SUBSPACE PARTITION WEIGHTED SUM FILTERS FOR IMAGE DECONVOLUTION

Yong Lin[1], *Russell C. Hardie*[1] and *Kenneth E. Barner*[2]

1. Department of Electrical and Computer Engineering
University of Dayton
300 College Park, Dayton OH 45469-0226
Phone: (937) 229-2436, Fax: (937) 229-2097
Email: yong.lin@notes.udayton.edu, rhardie@udayton.edu

2. Department of Electrical and Computer Engineering
University of Delaware
Newark DE 19716
Phone: (302) 831–6937, Fax: (302) 831–4316
Email: barner@udel.edu

ABSTRACT

The previously proposed partition-based weighted sum (PWS) filters combine Vector Quantization (VQ) and linear finite impulse response (FIR) Wiener filter concepts. By partitioning the observation space and applying a tuned Wiener filter to each partition, the PWS is spatially adaptive and has been shown to perform well in noise reduction applications. In this paper, we propose the subspace PWS (SPWS) filter and evaluate the efficacy of the SPWS filter applied to the image deconvolution problem. In the SPWS filter, we project the observation vectors into a subspace using principal component analysis (PCA) for partitioning. This subspace projection can dramatically reduce the computational burden associated with the large window size PWS filters that are needed for effective image deconvolution. In some cases, performance is also enhanced due to improved partitioning.

1. INTRODUCTION

One of the most commonly used approaches for image restoration with prior knowledge is Wiener filtering. The well-known Wiener filter is optimum, in a mean-squared error (MSE) sense, if the signal and noise are jointly Gaussian and stationary. It may, however, be suboptimal for a spatially-varying degradation function or with non-stationary signals and noise. Barner, Sarhan, and Hardie, [1] proposed the partition-based weighted sum (PWS) filter, which combines Vector Quantization (VQ) [2] and Wiener filter concepts. A blind deconvolution method using VQ partitioning, like the PWS filter was recently proposed by Nagasaki and Katsaggelos [3]. VQ has also been used in image superresolution [4] and other image restoration applications [5-6].

The PWS filter uses a moving window operation. The observation vector at each position in the image is quantized into one of the M partitions. Associated with each partition is a finite impulse response (FIR) Wiener filter that is "tuned" for data falling into that partition. After an observation vector is classified, the corresponding Wiener filter is applied. The Wiener filter can be considered a special case of the PWS filter with only one partition (M=1). Previous work has demonstrated the effectiveness of the PWS filter in an image de-noising applications using relatively small observation windows. However, the image deconvolution application may require significantly larger observation window sizes to achieve desired performance. This dramatically increases the computational cost of the VQ partitioning and VQ codebook generation to as point where it may become impractical.

In this paper, we propose a subspace PWS (SPWS) filter and evaluate its efficacy in an image deconvolution application with prior knowledge of the degradation point spread function (PSF). In the SPWS filter, we project the observation vectors into a subspace using principal component analysis (PCA). Vector quantization (VQ) based on the Linde-Buzo-Grey (LBG) algorithm [2] is used in the subspace for partitioning. This subspace projection can dramatically reduce the computational burden associated with the large window size PWS filters that may be needed for effective image deconvolution. In some cases, performance is also enhanced due to improved partitioning.

The remainder of this paper is organized as follows. The PWS and SPWS filters are defined in Section II.

Section III shows the experimental results comparing the SPWS with traditional Wiener filter deconvolution and the PWS. An analysis of computational complexity is also provided. Finally, conclusions are presented in Section IV.

