NON-ITERATIVE IMAGING ALGORITHM FOR CLSAR

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ABSTRACT

Curvilinear synthetic aperture radar (CLSAR), which aperture is formed via a curvilinear trajectory, is considered as a more practical three-dimensional (3-D) imaging system. The 3-D images obtained by using non-parametric methods, however, have little practical use because the data collected by CLSAR is sparse in the 3-D frequency space. Some parametric methods have been successfully applied into CLSAR for imaging but have expensive computational cost since they are iteration methods. In this paper, a non-iterative imaging (NII) algorithm is proposed. The new algorithm estimates the range parameters of all scatterers via modern spectrum method, and then using these range estimates and the received data to form the two-dimensional (2-D) data slices, from which the cross-range parameters are estimated. Once the position (range and cross-range) estimates are obtained, the radar cross section (RCS) can be calculated from the data. Simulation results show that the new algorithm can efficiently form the target's 3-D image via CLSAR.

1. INTRODUCTION

By transmitting signals with large bandwidth and utilizing high apertures in both azimuth and height, the curvilinear synthetic aperture radar (CLSAR) has the three-dimensional (3-D) imaging capability. Because the curvilinear aperture is formed by any maneuvering flight in cross-range, the data collected via CLSAR is limited on a curved face and is not the full volume but sparse data in the 3-D frequency space. Consequently, the image obtained via non-parametric methods has little practical use. The useful images must be rebuilt by some parametric methods[1][2][3].

Recently, an efficient algorithm, LODIPS, is proposed for CLSAR imaging system[3]. Based on the relaxation idea, the LODIPS algorithm extracts the scatterers one by one, and furthermore, with considering the loose coupling between the range and the cross-range parameters, the LODIPS algorithm reduce the 3-D FFT into one-dimensional (1-D) and two-dimensional (2-D) FFTs to obtain every scatterer's position estimate. Due to utilizing the lower dimensional FFTs, the LODIPS algorithm dramatically reduces the computational burden.

This work is supported by the Specialized Research Fund for the Doctoral Program of Higher Education under grant 20030003006.

In this paper, a non-iterative imaging algorithm, which is referred to as NII, is presented for the spotlight mode CLSAR. The new algorithm follows the main idea of LODIPS, which decouples the range and cross-range parameters and estimates them in sequence. NII algorithm, however, is not in iterative mode but in joint mode to estimate all range estimates at every look angle, which are obtained by utilizing modern spectrum method, such as MUSIC[4], ESPRIT[5], and so on. With these range estimates, the cross-range and complex amplitude estimates can be estimated from the received data.

2. PROBLEM FORMULATION

For far field, small patch imaging, the data received by a spotlight mode CLSAR, after dechirping and sampling, can be modelled as[2]

$$r(n,m) = \sum_{k=1}^{K} \alpha_k s_k(n,m) + e(n,m) \tag{1}$$

where K denotes the number of scatterers, α_k is the k^{th} scatterer's radar cross section (RCS), $e(n,m)$ denotes the noise and clutter, and

$$\begin{aligned} s_k(n,m) = \exp\{&jx_k(\tau_n \cos\theta_m \cos\phi_m) \\ &+jy_k(\tau_n \sin\theta_m \cos\phi_m) + jz_k(\tau_n \sin\phi_m)\} \end{aligned} \tag{2}$$

is a 3-D complex sinusoid with the frequency (x_k, y_k, z_k), which corresponds to the 3-D location of the k^{th} scatterer, τ_n $(n = 1, \cdots, N)$ denotes the wavenumber, $\{\theta_m, \phi_m\}$ $(m = 1, \cdots, M)$ are the look angles (the azimuth and elevation angles), N and M are the dimensions of available data samples in range and cross-range, respectively. It notes that the 3-D frequency (x_k, y_k, z_k) is not the scatterer's true position but very close to it for far field imaging[2].

Our problem of interest herein is to estimate $\{\alpha_k, x_k, y_k, z_k\}_{k=1}^{K}$ from $r(n,m)$, which can be achieved by minimizing the following nonlinear least squares (NLS) cost function

$$C(\{\alpha_k, x_k, y_k, z_k\}_{k=1}^K) = \sum_{n=1}^{N}\sum_{m=1}^{M}\left|r(n,m) - \sum_{k=1}^{K}\alpha_k s_k(n,m)\right|^2 \quad (3)$$

If the noise is a zero-mean white Gaussian process, the NLS appproach is equivalent to the maximum likelihood (ML) method. In case the noise is colored or non-Gaussian, it has been shown that the NLS approach can still have good statistical estimation performance[6].

