BLIND DECONVOLUTION USING A MONOTONICITY CONSTRAINT ON THE PSF

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ABSTRACT

It is well recognized that blind deconvolution is a severely ill-posed problem and proper constraints on the image and the system point spread function (PSF) should be applied to counteract the ill-poseness. In this paper we investigate a novel PSF constraint, the monotonicity, which means the value of the PSF monotonically decreases (or not increase) from the center of its support. We regard the monotonicity as a common property of many simplified but well accepted PSF models, such as the geometrical model of defocus, the Gaussian model and the synthetic model in astronomical imaging. The property is utilized as a PSF constraint in a novel iterative blind deconvolution algorithm RL-CLSE. Experiments on real microscopic data show that the proposed constraint can significantly improve the quality and stability of blind deconvolution.

1. INTRODUCTION

In practical imaging applications the recorded image is usually a noisy and blurred version of the original scene, which can be formulated as

$$g = f * d + n, \tag{1}$$

in which g stands for the recorded image, f the ideal 'clear' image, d the system point spread function (PSF), n the additive noise, and '*' the sign of convolution [1]. A lot of image restoration algorithms have been developed to retrieve the clear image with a known PSF [1], [2]. However, in many cases, the PSF is not known *a priori* so that both the clear image and the PSF have to be estimated from the recorded image. Such a problem is called blind deconvolution, one of the most challenging problems in image processing.

It is well recognized that blind deconvolution is severely ill-posed and proper constraints should be applied to get a reasonable and stable solution [3], [4]. Only a few of these constraints can be used both on the image and the PSF, such as the nonnegativity; while more of them are especially for the PSF, because the PSF is physically determined by the system and much less diversified than the image. For example, it is often assumed that the image degradation process caused by the PSF will not change the mean value of the image; therefore the integral of the PSF against its support region should be a unit [3]. Besides, the PSF should be band limited for diffraction limited imaging systems, and radically symmetric for systems with circular lens and apertures [5]. In addition, the PSF are sometimes assumed to be partially known and constrained to be the sum of a known deterministic part and an unknown random part [6].

In this paper we will address another PSF constraint, the monotonicity, which means that the value of the PSF monotonically decreases (or not increase) from the center of its support. In blind deconvolution, this constraint was first suggested by Holmes in 1992 [5]. To our knowledge, however, no further investigation or algorithm following this suggestion has been reported hitherto. One of the possible reasons, as we presume, might be that it is doubtful whether the practical PSF is really monotonic. It should be admitted that a physically determined PSF can be very complex and far from being monotonic; but in deconvolution applications simplified PSF forms are also widely accepted and many of them are monotonic. For example, the physical model of a defocus PSF has many side lobes, but it can also be geometrically modeled as simple as a uniform disk [7]. It is reported that under low SNR conditions such a monotonic geometrical model can be used in the place of the parameter-sensitive physical counterpart without significant loss in the deconvolution quality [8]. Another example of the simplified monotonic PSF is the Gaussian model, which often serves as the long exposure PSF caused by the unpredictable atmospheric turbulence in ground based astronomical imaging [9]. Moreover, sometimes astronomical images can also be deconvolved with a synthetic monotonic PSF which has a radically symmetric form:

$$\mathrm{PSF}(r) = \left(1 + \frac{r^2}{R^2}\right)^{-\beta}, \tag{2}$$

in which R and β are parameters obtained by fitting the model with some given stars, and r the distance from the original [2].

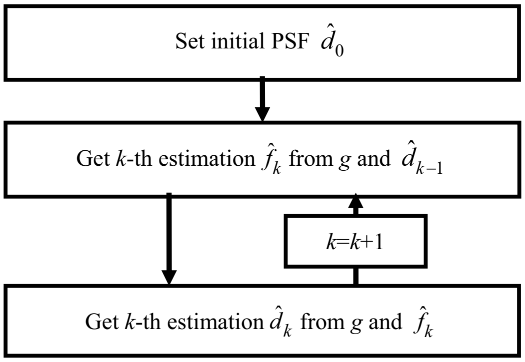


Fig. 1. The IBD framework.

