# A NEW FRAMEWORK FOR CHARACTERIZATION OF HALFTONE TEXTURES

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**ABSTRACT**

Characterization of halftone texture is important for quantitative assessment of halftone quality. In this paper, we develop a new framework based on directional local sequency analysis and a filter bank structure. We decompose a halftone image into subband images from which we can easily reconstruct the original halftone. Based on these subband images, we define the directional sequency spectrum which is analogous to the 2-D Fourier spectrum, and formulate several texture measures. Three test images are used to justify these measures.

## 1. INTRODUCTION

Characterization of halftone textures is challenging because of the difficulties in modeling all aspects of the human visual system (HVS), the non-ideal behavior of the rendering device, and the fact that a halftone texture is a very particular type of binary image with its spectral energy well-separated from the origin. Ulichney [1] identified a spectral property of potentially good halftone textures of constant-tone images, and termed the characteristic *blue noise*. He used the *radially averaged power spectrum* (RAPS) and *principal frequency* to describe an ideal spectrum for dispersed-dot halftone images. In addition, he formulated a measure of *anisotropy* and used it as a criterion of goodness. His work is based on the Fourier transform and has provided the basis for development of a number of novel halftoning algorithms [2].

In this paper, we propose a new framework for halftone texture analysis based on the idea of image decomposition according to different *sequencies* at a certain orientation. From the statistics of all the subband images, we define several characteristics to analyze halftone textures: In analogy to the 2-D power spectrum, the *directional sequency spectrum* (DSS) manifests the overall spectral characteristics of a given halftone. Similarly, the *radially averaged directional sequency spectrum* (RADSS) and the *directional asymmetry measure* reveal halftone characteristics which are, respectively, analogous to the RAPS and anisotropy formulated by Ulichney [1]. Our other metrics for *inhomogeneity* and *aperiodicity* can be found in [3].

## 2. THE NEW FRAMEWORK FOR HALFTONE IMAGE DECOMPOSITION

*Calculation of sequency*: There are limitations of Fourier analysis as applied to binary images. The notion of *sequency* [4][5] can overcome some of these problems. In this paper, we define sequency as the number of 0-1 and 1-0 transitions of a given binary sequence, normalized by $N_s - 1$, where $N_s$ is the length of the sequence. Specifically, let $b_0 b_1 \cdots b_{N_s-1}$ be a binary sequence with each $b_i,\ i \in \{0, \cdots, N_s - 1\}$, taking on the value 0 or 1. Its sequency $s$ is computed according to

$$s = \frac{1}{N_s - 1} \sum_{i=0}^{N_s-2} I(b_i \neq b_{i+1}), \qquad (1)$$

where $I(\cdot)$ is the indicator function taking on value 1 if its argument is true or 0 otherwise.

*The quasi filter bank structure*: The proposed framework for texture analysis, schematically illustrated in Fig. 1, consists of many channels, each tuned to a particular angle. Each angular channel represents the response of the HVS at a certain orientation. For a given halftone image $g[m,n]$ at angle $\theta_i,\ i \in \{0, \cdots, N_d - 1\}$, the sequency selection filter (SSF) decomposes $g[m,n]$ into subband images $h_s^{\theta_i}[m,n],\ s = 0,\ 1/(N_s - 1), \cdots,\ 1$, with each corresponding to a certain sequency band.

*Sequency selection filter* (SSF): The SSF is a 1-D nonlinear operation which processes a given binary sequence in a way analogous to that of a 1-D linear filter. Within a window of size $N_s$ from the given binary sequence, the SSF calculates the sequency by counting the number of $0 - 1$ and $1 - 0$ transitions according to (1). The dot addressing method explained in [3] provides a way to perform 1-D operations on a 2-D image. Therefore, we can easily apply

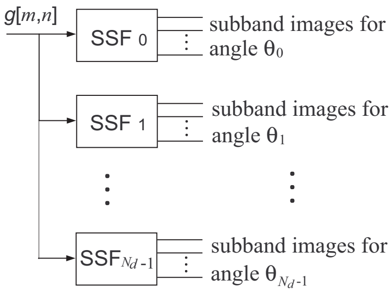


**Fig. 1**. The proposed framework for texture analysis with each channel performing an image decomposition according to different sequencies at a certain angle.

the SSF to a given halftone image in different angular directions. The sequency calculation for directions other than $0°$ and $90°$ is slightly modified and also described in [3]. We perform the SSF only at the locations of the minority pixels, with each being treated as the center point of a length-$N_s$ sequence. This will yield $N_s$ subband images. The number $N_s$ can be regarded as the filter length.