To minimize C is a highly nonlinear optimization problem. This problem, however, can be simplified by using the data's inherent characteristic. Based on the fact that there is loose coupling between the range and cross-range parameters for far field, small patch imaging, the NII algorithm estimates these parameters in sequence. Because the optimizing dimensions is decreased, the new algorithm can reduce the computational complexity.

3. NII ALGORITHM

Since the varying ranges of θ_m and ϕ_m are very small, $\cos\theta_m \approx 1$, $\cos\phi_m \approx 1$, $\sin\theta_m \approx \theta_m$ and $\sin\phi_m \approx \phi_m$. Therefore, eqn. (2) can be simplified as

$$s_k(n,m) = e^{j[x_k + y_k\theta_m + z_k\phi_m]\tau_n} \quad (4)$$

It is evident that $s_k(n,m)$ can be considered as a 1-D sinusoid at the m^{th} look angle with frequency

$$f_k(m) = x_k + y_k\theta_m + z_k\phi_m \quad (5)$$

Hence, the received data $r(n,m)$ at the m^{th} look angle can be considered as the sum of several 1-D sinusoids. These sinusoids' frequency $\{\hat{f}_k(m)\}_{k=1}^K$ can be estimated via MUSIC. Obviously, the frequency $f_k(m)$, as a function of look angle, slightly varies around the range parameter x_k. Therefore,

$$\hat{x}_k = \frac{1}{M}\sum_{m=1}^{M} f_k(m) \quad (6)$$

Hence, for the m^{th} look angle, the value at range $\hat{x}_k$ is calculated by

$$d_k(m) = \frac{1}{N}\sum_{n=1}^{N} r(n,m)\cdot e^{-j\hat{x}_k\tau_n} \quad (7)$$

Substitute $r(n,m)$ in eqn. (1) into eqn. (7),

$$d_k(m) = \sum_{l=1}^{K}\alpha_l e^{j\tau_c\Delta p_{l,k}(m)} P_{l,k}(m) + e(m) \quad (8)$$

where $\tau_c = \tau_1 + \Delta\tau(N-1)/2$ with $\Delta\tau$ denoting the wavenumber step, $\Delta p_{l,k}(m) = x_l + y_l\theta_m + z_l\phi_m - \hat{x}_k$, $P_{l,k}(m) = \frac{\sin(N\Delta\tau\Delta p_{l,k}/2)}{N\sin(\Delta\tau\Delta p_{l,k}/2)}$, and $e(m)$ is the noise in $d_k(m)$.

Assume that all scatterers can be separated at every look angle, then $P_{l,k}(m) \approx 0$ for $l \neq k$, $P_{l,k}(m) \approx 1$ for $l = k$, and eqn. (8) can be rewritten as

$$d_k(m) = \alpha_k e^{j\tau_c(x_k - \hat{x}_k)} e^{j\tau_c(y_k\theta_m + z_k\phi_m)} + e(m) \quad (9)$$

Obviously, the data $d_k(m)$ is a 2-D sinusoid with complex amplitude $\alpha_k e^{j\tau_c(x_k-\hat{x}_k)}$ and 2-D frequency (y_k, z_k). Eqn. (9) shows that the range error $(x_k - \hat{x}_k)$, which is constant, only results in the ambiguity between the phase error due to $(x_k - \hat{x}_k)$ and the phase of α_k, and does not introduce any effect in the 2-D frequency (y_k, z_k). Hence, the 2-D frequency (y_k, z_k) can be determined exactly by

$$(\hat{y}_k, \hat{z}_k) = \arg\max_{(y_k,z_k)}\left|\sum_{m=1}^{M} d_k(m) e^{-j\tau_c(y_k\theta_m + z_k\phi_m)}\right|^2 \quad (10)$$

Substitute the obtained position estimates $\{\hat{x}_k, \hat{y}_k, \hat{z}_k\}_{k=1}^K$ into eqn. (3), then minimizing the cost function is reduced into solving the least square problem, i.e., all amplitude estimates can be calculated by the following equation

$$\mathbf{A} = (\mathbf{S}^H\mathbf{S})^{-1}\mathbf{S}^H\mathbf{r} \quad (11)$$

where

$$\begin{aligned}
\mathbf{A} &= [\alpha_1, \alpha_2, \ldots, \alpha_K]^T \\
\mathbf{S} &= [\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_K] \\
\mathbf{s}_k &= [s_k(1,1), \ldots, s_k(N,1), s_k(1,2), \ldots, s_k(N,M)]^T \\
\mathbf{r} &= [r(1,1), \ldots, r(N,1), r(1,2), \ldots, r(N,M)]^T
\end{aligned}$$

Utilizing the above non-iterative scheme, we get all initial estimates $\{\hat{\alpha}_k, \hat{x}_k, \hat{y}_k, \hat{z}_k\}_{k=1}^K$. These estimates, however, contain some errors, which are introduced by the estimation scheme and interference between scatterers. To minimize these errors and get fine estimates, a local search is needed.

