\theta_{\mathbf{m}} = (\mu_{m,1}, \ldots, \mu_{m,K}, \sigma^2_{m,1}, \ldots, \sigma^2_{m,K})^T$ contains the distinct unknown mean and variance values of these K Gaussian distributions and $\mathbf{\Psi_m} = (\theta_{\mathbf{m}}{}^T, \pi_{m,1}, \ldots, \pi_{m,K})$ denotes all the unknown parameters, for image m. The vector $(\pi_{m,1}, \ldots, \pi_{m,K})$ represents the mixture proportions, where $\sum_{k=1}^{K} \pi_{m,k} = 1$, for each image (they can be interpreted as the prior probabilities of the pixel classes). Essentially, the problem becomes that of fitting the FMN model to the image histogram (“true” distribution). The k-th normal component density of the mixture, can be described by the gaussian kernel:

$$f_{m,k}(x|\theta_{\mathbf{m,k}}) = \frac{1}{\sqrt{2\pi}\sigma_{m,k}} exp\{-\frac{(x-\mu_{m,k})^2}{2\sigma^2_{m,k}}\}, \quad m = 1, \ldots, M \qquad (2)$$

For the problem under investigation, the estimation of the gaussian component parameters is performed using the popular *expectation-maximization* (EM) algorithm [4]. Afterwards, the multisensor images are classified using a two-step classification procedure and for each classified image, a measure of the total consistency level (TCL) is calculated [5]), based on the classification of the pixels in some local neighborhood. The TCL values are next injected as the fusing weights, leading to the formation of a fused image. Finally, the error between the fused image and each filtered output is used to control the adaptation of the corresponding FIR filter coefficients. The overall block diagram of the adaptive restoration system can be schematically seen in Fig. 1.

2. DESCRIPTION OF THE ALGORITHM

2.1. FMN modelling and parameter estimation

Let $g_1, g_2, \ldots, g_M$ be M different degraded sensor images of the same scene (assumed registered). Each acquired image g_m has undergone a blurring process according to the linear degradation model [1]:

$$g_m = s_m * h_m + n_m \qquad (3)$$

where g_m is the degraded image from the m-sensor, s_m is the original undistorted m-scan of the scene, h_m is a blurring point-spread function (PSF) and n_m is zero-mean additive noise. The $*$ operator denotes a convolution process.

At the iterative restoration process, each image is initially filtered with u_m, i.e. $f_m = g_m * u_m$. Next, each filtered output f_m is approximated using the finite mixture of normal densities, described in equation (1), thus producing $f_m^{FMN}, \ m = 1, \ldots, M$. The model parameters $(\mu_{m,k}, \sigma^2_{m,k}, \pi_{m,k})$ for each image need, next, to be estimated, for $k = 1, \ldots, K$ (K denotes the number of the classes representing the original scene and it is assumed to be fixed and identical for all degraded images). This estimation can be achieved by maximizing the joint likelihood function[1]:

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[1] It should be, also, noted that asymptotic (weak) independence of the image pixels has been assumed.