2. PARTITION WEIGHTED SUM DECONVOLUTION

A. PWS Filter

From Barner, Sarhan and Hardie [1], the output of a PWS filter can be expressed as

$$F_{PWS}(\mathbf{x}(\mathbf{n})) = \mathbf{w}^T_{p(\mathbf{x}(\mathbf{n}))}\mathbf{x}(\mathbf{n}), \quad (1)$$

where

$$\mathbf{x}(\mathbf{n}) = [x_1(\mathbf{n}), x_2(\mathbf{n}), ..., x_N(\mathbf{n})]^T \quad (2)$$

contains the N observation samples spanned by a moving window centered at $\mathbf{n} = [n_1, n_2]$ and $\mathbf{w}_i = [\mathbf{w}_{i,1}, \mathbf{w}_{i,2}, ..., \mathbf{w}_{i,N}]^T$ for $i = 1, 2, ...M$ are the filter weights for each of the M VQ partitions. The encoder or partition function $p(\cdot): R^N \mapsto \{1, 2, ..., M\}$ generates the partition index and is given by

$$p(\mathbf{x}(\mathbf{n})) = \arg\min_i \left\|\mathbf{x}(\mathbf{n}) - \mathbf{z}_i\right\|^2, \quad (3)$$

where $\mathbf{z}_i$ is a codeword from the codebook $C = \{\mathbf{z}_i, i = 1, ..., M\}$. As in [1], we use the LBG algorithm to generate the codebook from a training image.

The weight vectors for each corresponding partition are generally estimated with the aid of training data. It was shown in [1] that the optimum weights, in an MSE sense, are found by using the Wiener weights for each partition. This is given by

$$\mathbf{w}^*_i = \mathbf{R}^{-1}_i \mathbf{p}_i, \quad (4)$$

where $i = 1, 2, ...M$, $\mathbf{R}_i = E[\mathbf{x}(\mathbf{n})\mathbf{x}(\mathbf{n})^T \mid \mathbf{x}(\mathbf{n}) \in \Omega_i]$ is the auto correlation matrix and $\mathbf{p}_i = E[d(\mathbf{n})\mathbf{x}(\mathbf{n}) \mid \mathbf{x}(\mathbf{n}) \in \Omega_i]$ is the cross correlation vector for the i'th partition, Ω_i.

B. Subspace PWS Filter

To reduce the computational complexity of partitioning the R^N observation space, we propose projecting the observation vectors into an R^K subspace ($K<N$) through a linear transformation

$$\tilde{\mathbf{x}}(\mathbf{n}) = \mathbf{A}\mathbf{x}(\mathbf{n}), \quad (5)$$

where $\mathbf{A}$ is a $K \times N$ matrix. Thus the output of SPWS filter can be rewritten as

$$F_{SPWS}(\mathbf{x}(\mathbf{n})) = \mathbf{w}^T_{\tilde{p}(\tilde{\mathbf{x}}(\mathbf{n}))}\mathbf{x}(\mathbf{n}), \quad (6)$$

where $\tilde{p}(\cdot): R^K \mapsto \{1, 2, ..., M\}$. We now consider and describe three choices for $\mathbf{A}$.

The first case we consider is where $\mathbf{A}$ is based on PCA using Karhunen-Loeve (KL) transform. The PCA algorithm, or KL transform, is an elegant and efficient way to reduce data dimensionality. In this case, the rows of $\mathbf{A}$ are made up of the K eigen vectors with the largest eigen values of the covariance matrix for $\mathbf{x}(\mathbf{n})$. The bulk of the variation in the data can often be captured in a subspace using this method.

A simpler method selects K observation sample locations to form the subspace. In that case, $\mathbf{A}$ contains one one in each row located in a unique column (the other entries are zero). This serves as a simple selection function. In Section III, we consider the case where the most central K samples are selected. Finally, we consider combining the PCA and center selection method. In this case, $\mathbf{A} = \mathbf{A}_{PCA}\mathbf{A}_S$ where $\mathbf{A}_S$ is an $L \times N$ matrix that selects the L most central samples and $\mathbf{A}_{PCA}$ is a $K \times L$ PCA subspace transformation matrix where $K < L < N$. The emphasis on the center comes from the observation that the deconvolution partition filters tend to have the largest magnitude impulse response values at the center.