We regard the monotonicity as a common property of those simplified but well accepted PSF models and consider it a reasonable constraint on the PSF in blind deconvolution. In the rest of the paper, Section 2 presents an IBD based algorithm RL-CLSE to apply this constraint and Section 3 tests it with real data. Section 4 gives conclusions and perspectives of the paper.

2. ALGORITHM

Our presented algorithm is based on the iterative blind deconvolution (IBD) method, which is famous for its computational simplicity [3]. The IBD method alternately estimates the image and PSF in each iteration, as shown in Fig. 1. It is first proposed by Ayers and Dainty and later serves as a flexible framework in which various non-blind image restoration methods and PSF estimation methods can be easily integrated [10]-[17].

We employ the Richardson-Lucy (RL) algorithm and a constrained least square error (CLSE) method to restore the image and estimate the PSF respectively. The RL algorithm is formulated as

$$\hat{f}^{(l)}(x,y) = \hat{f}^{(l-1)}(x,y) \times \left[\frac{g(i,j)}{\hat{g}^{(l-1)}(i,j)} * d(-i,-j) \right](x,y), \tag{3}$$

where $\hat{f}^{(l-1)}$, $\hat{f}^{(l)}$ represent the l-1 and l-th estimation of the clear image respectively, $\hat{g}^{(l-1)}$ the convolution of $\hat{f}^{(l-1)}$ and d, and (x,y), (i,j) the discrete pixel index [18], [19]. The RL algorithm is popular in astronomy and medical image restoration for its robustness in the presence of noise and it can also be used in PSF estimation, as many IBD algorithms do [14]-[16]. However, we choose a CLSE method to estimate the PSF. The reason is that CLSE can naturally take the PSF monotonicity as one of its constraints, whereas the RL algorithm cannot. The least square error (LSE) criterion is expressed as

$$\min_{\mathbf{d}} \| \mathbf{g} - \mathbf{Fd} \|_2^2, \tag{4}$$

in which **g**, **d** are lexicographic forms of g, d respectively, and F stands for f in the form of convolution matrix [1]. Suppose a discrete PSF has s elements and they have been incrementally indexed according to their distance to the center, then the PSF monotonicity constraint can be easily expressed as

$$\mathbf{d}(1) \geq \mathbf{d}(2) \geq \ldots \geq \mathbf{d}(s), \tag{5}$$

or in the matrix-vector form:

$$\mathbf{Qd} \geq \mathbf{0}, \tag{6}$$

where

$$\mathbf{Q} = \begin{bmatrix} 1 & -1 & & & \\ & 1 & -1 & & \\ & & \ddots & \ddots & \\ & & & 1 & -1 \end{bmatrix}_{(s-1)\times s} \tag{7}$$

In the CLSE method we also used the nonnegativity and energy conservation constraints, respectively expressed as:

$$\mathbf{d} \geq \mathbf{0}, \tag{8}$$

$$[1,1,\ldots 1]_{s\times 1} \bullet \mathbf{d} = \mathbf{1}. \tag{9}$$

Our algorithm (RL-CLSE) calls the Matlab routine *deconvlucy* to implement the RL algorithm and *quadprog* to solve the constrained quadratic problem CLSE. The initial image for the RL iterations is set as the raw image g, and the initial value of **d** for *quadprog* is set as $\left[1/\mathrm{s}, 1/\mathrm{s}, \ldots 1/\mathrm{s}\right]^T$ which guarantees the satisfaction of (6), (8) and (9). Besides, three parameters should be appointed before the blind deconvolution starts: the initial PSF ($\hat{d}_0$), the iteration number for the IBD framework (*Kmax*) and the iteration number for the RL algorithm (*Lmax*).

