# TURBO ITERATIVE ESTIMATION OF SINGULARITY STRUCTURE IN SAR IMAGE BASED ON WAVELET-DOMAIN HIDDEN MARKOV MODELS

*Bao Guan, Hong Sun*

Laboratory of Signal Processing, Electronic Information School, Wuhan University
Wuhan, Hubei, 430072, China, hongsun@whu.edu.cn

## ABSTRACT

Wavelet-domain hidden Markov models (HMM's) have been widely applied to image processing, e.g., image restoration. The models provide great promise of detecting image singularity structures with some hidden states. However, these hidden states are rather difficult to be estimated, especially under the influence of the multiplicative speckle noise in SAR image, no efficient estimation method is developed yet. By using the principle of turbo iterative decoding, we propose a new turbo iterative method to estimate the hidden states of the wavelet-domain HMM's for SAR image. In our method, hidden states are estimated alternatively in two orthogonal sub-spaces with a soft estimation scheme, and the posterior probability is exchanged between the two sub-spaces. The experimental results of the proposed method illustrate rather an impressive performance.

## 1. INTRODUCTION

Real-world images are well characterized by their singularity structures, such as edges and ridges [1]. These features are often particularly meaningful for many applications involving image processing. Wavelet analysis has been proved to be a powerful tool for detecting the singularity structures of image, and also been successfully applied to a variety of optical image processing. However, the wavelet-based detection of the singularity structures in SAR image becomes rather difficult because of the effect of the multiplicative speckle noise.

In fact, most of the early wavelet-based detection methods implicitly treat each wavelet coefficient as though it were independent of all others, which is unrealistic for many real-world signals. S.Crouse et al [2] developed new wavelet-domain HMM's to match the non-Gaussian statistics of wavelet coefficients as well as the statistical dependencies between them. The models have also been applied to the despeckling of SAR image in [3], and in that study, a binary hidden state is defined to capture the insignificant or significant coefficient property. The state can take on the value 0 (insignificant coefficient) or 1 (significant coefficient). The former corresponds to those homogeneous areas and the latter to those occasional transitions such as edges and other heterogeneity structures. In [3], the estimation results of hidden state show a great potential of the models for detecting the singularity structures in speckled image. But, with the effect of the multiplicative speckle noise, an efficient and practical estimation method is hard to be devised yet.

In another domain, turbo codes proposed by C.Berrou et al in [4] are among the most promising developments in the field of coding theory in recent years, and the turbo iterative principle has been introduced into the speckled image processing by Sun and Maître et al [5]. Just like the decoding of binary codes, the estimation of hidden state is essentially to compute the posterior probability of a binary random variable given the observation. Consequently, with the same essence, the turbo iterative principle can be exploited to estimate the hidden state under the influence of the multiplicative speckle noise.

By using the turbo iterative principle, the objective of this paper is to develop a novel turbo iterative method to estimate the hidden state of the wavelet-domain HMM's for SAR images. In section II, we review the wavelet-domain HMM's for the further development. In Section III, we present a complete turbo iterative scheme to estimate hidden state. The proposed method is tested on the synthetic speckled images as well as the real SAR image in Section IV. Finally, we conclude the paper.

## 2. REVIEW OF WAVELET-DOMAIN HMM'S

Besides the properties of locality, multiresolution and compression, wavelet transform also has the properties of clustering and persistence [2], and they imply that a residual dependency structure always remains between the wavelet coefficients.

The wavelet-domain HMM's can be used to describe this residual dependency structure by assigning a hidden state for each wavelet coefficient. Generally, the wavelet-domain HMM's consist of three sub-models: independent mixture (IM) model, hidden Markov chain (HMC) model and hidden Markov tree (HMT) model [2]. The IM model describes the marginal statistics of coefficients in the case of ignoring any intercoefficient dependencies. The HMC and HMT model characterize the intra- and interscale dependencies between wavelet coefficients respectively.

As for SAR images, the interscale dependencies become very weak owing to the effect of speckle noise. Thus, we can only consider the first two sub-models in this case [3].