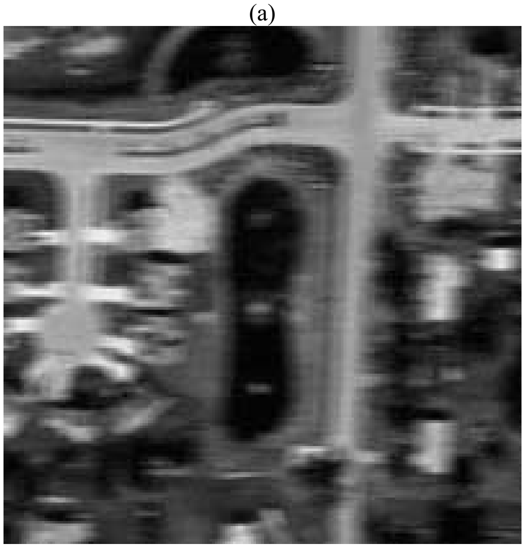
3. EXPERIMENTAL RESULTS

In this section, the results of SPWS applied to motion-blurred images are compared with FIR Wiener filter and PWS filter results to evaluate its performance. Separate 600×400 training and testing images are obtained from different parts of a single high-resolution aerial image. The images are consider ideal and are artificially degraded to allow for quantitative error analysis. The training image is used to generate the VQ codebook and to determine the weights of the SPWS filters for each partition. The other image is used for testing.

The image results are shown in Fig. 1 (center portions magnified). Figure 1(a) shows the true test image, Fig. 1(b) shows the motion blurred image with Gaussian noise (standard deviation is 1% of the dynamic range, MSE=36.68), Fig. 1(c) shows the Wiener filter output (MSE=20.41). The PWS output is shown in Fig. 1(d) (MSE=20.03). The SPWS filter outputs using the PCA method (MSE=20.04) and center PCA method (MSE=19.86) are shown in Figs. 1(e) and 1(f), respectively. Note that in all of these results $N = 1 \times 61$, $M = 50$, $K = 5$, and $L = 31$. The SPWS and PWS images appear to be very similar, but the computational complexity of the SPWS filters is significantly lower, making it an attractive choice.

Quantitative error analysis is shown in Fig. 2. In particular, MSE is plotted as a function of the number of partitions, M, for the various filters. Note that for M=1, the PWS and SPWS filters are equivalent to the Wiener filter. The results show improvement with an increase in

(a)



(b)

(c)

(d)

(e)

(f)

Figure 1: (a) True test image, (b) motion blurred image with noise, (c) Wiener filter, (d) PWS, (e) SPWS filter using the PCA method, (f) SPWS filter using center PCA method.

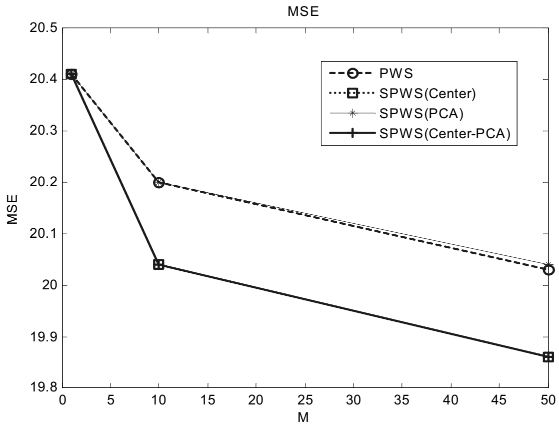


Figure 2: MSE for the various filters applied to a motion blurred image with Gaussian noise.

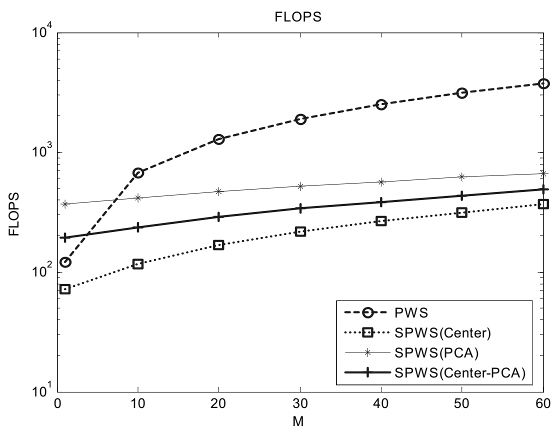


Figure 3: Floating point operation count for the various filters.