We assume, without loss of generality, that the pixel values of minority and majority pixels are 1 and 0, respectively. From the SSF, we can obtain the sequency for each pixel at the angle $\theta_i$, denoted as $s_{\theta_i}[m,n]$, and create the subband images $h_s^{\theta_i}[m,n],\ s = 0,\ 1/(N_s-1), \cdots,\ 1$, according to

$$h_s^{\theta_i}[m,n] = \begin{cases} 1, & \text{if } g[m,n] = 1 \text{ and } s_{\theta_i}[m,n] = s, \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

Then, for any $\theta_i,\ i = 0, \cdots, N_d - 1$, the original halftone $g[m,n]$ can be easily reconstructed by superposing the subband images.

We consider the number of minority pixels in each subband image as the energy of the corresponding sequency band. We can plot the energy as a function of the sequency for each orientation. This plot is referred to as the *directional sequency spectrum* (DSS) $P_{\theta_i}(s)$, which can be expressed as

$$P_{\theta_i}(s) = \sum_m \sum_n h_s^{\theta_i}[m,n]. \tag{3}$$

Again, we assume that the value of the minority pixel is 1.

## 3. CHARACTERIZATION OF HALFTONE TEXTURES

Based on the directional sequency spectrum $P_{\theta_i}(s)$ defined by (3), we formulate two additional halftone characteristics in this section.

*Radially averaged directional sequency spectrum*: We average the directional sequency spectra $P_{\theta_i}(s)$ over the $N_d$ different orientations to yield the *radially averaged directional sequency spectrum* (RADSS)

$$\bar{P}(s) = \frac{1}{N_d} \sum_{i=0}^{N_d-1} P_{\theta_i}(s). \tag{4}$$

This spectrum is analogous to the RAPS defined in [1]. The function $\bar{P}(s)$ manifests the energy distribution among different sequencies of a given halftone image; but it does not provide information about directional or spatially local structures in the halftone texture.

*Directional asymmetry*: We define a *directional asymmetry* measure $\mathcal{D}(\delta)$ as the average weighted energy difference between directions which differ by a certain angle. To be precise, let $\delta$ denote the difference in the angle index over which we wish to measure the asymmetry. We define

$$\mathcal{D}(\delta) = \frac{1}{N_s N_d} \sum_{i=0}^{N_s-1} \sum_{j=0}^{N_d-1} I_{T_P} \frac{|P_{\theta_j}(s_i) - P_{\theta_{[j+\delta]_{N_d}}}(s_i)|}{P_{\theta_j}(s_i) + P_{\theta_{[j+\delta]_{N_d}}}(s_i)}. \tag{5}$$

Here, $s_i = i/(N_s - 1)$; $[x+y]_{N_d}$ denotes $(x+y)$ mod $N_d$ so that the angle restarts from the beginning after $2\pi$ is passed; and $I_{T_P} = I(P_{\theta_j}(s_i) + P_{\theta_{[j+\delta]_{N_d}}}(s_i) > T_P)$ is the indicator function where $T_P$ is a predetermined threshold. The indicator function is intended to prevent terms for which the denominator is very small from unduly influencing the overall summation. The threshold $T_P$ is chosen to be 0.2 percent of the total energy.

## 4. EXPERIMENTAL RESULTS

To compare our formulation with that in [1], we use three images, shown in Fig. 2 and generated by applying, respectively, dual-metric direct binary search (DBS) [6], Floyd and Steinberg error diffusion (FSED) [7], and error-quantized tone-dependent error diffusion (EQ-TDED) [8] to the same constant-tone patch. For the DBS texture, we use an initial halftone generated by a $128 \times 128$ dispersed-dot screen designed using DBS. For the EQ-TDED texture, the quantizer reproduction levels are adjusted in such a way that the resulting halftone image contains limit-cycle artifacts.