### 2.1. IM sub-model

The distribution of the noise-free wavelet coefficients in each subband is usually sharply peaked at zero and heavy tailed [3,6]. For this special statistics, the mixture of two normals is often considered as a simple yet effective choice of the marginal prior of an individual coefficient [6]. In this case, we define a binary hidden state $s_i$ for each coefficient $w_i$ and $s_i$ can take on the value 0 or 1 (representing significant or insignificant coefficient). The marginal probability density function (pdf) of $w_i$ can be written as follow:

$$p(w_i) = \sum_{k=0,1} p(s_i = k) \cdot p(w_i \mid s_i = k) \tag{1}$$

$$p(w_i \mid s_i = k) \sim \mathrm{N}(0, \sigma_{wk}^2) \qquad k = 0,1 \tag{2}$$

where $\mathrm{N}(0,\sigma_{wk}^2)$ stands for a Gaussian distribution with zero mean and variance $\sigma_{wk}^2$. When the signal is corrupted by AWGN, the noisy wavelet coefficients also obey the mixture density of two normal distributions with zero mean, but with an increased variance that depends on the noise level $\sigma_n^2$. Given $\sigma_n^2$, the model is fully parameterized by $\sigma_{w0}^2$, $\sigma_{w1}^2$ and $p(s_i=1)$, which can be estimated from the noisy coefficients with the EM algorithm described in [3].

### 2.2. HMC sub-model

To model the intrascale spatial dependence between the wavelet coefficients in each individual subband, we introduce an Markov random field (MRF) into hidden state. A two-state Potts model with a second-order neighborhood system is used here, and moreover, only single-site and pair-site cliques are considered. Under such an assumption, the MRF prior of hidden state configuration S can be defined as a Gibbs random field:

$$p(S) = \frac{1}{Z} \cdot \exp(-U(S)) \tag{3}$$

$$U(S) = V_1(S) + V_2(S) \tag{4}$$

$$\exp(-V_1(S)) = \prod_{i=1}^{N} p(s_i) \quad V_2(S) = -\beta \sum_{i=1}^{N} \sum_{j \in \eta(i)} (\delta(s_i - s_j) - 1) \tag{5}$$

where $Z$ is a normalizing constant; $\beta$ is the parameter that controls the local smoothness; $\eta(i)$ is the neighborhood of pixel $i$; $\delta(\cdot)$ is the discrete delta function.

In terms of the Hammersley-Clifford theorem [7], the conditional pdf can be derived as:

$$p(s_i \mid S_{\eta(i)}) = \frac{p(S)}{p(S_{\eta(i)})} \qquad S = \{s_i \bigcup S_{\eta(i)}\} \tag{6}$$

Taking (3)~(5) into (6), we can get the following form:

$$p(s_i \mid S_{\eta(i)}) \propto p(s_i) \cdot \exp(2\beta \sum_{t \in \eta(i)} (\delta(s_i - s_t) - 1)) \tag{7}$$

## 3. TURBO ITERATIVE ESTIMATION OF HIDDEN STATE

The optimal estimation of a binary hidden state, like the turbo decoding, is to seek the posterior probability of a binary random variable given observation. As for Turbo codes, there are two essential contributing factors to explain their success [5]: (I) the presence of the pseudo-random interleavers between the two component codes makes them uncorrelated; (II) the two independent codes work as a loopy Bayesian belief network with exchanging extrinsic information.

Accordingly, the turbo iterative estimation of hidden state should obey the following principles: (I) hidden state should be estimated alternatively in two orthogonal sub-spaces; (II) the soft estimation that offers a scheme of soft-input and soft-output, like the soft decoding in Turbo codes, should be adopted in both of two sub-spaces; (III) the appropriate information should be exchanged between the two estimated results. The framework of our method is shown in Fig.1, where the observed amplitude image $A$ is the input and the posterior probability of hidden state is the output.

### 3.1. Pre-transform

The multiplicative model holds for an amplitude SAR image, which expresses the observed amplitude $A$ as the product of the scatterer amplitude $R$ and the speckle noise amplitude $U$ [3]:

$$A = R \cdot U \tag{8}$$

In theory, $R$ is statistically independent of $U$, which offers the turbo iterative estimation method two orthogonal sub-spaces: signal sub-space $R$ and noise sub-space $U$.