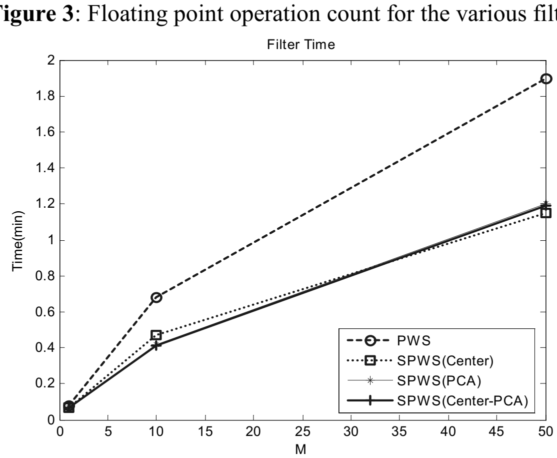


Figure 4: Filter implementation time.

the number of VQ partitions. The SPWS (Center and Center-PCA) had lower MSE than the PWS filter.

By reducing the dimensionality of the observation space, the computational demands of the VQ partitioning are greatly reduced. This *dramatically* speeds up training and significantly speeds up filtering. To see this, we estimated the number of floating point operations (flops) per pixel required to implement the various filters. The results for $N = 61$, $M = 50$, $K = 5$, and $L = 31$ are shown in Fig. 3. For small values of *M*, the PCA methods actually increase the computational complexity because of the matrix multiplication in (5). However, this quickly "pays" for itself and we see a dramatic reduction in flops for larger values of *M*. The SPWS (Center) method has the lowest computational complexity.

In our MATLAB implementation on a Pentium IV 2.4 GHz PC, the total computation time including training and testing for the PWS was 826 minutes (*M*=50, *N*=61). The SPWS (Center) method took only 34.8 minutes – a 95.7% improvement. The PCA and center PCA methods required a similar time. Figure 4 shows the filtering times.

4. CONCLUSIONS

We have proposed a subspace PWS filter for image deconvolution that dramatically reduces the computational complexity of the PWS filter and allows for large window sizes. We demonstrate the efficacy of the SPWS filter in an image deconvolution application. The performance of the SPWS and PWS filters depends very much on the VQ codebook that defines the observation space partitioning. We have observed that the LBG algorithm appears to produce superior results in this application in a lower dimensional space. Thus, in addition to reducing the computational complexity, the subspace projection method improves the PWS filter performance in some cases.

REFERENCES

[1] K. E. Barner, A. M. Sarhan, and R. C. Hardie, "Partition-Based Weighted Sum Filters for Image Restoration," *IEEE Trans. Image Processing*, vol. 8, pp. 740–745, May 1999.
[2] Y. Linde, A. Buzo, and R. M. Gray, "An Algorithm for Vector Quantization," *IEEE Trans. Commun. Theor*y, vol. COMM-28, pp. 84–95, Jan 1980.
[3] R. Nakagaki, and A. K. Katsaggelos, "A VQ-Based Blind Image Restoration Algorithm," *IEEE Trans. Image Processing*, vol. 12, pp. 1044–1053, Sep 2003.
[4] D. G. Sheppard, K. Panchapakesan, A. Bilgin, B. R. Hunt, and M. W. Marcellin, "Lapped Nonlinear Interpolative Vector Quantization and Image Super-Resolution," *IEEE Trans. Image Processing*, vol. 9, pp. 295–298, Feb 2000.
[5] K. E. Barner, G. R. Arce, and J.-H. Lin, "On the performance of stack filters and vector detection in image restoration," *Circuits, Systems, and Signal Processing*, vol. 11, pp. 153–169, Jan. 1992.
[6] M. Shao, K. E. Barner, and R. C. Hardie, "Partition-based interpolation for image demosaicking and super-resolution reconstruction," *Optical Engineering*, July 2004. Submitted for publication.