OPTIMAL CO-DESIGN OF COMPUTATIONAL IMAGING SYSTEM

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ABSTRACT

In this paper, we propose a novel method for the optimal co-design of the optical and reconstruction filters in a computational imaging system. Closed form solutions are presented for design of optimal observation matrix for a fixed reconstruction matrix. An iterative method for computing global optimal filters is then proposed based on the derived analytical solution and the well known Wiener filter. The performance of the proposed optimal filters represents a universal bound on the performance of any physically realizable computational imaging system.

1. INTRODUCTION

Traditional imaging systems utilize a lens or mirror in the front end to form the image that is then sampled onto a detector array. Examples of such systems include the familiar optical cameras, and telescopes. The term "computational imaging" describes the emerging field of optical systems in which a true image is not formed by a lens and simply sampled onto a detector but rather the process of image formation is shared between the power of the optical elements and signal processing of the sampled amplitudes. Several point solutions serve to illustrate the utility of this approach as it can obtain system level performance exceeding separately optimized optical and signal processing designs [1,2]. This effort seeks to determine a global bound on the computational imaging approaches. An optical system with no physical constraints is considered, and then increasing levels of constraints are applied. The types of constraints which can be considered are (in order of how much they constrain the solution space): energy conservation of the optical system (photons only detected once), isoplanatism of the optical field (rows of the observation matrix are shifted versions of each other), low pass response of the optical system (corresponding to the limiting physical aperture), and more. This paper takes steps towards the physical constraint of photons only being sensed once as well as considering the unconstrained system.

In this paper, we propose a framework for the design of a computational imaging system that incorporates an end-to-end performance metric. Specifically, we compute the observation filter (defined in Section 2) that minimizes the mean squared error (MSE) performance between the final output image and the original scene. Coupled with the well known optimal Wiener reconstruction filter [3], we propose an iterative method to arrive at the jointly optimal observation and reconstruction filter. Based on the empirical observations of the output of the iterative process after convergence, we propose a simple analytical method for the construction of an observation matrix and the associated reconstruction matrix. The proposed construction technique applies equally to the case of white and colored noise models.

Potential applications of the proposed designs include hyperspectral imaging, and computational tomography. Specifically the proposed design methods are being considered for the design of a pervasive flat form factor imaging system [4]. The co-design of the optimal observation and reconstruction matrices uses tools from vector optimization and principal component analysis [5]. Most of the existing signal processing methods focus on computing reconstruction filters under various optimality constraints. Analogous work in the communications applications includes the design of optimal equalizers for fading channels [6].

The remainder of this paper is organized as follows. Section 2 presents the system model and description. Section 3 considers the co-design of the observation and reconstruction filter and Section 4 concludes the paper.

2. SYSTEM MODEL

We consider a simplified model of the imaging system of interest. A functional block diagram of the system is given in Fig. 1. The two dimensional scene of interest is represented by column vector x, which represents the scene information in row ordered form. The output of the detector array y is given by

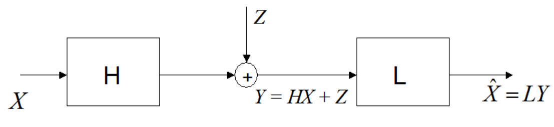

Fig. 1: Simplified Block Diagram of Computational Imager. The combined effect of optical lens and detector array is represented by observation matrix H and the reconstruction process is represented by matrix L.

$$y = Hx + z \quad (1)$$

where, H is the observation matrix that represents the combined effect of the optical lens and detector array, and z represents the measurement noise. We assume that the noise vector z is circularly symmetric Gaussian random variable with zero mean and autocorrelation R_z. The dimensions of H equal m x n. Further, we assume that $m \leq n$, which corresponds to an imager that has lesser number of observation pixels than in the original scene at the highest resolution. The observation vector is processed in linear manner by reconstruction matrix L. The goal of the computational imaging system is to accurately reconstruct the image x at the output of the digital processing. The reconstructed image $\hat{x}$ is represented by

$$\hat{x} = Ly = L(Hx + z) \quad (2)$$

The objective of the proposed design methods is to jointly construct H and L to optimize end-to-end system performance.