A logarithm transform is usually applied to converting the multiplicative model into an additive model before the stationary wavelet transform (3-order B-spline wavelet kernel is used in this study). Here, the logarithm transform and the wavelet transform are together taken as the pre-transform. Since the interscale dependence can be ignored, we only consider the second level in the signal sub-space and the first level in the noise sub-space.

### 3.2. Soft estimation of hidden state

The soft estimation of hidden state is essentially to compute its posterior probability in terms of its prior $p(s_i)$, the observation data $w_i$ and the estimated variances $\sigma_{w0}^2, \sigma_{w1}^2$.

The iterated conditional modes (ICM) algorithm is used to maximize the posterior probability of $s_i$, and the convergence rate and final state of this optimization method are strongly dependent on the initial condition. Using the MAP criterion based on the marginal prior in the IM model, a more reasonable initial configuration of $s_i$ is given with the following thresholding operation [3]:

$$\hat{S}_i = \begin{cases} 1 & |w_i| \geq Q \\ 0 & |w_i| < Q \end{cases} \qquad Q = \sqrt{\frac{2{\sigma_{w0}}^2 \cdot {\sigma_{w1}}^2}{{\sigma_{w0}}^2 - {\sigma_{w1}}^2} \log \frac{p(s_i = 1) \cdot \sigma_{w0}}{p(s_i = 0) \cdot \sigma_{w1}}} \tag{9}$$

With the initial condition (9), the posterior probability of $s_i$ can be computed based on the HMC sub-model:

$$p(s_i \mid w_i, \hat{S}_{\eta(i)}) \propto p(w_i \mid s_i) \cdot p(s_i \mid \hat{S}_{\eta(i)}) \tag{10}$$

with (2) and (7), the logarithm posterior probability of hidden state can be written as follow:

$$\log p(S_i \mid w_i, \hat{S}_{\eta(i)}) \propto G(S_i \mid w_i, \hat{S}_{\eta(i)}) = -\frac{1}{2}\log \sigma_{S_i}{}^2 - \frac{1}{2}\frac{w_i^2}{\sigma_{S_i}{}^2} + \log p(s_i) + 2\beta \sum_{u\in\eta(i)} (\delta(s_i - \hat{s}_u) - 1) \tag{11}$$

We define the normalized posterior probability:

$$p(S_i = 0 \mid w_i, \hat{S}_{\eta(i)}) = \frac{\exp\left\{G(s_i = 0 \mid w_i, \hat{S}_{\eta(i)})\right\}}{\exp\left\{G(s_i = 0 \mid w_i, \hat{S}_{\eta(i)})\right\} + \exp\left\{G(s_i = 1 \mid w_i, \hat{S}_{\eta(i)})\right\}} \tag{12}$$

$$p(S_i = 1 \mid w_i, \hat{S}_{\eta(i)}) = \frac{\exp\left\{G(s_i = 1 \mid w_i, \hat{S}_{\eta(i)})\right\}}{\exp\left\{G(s_i = 0 \mid w_i, \hat{S}_{\eta(i)})\right\} + \exp\left\{G(s_i = 1 \mid w_i, \hat{S}_{\eta(i)})\right\}} \tag{13}$$

### 3.3. Information exchange

The acquirement of the prior $p(s_i)$ is very crucial for computing the posterior probability of $s_i$. In general, $p(s_i)$ can be initialized by the EM algorithm at the first iteration in the sub-space *R*. Then, in the later iterations, the prior $p(s_i)$ in one sub-space can be updated by the last posterior in the other sub-space, which is the procedure of information exchange, illustrated as Fig.1.

In the sub-space *R* where the signal is dominant, the configuration of the significant coefficients with 1-valued state represents the singularity structures since the coefficients containing primarily noise have become very weak at the second level, illustrated in Fig.2 (a); on the contrary, in the sub-space *U* where the noise is dominant, it's the configuration of the insignificant coefficients with 0-valued state that represents the singularity structures since the coefficients with the weak textural signal are often very small comparing with the strong noise at the first level, illustrated in Fig.2(b). Consequently, in the sub-space *R*, the prior $p(s_i=1)$ should be updated by $p(s_i = 0 \mid w_{Ui}, \hat{S}_{\eta(i)})$ (the last posterior in the sub-space $U$ ), while in the sub-space *U*, the prior $p(s_i=0)$ should be updated by $p(s_i = 1 \mid w_{Ri}, \hat{S}_{\eta(i)})$ (the last posterior in the sub-space *R*).