For refining the l^{th} scatterer's position estimate $(\hat{x}_l, \hat{y}_l, \hat{z}_l)$, we define

$$r_l(n,m) = r(n,m) - \sum_{k=1,k\neq l}^{K}\hat{\alpha}_k\hat{s}_k(n,m) \quad (12)$$

where $\hat{s}_k(n,m)$ has the same form as $s_k(n,m)$ in eqn. (2) except that (x_k, y_k, z_k) are replaced by $(\hat{x}_k, \hat{y}_k, \hat{z}_k)$.

With the definition in eqn. (12), the cost function C in eqn. (3) can be rewritten as

$$C_l(\alpha_l, x_l, y_l, z_l) = \sum_{n=1}^{N}\sum_{m=1}^{M}|r_l(n,m) - \alpha_l s_l(n,m)|^2 \quad (13)$$

Minimizing C_l yields

$$(\hat{x}_l, \hat{y}_l, \hat{z}_l) = \arg \max_{(x_l, y_l, z_l)} \left| \sum_{n=1}^{N} \sum_{m=1}^{M} s_l^H(n,m) r_l(n,m) \right|^2 \tag{14}$$

which can be obtained via a local search starting the initial position estimate of the l^{th} scatterer. After refining the l^{th} position estimate, all the amplitude estimates is recalculated by using eqn. (11).

With the above preparations, the new algorithm can be summarized as follows.

Step 1: Estimating the initial estimates.

Substep 1: Extract $\{\hat{f}_k(m)\}_{m=1}^{M}$ at every look angle via MUSIC. Using eqn. (6), calculate the range estimates $\{\hat{x}_k\}_{k=1}^{K}$.

Substep 2: Form the 2-D sinusoids $\{d_k(m)\}_{k=1}^{K}$ via eqn. (7) by using $\{\hat{x_k}\}_{k=1}^{K}$, and obtained the cross-range estimates $\{\hat{y}_k, \hat{z}_k\}_{k=1}^{K}$ by using eqn. (10).

Substep 3: With all position estimates $\{\hat{x}_k, \hat{y}_k, \hat{z}_k\}_{k=1}^{K}$, calculate the complex amplitude $\{\hat{\alpha}_k\}_{k=1}^{K}$ from the data $r(n,m)$ by using eqn. (11).

Step 2: Refining the estimates.

Substep 1: Calculate the l^{th} residual data $r_l(n,m)$ by eqn. (12), and refine the l^{th} scatterer's position estimate $(\hat{x}_l, \hat{y}_l, \hat{z}_l)$ with $r_l(n,m)$ via a local search.

Substep 2: Redetermine the amplitude estimates $\{\hat{\alpha}_k\}_{k=1}^{K}$ with the position estimates $\{\hat{x}_k, \hat{y}_k, \hat{z}_k\}_{k=1}^{K}$ by using eqn. (11).

Repeat the above two substeps for $l = 1, \ldots, K$.

Repeat the step 2 until "practical convergence", which is determined by checking the relative change of the cost function C in eqn. (3) between two consecutive iterations.

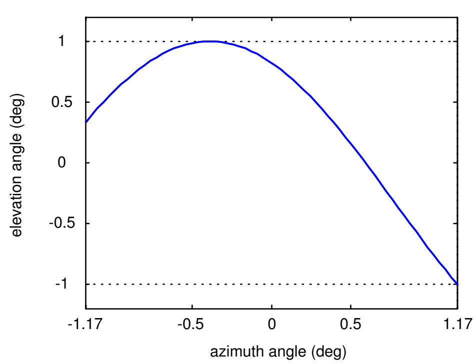


Fig. 1. Curvilinear aperture for simulation examples.

4. SIMULATION RESULTS

In this section, some simulation examples are provided to indicate the performance of the NII algorithm. The curvilinear aperture considered herein is shown in Fig. 1, which consists of $M = 64$ look angles, and the spans of azimuth and elevation are $-1.17° \sim +1.17°$ and $-1.0° \sim +1.0°$, respectively. Assume that there are thirteen scatterers in the scene, which image, as shown in Fig. 2, are obtained by FFT with using the full volume data, which is backscattered by these scatterers.