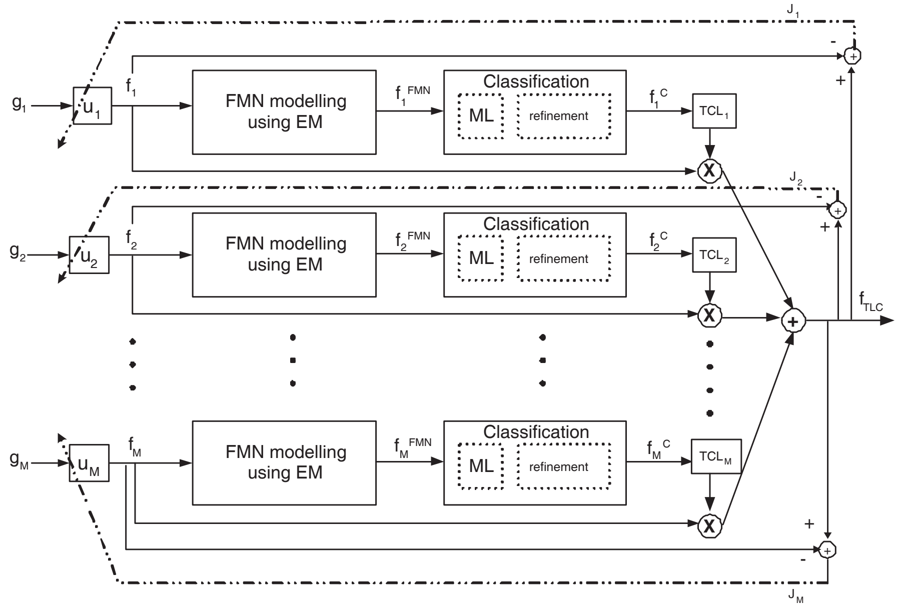


Figure 1: Block diagram of the proposed recursive restoration technique.

$$L_m(\mathbf{\Psi_m}) = \prod_{i=1}^{N} \left[\sum_{k=1}^{K} \pi_{m,k} f_{m,k}(x_i|\theta_{\mathbf{m,k}}) \right], \quad m = 1, \ldots, M \tag{4}$$

where N denotes the number of the pixels of the image f_m^{FMN}. Numerical estimation of the gaussian mixture parameters is performed using the popular *expectation-maximization* (EM) algorithm [4]. The final updating rules for the ML estimates of the FMN model (the mean, variance values and the mixing factors of the clusters) for $m = 1, \ldots, M$, are formulated as follows:

$$\mu_{m,k}^{r+1} = \sum_{i=1}^{N} x_i p(\theta_{\mathbf{m,k}}^{\mathbf{r}}|x_i) / \sum_{i=1}^{N} p(\theta_{\mathbf{m,k}}^{\mathbf{r}}|x_i)$$

$$var_{m,k}^{r+1} = \sum_{i=1}^{N} p(\theta_{\mathbf{m,k}}^{\mathbf{r}}|x_i)(x_i - \mu_{m,k}^{r+1})(x_i - \mu_{m,k}^{r+1})^T / \sum_{i=1}^{N} p(\theta_{\mathbf{m,k}}^{\mathbf{r}}|x_i)$$

$$\pi_{m,k}^{r+1} = \frac{1}{N} \sum_{i=1}^{N} p(\theta_{\mathbf{m,k}}^{\mathbf{r}}|x_i), \qquad k = 1, \ldots, K \tag{5}$$

where $var_{m,k}$=$\sigma_{m,k}^2$, the current iteration is denoted by r and $p(\theta_{\mathbf{m,k}}|x_i)$ represents the posterior probability, described by the Bayes rule[2]. Initial values $\mu_{m,k}^0, var_{m,k}^0$ and $\pi_{m,k}^0$ are approximated using the K-means algorithm.

2.2. Two-step image classification and fusion

Given the estimated FMN parameters, the process of *classification* involves the assignment of each pixel of the images $f_m, \ m = 1, \ldots, M$, to the appropriate k-th component of the gaussian mixture model, for $k = 1, \ldots, K$. The goal of this classification is, essentially, the derivation of the appropriate fusing weights that will be used to effectively fuse the filtered outputs and produce an improved global image.

(i) A single-pass (soft) maximum-likelihood (ML) classification is performed, where each image pixel x_i is assigned to the k-th gaussian component with the highest individual likelihood. Equivalently, based on (2), each pixel x_i is assigned a label (class) k, such that: $log(\sigma_{m,k}) + (x_i - \mu_{m,k})^2/2\sigma_{m,k}^2$ yields its minimum value for $k = 1, \ldots, K$.