### 3.4.The transform between two orthogonal sub-spaces

According to the multiplicative model as (8), one sub-space can be transformed into the other sub-space through a ratio operation as follow:

$$U = \frac{A}{R} \tag{14}$$

In order to perform the transform between these two sub-spaces, it's necessary in each iteration to estimate the restored image $\hat{R}$ in terms of the observation amplitude *A* and the posterior probability of hidden state $s_i$ by using an optimum filter.

In the classic Lee or Kuan filter, the estimated result is in nature the weighted average of the local mean and the individual pixel value, and the weighted factor depends on the heterogeneity degree of local texture. In fact, the posterior probability of hidden state is more reasonable to be used to describe the heterogeneity of local texture than the normalized variance used in the classic filters. Now, we develop a new weighted filter by using this posterior probability as the weighted factor *k*:

$$\hat{R}_A = A \cdot k + \overline{A}(1-k) \tag{15}$$

where *A* and $\overline{A}$ denote the observation amplitude and the local mean, respectively, and in the sub-space *R*, the weighted factor $k = p(s_i = 1 \mid w_{Ri}, \hat{S}_{\eta(i)})$ , while in the sub-space *U*, the weighted factor $k = p(s_i = 0 \mid w_{Ui}, \hat{S}_{\eta(i)})$.

The posterior probabilities of hidden state in the two orthogonal sub-spaces converge very rapidly. In general, the posterior $p(s_i = 1 \mid w_{Ri}, \hat{S}_{\eta(i)})$ and $p(s_i = 0 \mid w_{Ui}, \hat{S}_{\eta(i)})$ are very close to each other after 3 or 4 iterations. At this time, according to the convergent posterior probabilities, we can make a hard decision of hidden state to obtain its optimal estimation, that is, to say $s_i$=0 or $s_i$=1.

## 4. EXPERIMENTAL RESULTS AND ANALYSIS

We corrupted the standard $256 \times 256$ Lena image by 4-looks multiplying speckle noise (spatially uncorrelated and Gamma distributed) to obtain the synthetic speckled image (SNR=3.15db), shown in Fig.2 (a). To make a comparison, we assess an auto-iterative estimation method, in which the estimation is performed only in the signal sub-space, and its normalized probabilities $p(s_i = 1 \mid w_i, \hat{S}_{\eta(i)})$ in the 1st and 4th iteration are shown in Fig.2 (c) and (d) respectively. It's clear that the auto-iterative estimation have no ability to correct the errors produced in the last iteration, and thus, the estimation errors may be accumulated in the whole procedure of iteration.

The 1st and 4th results of the turbo iterative estimation are shown in Fig.2 (e) and (f), and they indicate that the soft-estimation in one sub-space can efficiently correct the errors produced at the last soft-estimation in the other sub-space, which is because the exchanging information in the two orthogonal sub-spaces is complementary. Here, $\beta = 0.1$ is used for the Potts model to control local smoothness ( $\beta$ is adaptive to the tested images).

Then, the proposed method is also evaluated on a 6-looks X-band airborne SAR image, shown in Fig.3 (a), and the final estimation result of hidden state is shown in Fig.3 (b), which well represents the singularity structures in the image: the edges of farmlands are very clear in the result; the left-bottom heterogeneous areas, which contain many sharp transitions, also outstand in the result. Here, the control parameter $\beta = 0.5$ is used.

Clearly, the above results can well describe the heterogeneities of local texture and represent the singularity structures in the speckled images. The detection of singularity structures is very useful for many other applications such as restoration, segmentation as well as edge detection et al.