3. OPTIMAL CO- DESIGN OF H AND L

The overall error vector, e, in the image formation task is defined as $e = x - \hat{x}$. The mean squared error (MSE) can be written as

$$E(e^t e) = E\{Tr(ee^t)\}$$

Further, this MSE can be simplified as

$$E\{Tr(ee^t)\} = E\{Tr[(x - Ly)(x - Ly)^t\} \quad (3)$$

$$E\{Tr(ee^t)\} = Tr(R_x - 2LHR_x + LHR_x H^t L^t + LR_z L^t) \quad (4)$$

where, $Tr(A)$ is the trace of A and $R_x = E\{xx^t\}$ is the autocorrelation matrix of the input signal.

The optimization problem of interest can be formally stated as

$$\min_{\{H,L\}} Tr(R_x - 2LHR_x + LHR_x H^t L^t + LR_z L^t) \quad (5)$$

Solving (5) analytically is intractable. Hence, we use an approach analogous to the double minimization algorithm proposed by Blahut-Arimoto [7] and rewrite optimization problem (5) into two simpler optimization problems. We then use a simple iterative process to compute the observation and reconstruction matrices.

The two optimization problems are posed as

$$\min_{\{H\}} Tr(R_x - 2LHR_x + LHR_x H^t L^t + LR_z L^t) \quad (6)$$

for a fixed reconstruction filter L and,

$$\min_{\{L\}} Tr(R_x - 2LHR_x + LHR_x H^t L^t + LR_z L^t) \quad (7)$$

for a fixed observation filter H. The optimal solution to (7) is given by the Wiener filter,

$$L^* = R_x H^t (HR_x H^t + R_z)^{-1} \quad (8)$$

It can be easily derived that the optimal H^* for a given L matrix, i.e., the solution to (6), equals the pseudo inverse

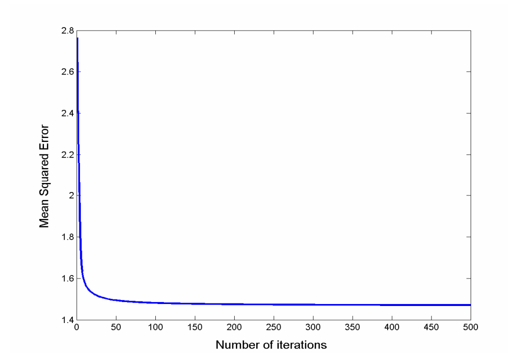

Fig. 2: Plot of Error Vs number of iterations

of L, and is given by

$$H^* = (L^t L)^{-1} L^t \quad (9)$$

Computing a H^* and L^* that jointly satisfy (8) and (9) leads to the result that the entries of H increases without bound. The effect of the unbounded H is to mask out the effect of noise. This problem of unbounded growth is naturally overcome by considering the design of H with physical constraints. Currently, no constraint on L is imposed, since it's a computational block.

3.1. Optimization under physical realizability constraints

To ensure that photons are assumed to be sensed only once implies that the RMS sum of each row of the observation matrix is equal to one. If it were less than one, then some photons were not collected by the system leading to sub-optimum performance. To achieve a value greater than one would require optical "gain" in the sensing system – the amount of light collected from a single object point exceeds the amount present. We chose to approach this physically significant constraint in a series of small steps, thereby allowing us to examine the implications of each. As a first step we imposed the constraint that required the sum of all rows in the observation matrix to equal unity. The constraint set $\mathcal{H}$ is defined as

$$\mathcal{H} = \{H : Hj_{n\times1} = j_{m\times1}\}$$

where, $j_{a \times b}$ is a matrix of size a x b with all elements being one. The optimization problem can now be stated as

$$\min_{\{H\in\mathcal{H}\}} Tr(R_x - 2LHR_x + LHR_xH^tL^t + LR_zL^t) \quad (10)$$