(ii) A stochastic refinement follows afterwards, where each pixel x_i is randomly visited and is classified in class k which minimizes

$$log(\sigma_{m,k}) - log p_m(c_i = k|nbh_i) + (x_i - \mu_{m,k})^2/2\sigma_{m,k}^2 \tag{6}$$

where $p_m(c_i = k|nbh_i)$ is analogous to the cluster prior probability $\pi_{m,k}$. However it has a local nature and it can be considered as the conditional prior of the class c_i of a pixel x_i, given the classification of its neighboring pixels nbh_i (e.g. 3x3). In fact, this conditional probability can be simply described by the proportion of the pixels in this neighborhood, that have been assigned to the same class as x_i. The above minimization resulted from attempting to maximize the joint likelihood $p_{m,k}(x_i, c_i|nbh_i)$[3] of a pixel x_i and its class label c_i, conditioned by the local classification information nbh_i. After convergence to a steady classification point, the final classified images $f_m^c, \ m = 1, \ldots, M$ are generated.

In the subsequent fusion of the filtered images $f_m, m = 1, \ldots, M$, the more salient features are expected to be extracted, in order to produce a more regularized global result, consistent to the normal mixture model of the original scene. In order to generate appropriate fusing weights, measures of the total consistency level TCL_m are calculated for each classified image f_m^c [5], such that images with higher such values will affect the final formation of a global

[2] $p(\theta_{\mathbf{m,k}}^{\mathbf{r}}|x_i) = \frac{f_{m,k}(x_i|\theta_{\mathbf{m,k}}^{\mathbf{r}})\pi_{m,k}^r}{f_m(x_i|\mathbf{\Psi_{m,k}^r})}$

[3] $p_{m,k}(x_i, c_i|nbh_i) = p_m(c_i = k|nbh_i) f_{m,k}(x_i|\theta_{\mathbf{m,k}})$

result to a greater extent. Specifically, the total consistency level for image f_m^c is described by:

$$TCL_m = \sum_{i=1}^{N} \left[\sum_{k=1}^{K} \delta(c_i, k) p_m(c_i = k | nbh_i) f_{m,k}(x_i | \theta_{\mathbf{m,k}}) \right] \tag{7}$$

where $\delta(c_i, k)$ is the *Kronecker delta*. This formula is local in nature, but gives -overall- a global assessment for the consistency of the entire image classification, by summing over all pixels. For these TCL metrics, the aforementioned joint likelihoods are maximized. Finally, the fused image f_{TCL} is formed as a weighted sum:

$$f_{TCL} = \frac{\sum_{m=1}^{M} TCL_m \cdot f_m}{\sum_{m=1}^{M} TCL_m} \tag{8}$$

3. OPTIMAL FILTER ADAPTATION

The last step of the proposed algorithm involves the adaptation of each filter u_m, of size N_uxN_u, that will be fed back into the recursive deconvolution system. For this reason, the cost function J_m used in the restoration process, is defined as:

$$J_m = \sum_{x_i} (f_{TCL}(x_i) - f_m(x_i))^2, \quad m = 1, \ldots, M \tag{9}$$

for all the pixels x_i. Taking into account that $f_m = g_m * u_m$, equation (1) can be written as follows:

$$f_m(x_i) = \sum_{k=1}^{K} \pi_{m,k} \{u_m(x_i) * g_{m,k}(x_i | \theta_{\mathbf{m,k}})\} \tag{10}$$

where $g_{m,k}$ denotes the k-th normal component of the mixture approximation for the degraded image prior to filtering (note also that the term $\mathbf{\Psi_m}$ has been dropped for convenience of notation). Similarly, by combining (1), (8) and the convolution equation, the fused image $f_{TCL}(x_i)$ can be expressed by the following formula:

$$f_{TCL}(x_i) = \sum_{s=1}^{M} w_s \left[\sum_{k=1}^{K} \pi_{s,k} \{u_s(x_i) * g_{s,k}(x_i | \theta_{\mathbf{s,k}})\} \right] \tag{11}$$

where w_s denotes the normalized TCL_s fusing weights, such that $\sum_{s=1}^{M} w_s = 1$.