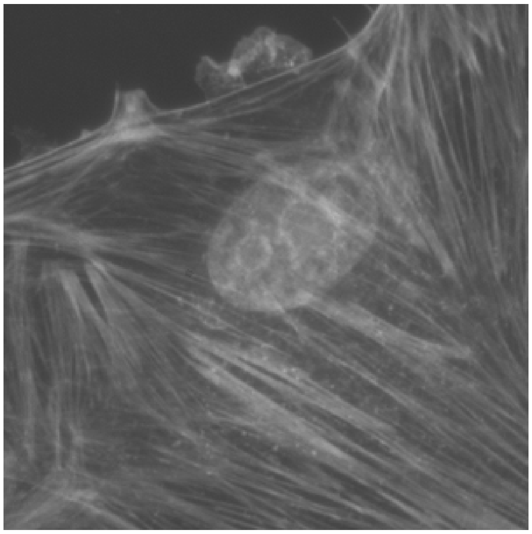
3. RESULTS

As an example, we deconvolved a microscopic image shown in Fig. 2(a). By experience we set $\hat{d}_0$ a 7×7 matrix with each element being 1/49, $Kmax = 10$ and $Lmax = 10$, and obtained Fig. 2(b). To make a comparison, we removed the monotonicity constraint (6) and obtained Fig. 2(c). It is evident that with the PSF monotonicity constraint the deconvolution process improved the quality of the raw image significantly, while without the constraint the restored image had many artifacts and lost important details (such as the small granules).

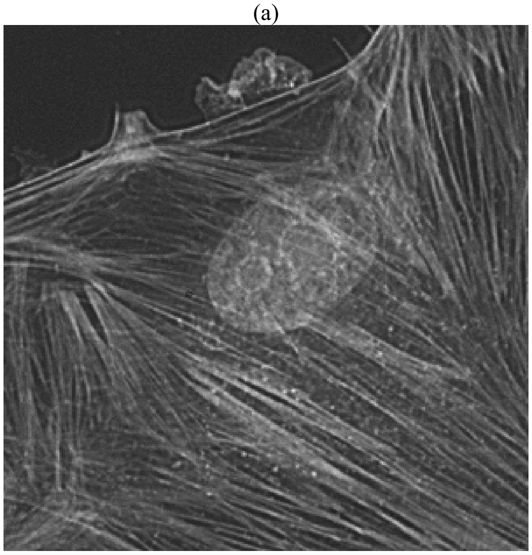
We also investigated the case in which the monotonicity constraint was not strictly enforced. We modified (6) to

$$\mathbf{Qd} \geq -\mathbf{b}, \tag{10}$$

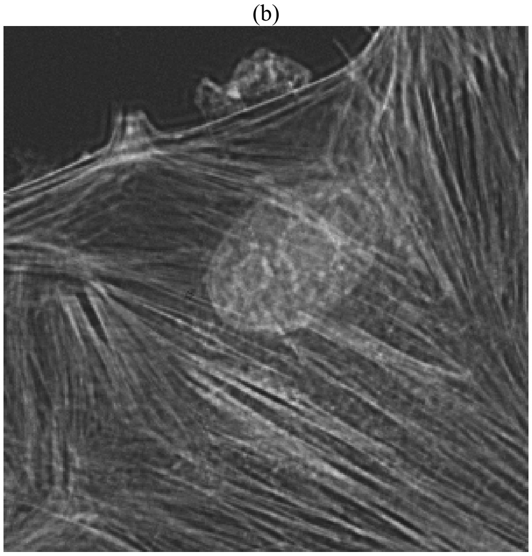
where **b** is a relaxation vector controlling the strictness of the constraint. If **b**=**0**, the monotonicity constraint is strictly enforced; if **b**=**1**, (10) will be redundant in the