This constrained optimization problem can be solved using the method of Lagrange multipliers. Define the new objective function as

$$J = Tr(R_x - 2LHR_x + LHR_xH^tL^t + LR_zL^t) + \lambda(Hj_{n\times 1} - j_{m\times 1}) \quad (11)$$

where, λ is the Lagrangian operator. The partial derivative of J with respect to H may be computed as

$$\frac{\partial J}{\partial H} = -2L^tR_x^{\ t} + 2L^tLHR_x + \lambda^t j_{1\times n} = 0 \quad (12)$$

$$\Rightarrow H = \frac{1}{2}(L^tL)^{-1}(2L^tR_x - \lambda^t j_{1\times n})R_x^{-1} \quad (13)$$

Substituting in the constraint, we find the Lagrangian operator as

$$\lambda^t = \frac{2}{\alpha}(L^t j_{n\times 1} - L^tLj_{m\times 1}) \quad (14)$$

where, $\alpha = j_{1\times n}R_x^{-1}j_{n\times 1}$ is a scalar constant that depends on noise autocorrelation. Hence,

$$H = (L^tL)^{-1}L^t(R_x - \frac{1}{\alpha}j_{n\times n} + \frac{1}{\alpha}Lj_{m\times n})R_x^{-1} \quad (15)$$

Thus, the optimal observation matrix H in the set $\mathcal{H}$ for a given reconstruction filter L is given by (15).

3.2. Iterative method

The solutions to the two optimization problems (6) and (7) results in a set of two equations (8) and (15) in two variables L and H. However, solving for the optimal H and L in terms of R_x and R_z is intractable. Instead a standard iterative process that mimics the well known double minimization method [7] is used to compute jointly optimal H and L, i.e., to find an H and L that satisfy both (8) and (15). The iterative process consists of the following steps:

1. Start with any arbitrary H_1 matrix that belongs to $\mathcal{H}$ Set the iteration number i =1.
2. Compute L_i for this observation matrix H_i using (8)
3. Compute H_{i+1} for the reconstruction matrix L_i using (15)
4. After the i^th^ iteration, the MSE is given by $\gamma_i = Tr(R_x - 2L_iH_iR_x + L_iH_iR_xH_i^tL_i^t + L_iR_zL_i^t)$
5. Set $i = i + 1$. Repeat steps 2 to 5, until a desired stopping condition is met. A standard threshold stopping criteria based on successive difference in γ_i is used.

The convergence of this iterative process is depicted in Fig. 2, which plots the variation of the MSE with number of iterations. Clearly, the algorithm converges within a few iterations to the optimal solution. We found that the iterative process converges for all initial values of H and for both colored and white noise cases.

Interestingly, we noticed that for a fixed R_x and R_z, the iterative process converges to a constant value for the product of L and H, for all initial values of H. This observation that the product of the optimal observation and reconstruction matrix is a constant motivates the following design methodology for the observation matrix H.

3.3. Design of *H* and *L*

Denote by M the matrix product of L and H after the convergence of the iterative process. i.e.,

$$\lim_{i\to\infty} L_iH_i = M \quad (16)$$

Substituting for L_i from (8) into (16) we obtain

$$R_xH^t(HR_xH^t + R_z)^{-1}H = M \quad (17)$$

Post multiplying (17) by R_x and then subtracting both sides from R_x results in,

$$R_x - R_xH^t(HR_xH^t + R_z)^{-1}HR_x = R_x - MR_x \quad (18)$$

Invoking the Matrix Inversion lemma, we obtain

$$R_x^{-1} + H^tR_z^{-1}H = [R_x(I-M)]^{-1} \quad (19)$$

$$\Rightarrow H^tR_z^{-1}H = [R_x(I-M)]^{-1} - R_x^{-1} \quad (20)$$

An optimal H can be now designed using standard principal component analysis. Express H in terms of its singular values and singular vectors as $H = Q_1\Lambda_HQ_2^{\ t}$, where, Q_1 and Q_2 are orthogonal matrices and Λ_H is a diagonal matrix that contains the singular values of H. Similarly, we can express autocorrelation matrix R_z and matrix $N = [R_x(I-M)]^{-1} - R_x^{-1}]$ in terms of their eigen value decomposition as $R_z = Q_3\Lambda_{R_z}Q_3^{\ t}$ and $N = Q_4\Lambda_NQ_4^{\ t}$. Rewriting (20), we obtain

$$\begin{aligned} H^tR_z^{-1}H &= (Q_2\Lambda_HQ_1^{\ t})(Q_3\Lambda_z^{-1}Q_3^{\ t})(Q_1\Lambda_HQ_2^{\ t}) \\ &= Q_4\Lambda_NQ_4^{\ t} \end{aligned} \quad (21)$$