A steepest descent minimization routine is, next, adopted:

$$u_m^{t+1}(l) = u_m^t(l) - d \cdot \nabla J_m[u_m(l)] \tag{12}$$

where $\nabla J_m[u_m(l)]$ denotes the gradient of the cost function J_m with respect to the filter coefficients $u_m(l)$ and $d > 0$ is an appropriate update step-size. The recursive law for the coefficients $u_m(l)$ of each filter is, therefore, formulated as:

$$\begin{aligned} u_m^{t+1}(l) \quad &= \quad u_m^t(l) - 2d \sum_{x_i} [(f_{TCL}(x_i) - f_m(x_i)) \\ &\cdot \quad \left(\sum_{s=1}^{M} \sum_{k=1}^{K} w_s \pi_{s,k} g_{s,k}(x_i - l | \theta_{\mathbf{s,k}}) \right. \\ &- \quad \left. \left. \sum_{k=1}^{K} \pi_{m,k} g_{m,k}(x_i - l | \theta_{\mathbf{m,k}}) \right) \right], \; m = 1, \ldots, M \end{aligned} \tag{13}$$

where $t \geq 0$ represents the current iteration and the filters are initialized with a value of unity in the center of the FIR window

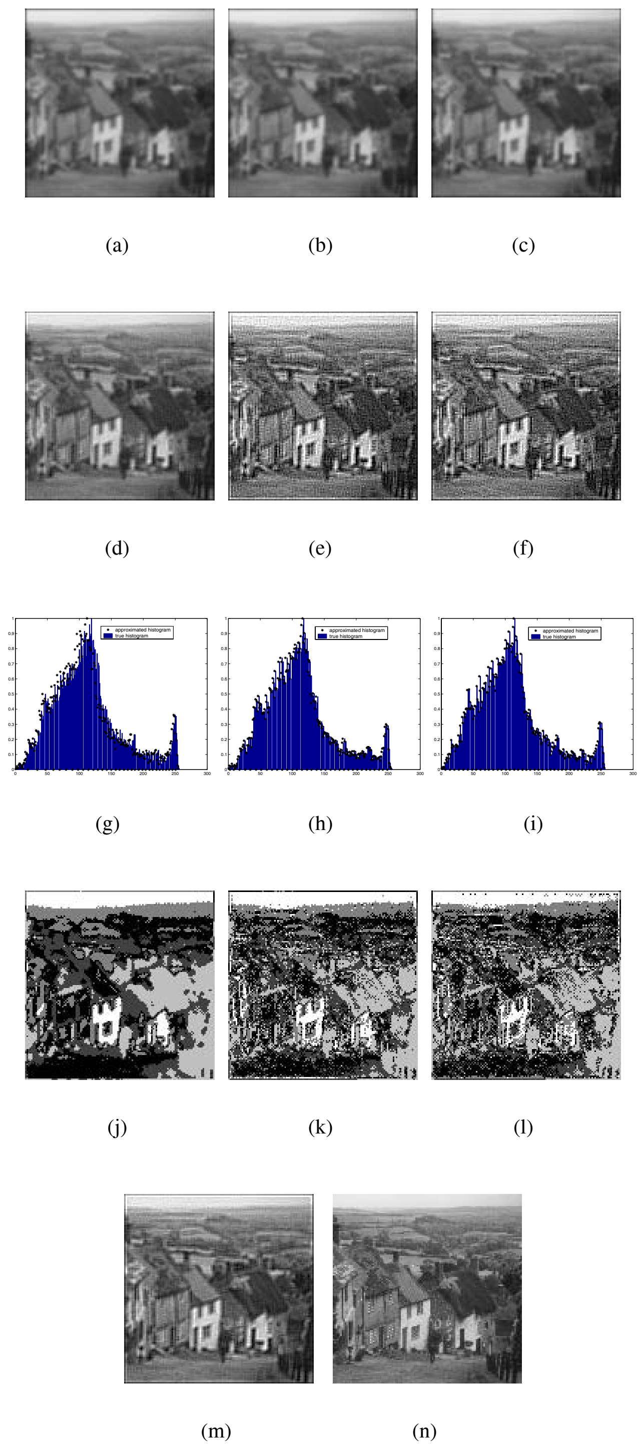
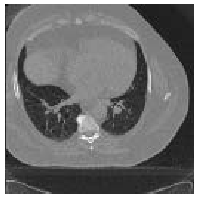
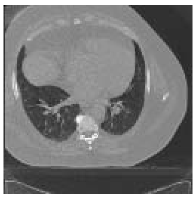


Figure 2: (a)-(c) Degraded images, (d)-(f) individual restorations after 5 iterations, (g)-(i) true and approximated histograms using the FMN model (shown with solid bars and the star ($*$) symbol, correspondingly), (j)-(l) classified images, (m) enhanced fused image at the 5-th iteration and (n) undistorted image.