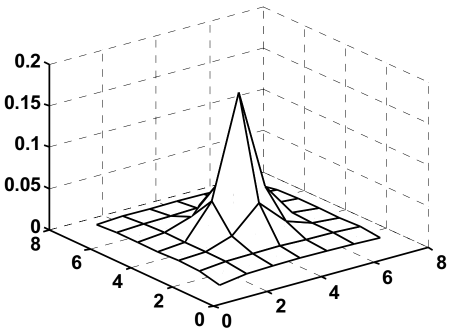
(a)



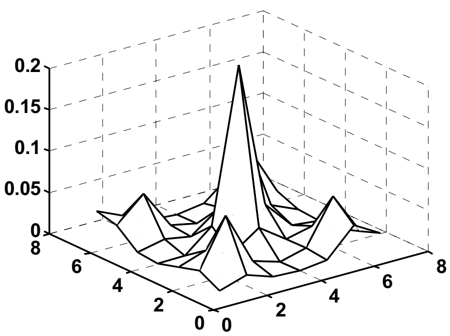
(b)



(c)



(d)



(e)

Fig. 2. (a) The raw image. Courtesy of Leica Microsystems Ltd., Cambridge, UK. (b)(d) Restored image and PSF with the monotonicity constraint. (c)(e) Restored image and PSF without the monotonicity constraint.

presence of (8) and (9). Note that the modification from (6) to (10) will not change the structure of the algorithm. The restoration quality is evaluated by the percentage mean square error (MSE) [3]:

$$MSE(g) = \frac{\| g - a\hat{g} \|_2^2}{\| g \|_2^2} \times 100\% , \tag{11}$$

where

$$\hat{g} = \hat{f}_{Kmax} * \hat{d}_{Kmax\text{-}1} , \tag{12}$$

$$a = \frac{\sum_{(x,y)} g(x,y) \times \hat{g}(x,y)}{\sum_{(x,y)} \hat{g}^2(x,y)} . \tag{13}$$

For the sake of simplicity, we set $\mathbf{b} = b \bullet \mathbf{1}$ where b is a scalar ranging from 0 to 1. Fig. 3 shows some of the curves of $MSE(g)$ against $Kmax$ with different b. We found that with any $Kmax$>2, $MSE(g)$ decreased monotonically with b, reaching its minimal at b=0. Moreover, the iterative process was generally more stable with b<1 than that with b=1, and the most stable one also occurred at b=0. Such results confirmed the validity of using the monotonicity as a PSF constraint in blind deconvolution.

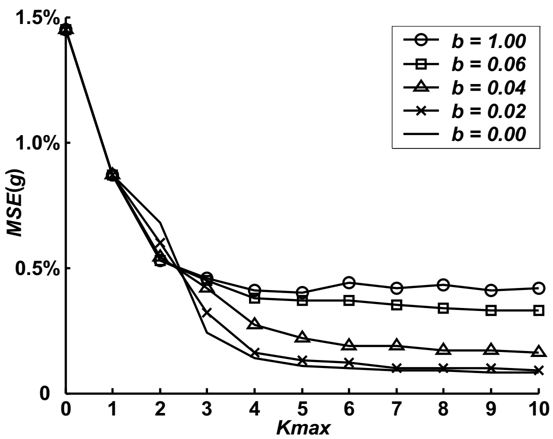


Fig. 3. The curves of the restoration error against the iteration number with different *b*.

4. CONCLUSIONS

The motivation of this paper is to improve blind deconvolution by using prior knowledge of the PSF. We considered the monotonicity a common property shared by many simplified but widely accepted PSF models, and utilized this property as a constraint in a novel algorithm RL-CLSE. Our experiment on a real microscopic image showed that such a PSF monotonicity constraint improved both the restoration quality and stability of blind deconvolution. We based the presented algorithm on the IBD framework, considering it the easiest way to apply the proposed constraint. In future work we will try the monotonicity constraint in more rigorous frameworks other than IBD, such as regularized least square error [20] and maximum likelihood [5].

5. ACKNOWLEDGEMENTS

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