Now selecting $Q_1 = Q_3$, (21) simplifies to

$$Q_2\Lambda_H\Lambda_z^{-1}\Lambda_HQ_2^{\ t} = Q_4\Lambda_NQ_4^{\ t} \quad (22)$$

An obvious solution to this equation is given by $Q_2 = Q_4$, $\lambda_{H_i} = \sqrt{\lambda_{N_i}\lambda_{z_i}}$ where, λ_{H_i} represents the i^th^ singular value of H, and $\lambda_{N_i}, \lambda_{z_i}$ represent the i^th^ eigen value of N and z respectively.

The proposed design of H. and L matrices may be succinctly summarized in the following steps. For given signal and noise autocorrelation matrices, compute the optimal product of LH using the proposed iterative method. Choose the eigenvectors Q_4 of N as the right singular vectors Q_2, of H; and choose the eigenvectors Q_3 of z as the left singular vectors Q_1 of H. Also, choose the i^{th} singular value of H as $\lambda_{H_i} = \sqrt{\lambda_{N_i} \lambda_{z_i}}$. Once H is computed, find L as the optimal Wiener filter (8).

Recognize that the proposed H is not a unique solution to the MSE minimization problem. Any H and associated L that satisfies (16) is a potential solution. This observation that there exists a set of optimal H matrices raises the possibility that a physically realizable filter can be generated that is optimal.

To illustrate the performance of the proposed method an observation matrix H and reconstruction matrix L were constructed. This observation matrix H was applied to a standard Lena image and the resulting observation are given in Fig. 3a. As with any typical computational imager, the image of the scene is indiscernible at the output of the detector. The reconstructed image (given in Fig. 3b) though has very high visual quality and also has a high peak signal to noise ratio (PSNR) of 37.05 dB.

We also constructed a family of physically realizable observation filters that can be succinctly represented as a compact set. For this constraint set on H, we computed the nearest realizable filter to the optimal H using the method of projection onto convex sets (POCS) [8]. Recognize that the POCS method does not generate the optimal H with the required constraints; the POCS method generates a point in the constraint set that is closest in MSE sense to the initial point.

The robustness and gains that are achieved using an adaptive H matrix is now quantified using numerical simulations. Consider an imaging system in which the optimal observation matrix is designed for a given R_x and R_z. Now let the same observation matrix be used in cases where the noise characteristics are changed. For each of the different noise cases, the optimal reconstruction matrix is used. The variation of the PSNR for different noise variances is given in Fig. 4. For comparison, the performance of a fully adaptive imaging system in which both the observation and reconstruction matrices are modified based on the different noise vectors is also given in Fig. 4. The gains in adapting the observation filter are evident from Fig. 4. It should also be noted that the performance of the proposed adaptive method serves as an upper bound on performance of any physically realizable computational imaging system

(a) (b)

Fig. 3: The output of the observation filter and the reconstructed image. The PSNR of the reconstructed image equals 37.05

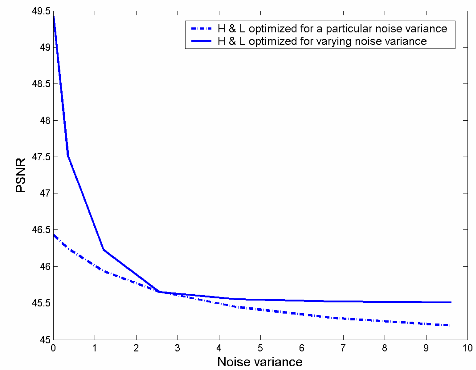

Fig. 4: Plot of PSNR versus noise variance

4. CONCLUSION

This paper presents preliminary results on the joint design of the observation and reconstruction filters in a computational imaging system. The proposed framework can be used to design series of physically realizable optical filters under various constraints. Specific applications of the proposed designs are being considered for the design of a novel flat form factor imaging system.

5. REFERENCES

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