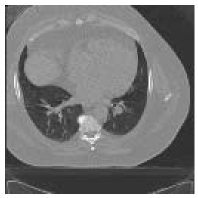
(a)



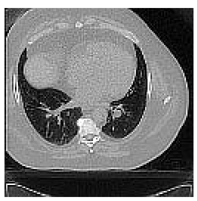
(b)



(c)



(d)



(e)

Figure 3: (a)-(d) Real degraded MR images and (e) the enhanced fused version of the organ after 6 iterations.

and zero elsewhere (discrete unit impulse). Convergence of the algorithm can be achieved using several numerical optimization techniques [6]. Alternatively, the conjugate gradient method can be, also, implemented for faster convergence. However, due to the normalization of $\{u_m\}$ at every iteration in order for the filter coefficients to sum up to unity, conjugate gradient minimization is computationally inefficient for such a method of constraint[4].

4. EXPERIMENTAL RESULTS

In this section, various experiments were performed to illustrate the efficiency of the proposed restoration scheme, both with synthetic and real data. To simulate a multiframe environment, the 128 x 128 grayscale *goldhill* image, seen in Fig. 2(n), was bilinearly interpolated to get a 256 x 256 image, which was next blurred using a 7 x 7 Gaussian PSF and white Gaussian noise was added, producing a blurred signal-to-noise ratio (BSNR) approximately equal to 60 dB. The degraded image was, then, downsampled so as to generate 4 lower-resolution images of size 128 x 128. Three of these degraded images were finally utilized, seen in Fig. 2(a)-(c), where each one was subject to different blurring, noise and warping (constant shift) artifacts.

[4]Convexity and uniqueness of the minimum have been shown in [2].

A 5-component mixture of normal densities ($K = 5$ classes) was assumed, as it was empirically shown that it approximated reliably the true histogram of these images (see Fig. 2(g)-(i)) and produced the best restoration results. The proposed recursive method converged to very good estimates of the solution in only 5 iterations. Termination of the algorithm was decided when excessive noise amplification started being observed visually. The individual restorations at the 5-th iteration are shown in Fig. 2(d)-(f), correspondingly, while the finally enhanced fused image can be seen in Fig. 2(m), exhibiting evident sharpening results. Regarding the classification process after convergence of the EM algorithm, the local-based refinement following the initial rough ML classification, removed most of the misclassifications of the first pass and converged at its second pass, such that no further pixels needed to update their class labels (see Fig. 2(j)-(l)). Finally, the three FIR 3 x 3 filters were recursively updated according to the optimization analysis of Section 3, simulated by a steepest-descent routine (an appropriate step-size of $d = 0.0003$ was set).

Restoration results were also obtained for four real MR scanned images, shown in Fig. 3(a)-(d). A choice of $K = 2$ was adopted and d was set equal to 0.001. The restoration algorithm converged after 6 iterations, generating the fused image of Fig. 3(e), where the sharpened details can be easily observed (note that the image histograms and the corresponding individual restorations are not shown here due to lack of space).

Finally, it should be noted that a wrong model assumption (inappropriate choice of K) may render the algorithm instable and divergent. Future work involves elaboration on automatic methods for the adoption of the most accurate FMN model.

5. CONCLUSIONS

An efficient adaptive fusion-and-filtering technique was introduced in this paper, for improving the quality of an image scene when multiple blurred and noisy acquisitions were available, using finite mixtures of normal densities. An enhanced fused version was generated, based on ML analysis and a two-step image classification. The adaptation of the filter coefficients was controlled by a steepest descent optimization routine. Both simulated and real experiments demonstrated efficient restoration and sharpening of the degraded multisensor imagery.

6. REFERENCES

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