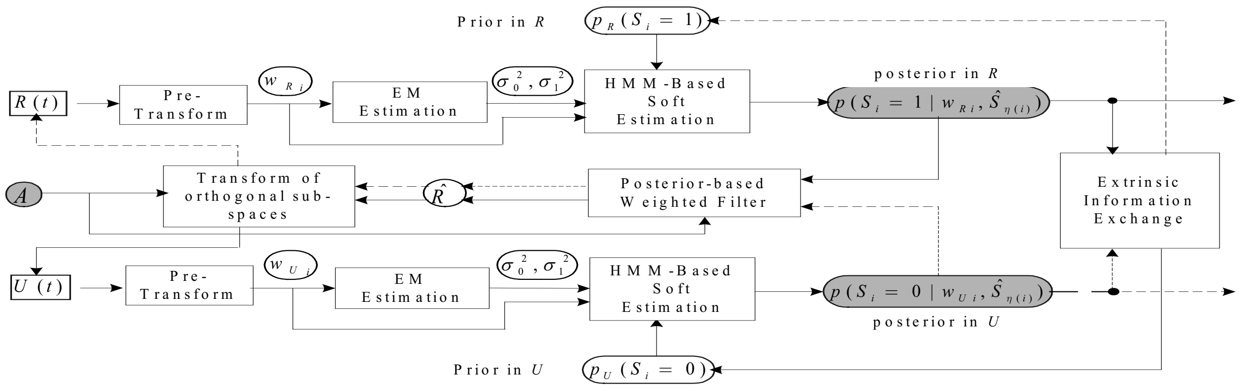
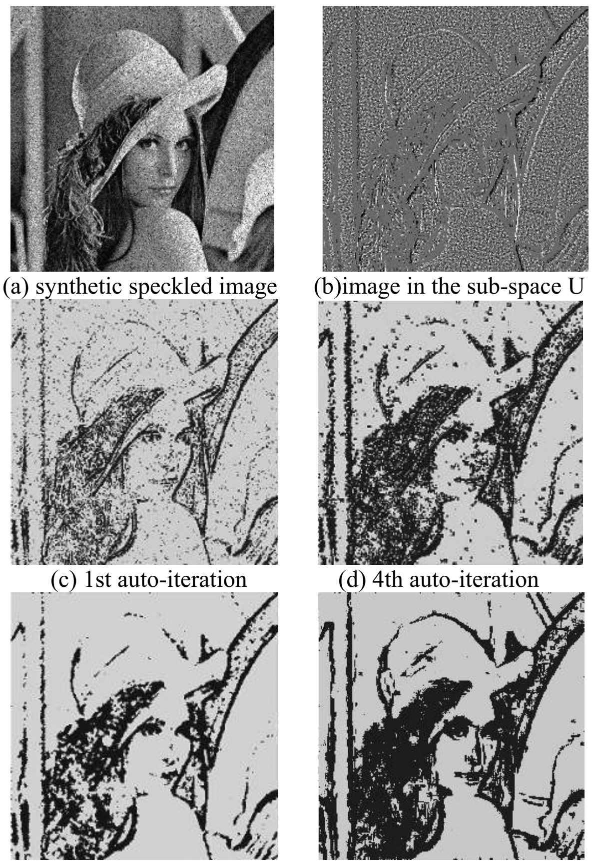



Fig.1 Framework of the proposed turbo iterative estimation method.



(a) synthetic speckled image (b)image in the sub-space U

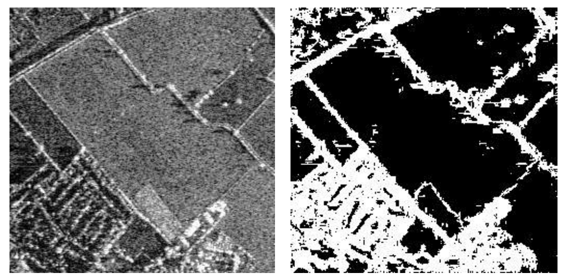
(c) 1st auto-iteration (d) 4th auto-iteration

(e) 1st Turbo-iteration (f) 4th Turbo-iteration

Fig.2 The synthetic speckled image and experimental results – soft estimation of the singularity structures

## 5. CONCLUSION

Wavelet-domain HMM's have a promising ability to detect the SAR image singularity structures. Based on the models, the singularity structures can be described by the hidden states. An efficient and practical method to estimate these hidden states for SAR image is hard to be developed due to the influence of the multiplicative speckle noise. We proposed a novel turbo iterative method for estimating hidden state in such an intricate case. Some excellent experimental results confirm that the turbo iterative principle developed in the field of coding theory is efficient for image processing.



(a) Real airborne SAR image (b) Estimate of hidden state

Fig.3: Experimental result for real SAR image

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