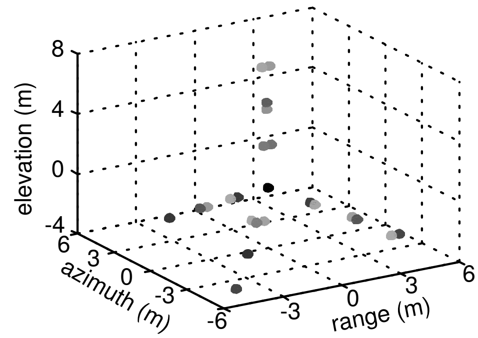
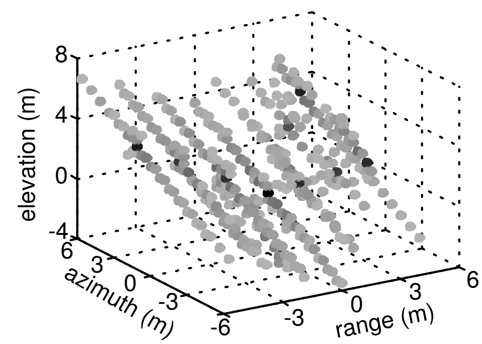
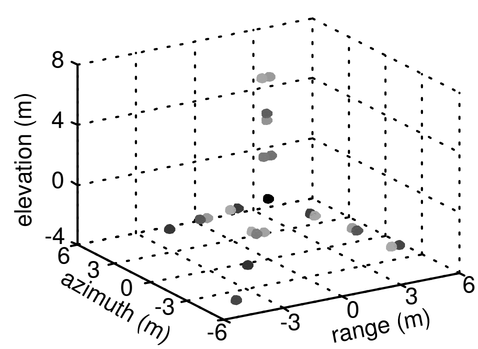


Fig. 2. Original target image.



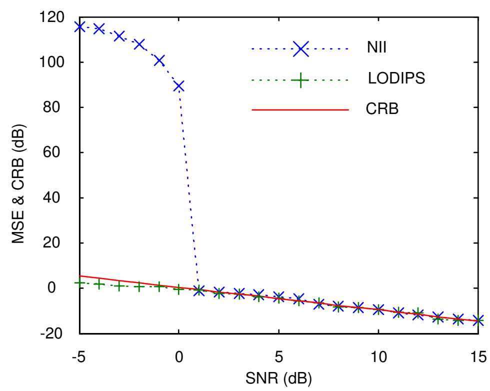
(a) Obtained directly by FFT



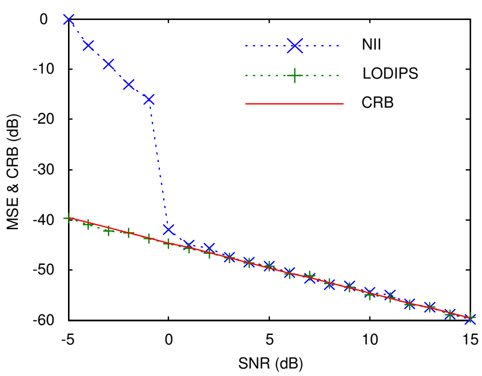
(b) Obtained by NII

Fig. 3. Images obtained by FFT and NII.

We now first show that the NII algorithm has the 3-D imaging capability. In case of noise free, the data collected via curvilinear aperture is processed to obtain the target images, as shown in Fig. 3(a) and (b), by using the non-parametric FFT method and NII, respectively. As compared to Fig. 2, the image in Fig. 3(b) is obviously better than that in Fig. 3(a).



(a) Amplitude estimate



(b) Position estimate

Fig. 4. Comparison of MSEs obtained via NII and LODIPS with corresponding CRBs for amplitude and position estimates.

As the above mentioned, the NII algorithm changes the imaging problem into a parameter estimation problem. Hence, Cramer-Rao bound (CRB), as an evaluation criterion for parameter estimator, can be used to evaluate the new algorithm. Assume that the additive noise is a zero-mean white Gaussian random process, with variance δ^2, and define signal-noise-ratio (SNR) as $||\alpha_{min}||^2/\delta^2$, where α_{min} is the faintest scatterer's RCS. The position error is defined as the distance between the estimated and true positions of scatterer. The mean square errors (MSEs) of the amplitude and position estimates of a scatterer obtained via NII and LODIPS are compared with the corresponding Cramer-Rao bounds (CRBs) in Fig. 4 as a function of SNR (The results for the other point scatterers are similar and hence are not shown here due to the space limitation). The MSEs are obtained through 200 Monte-Carlo trials. From Fig. 4, it can be noted that the MSEs of the estimates obtained via both algorithms can reach the CRBs.

5. CONCLUSION

In this paper, the NII algorithm is proposed for 3-D image formation via CLSAR. The new algorithm transforms the imaging problem into the parameters estimation problem for 3-D sinusoids, which can be performed by minimizing the cost function. With utilizing the received data's inherent characteristic, the NII algorithm estimates the range, cross-range, and RCS of scatterers in sequence. Due to employing the lower dimensional optimization in non-iterative mode, the new algorithm can dramatically reduces the computational cost. Simulation results show that the NII algorithm is an efficient estimator for extracting the target's features in CLSAR.

6. REFERENCES

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