Figure 3 shows the corresponding power spectra for the halftones in Fig. 2. We also show the RAPS and anisotropy measures, respectively, in Figs. 4 and 5. Throughout this paper, we use window size $N_s = 19$ and number of orientations $N_d = 32$ for the SSF operation. This is chosen to yield a reasonably fine granularity in the characterization of the local structure in the images.

*Directional sequency spectrum*: Figure 3 also shows the DSS for the halftone textures in Fig. 2. Here each point in the DSS represents the sequency given by the length of a line drawn from that point to the origin, and a scan path angle $\theta$ given by the angle that the line makes with the positive

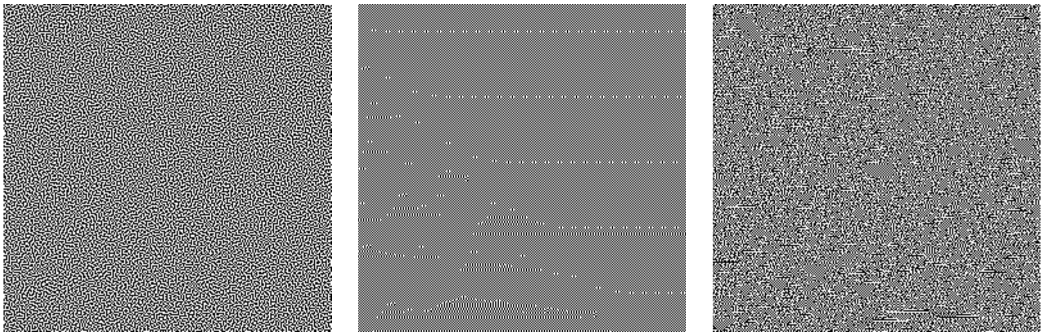

**Fig. 2**. Halftone images of a constant-tone patch of absorptance level 127/255. These images are generated by (from left to right) dual-metric direct binary search (DBS), Floyd and Steinberg error diffusion (FSED), and error-quantized tone-dependent error diffusion (EQ-TDED) with deliberate degradation.

horizontal axis. The maximum sequency value of one corresponds to the distance from the center of the image to the center of the left, right, top, or bottom edges of the image.

Comparing Figs. 2 and 3, we see that the dominant sequency varies very directly with the reciprocal of the mean minority pixel spacing. This is in contrast to the Fourier spectra which do not show the expected amount of change in actual principal frequency. This failure of the equation for ideal principal frequency near the midtones is well known [9]. It is not surprising that DSS overcomes this limitation since it is calculated directly in the spatial domain by looking at inter-pixel spacings.

*Radially averaged directional sequency spectrum*: Figure 4 shows the RADSS of the three test images. The sequency where the largest peak is located plays a role analogous to that of the principal frequency. We refer to this sequency as the *principal sequency*. The principal sequency can easily be understood as the most frequently occurring sequency of the halftone texture observed through a sliding window and scanned from all directions. Comparing RAPS and RADSS in Fig. 4, we see that the point of transition from low energy to high energy in the RAPS, which defines the actual principal frequency, is not nearly as consistent for the DBS and EQ-TDED textures, as is the peak in the corresponding RADSS, which we call the principal sequency.

*Directional asymmetry measure*: In contrast to the RAPS and RADSS which are based on analogous computations with the Fourier spectrum and DSS, respectively, our measure $\mathcal{D}(\delta)$ for directional asymmetry given by (5) is quite different than the anisotropy measure. Whereas anisotropy is a measure of the normalized variance in the power spectrum at each radial frequency $\rho$, $\mathcal{D}(\delta)$ is a measure of the normalized difference between the DSS and its $\delta$-rotated version, integrated over all directions and sequencies. As shown in Fig. 5, the anisotropy does not provide as meaningful a measure of asymmetry. In particular, DBS shows much higher anisotropy than EQ-TDED. The high anisotropy of this DBS texture is due to a residual periodicity remaining from the $128 \times 128$ screen used to initialize the DBS halftone, which is nonetheless imperceptible. On the other hand, our $\mathcal{D}(\delta)$ measure agrees very well with the visual impression of the three textures.

## 5. CONCLUSION

We have developed a new framework for halftone texture characterization that is based on the concept of sequency, rather than frequency. We introduced the sequency selection filter (SSF) that for a given angular orientation decomposes the halftone image into a set of sequency bands. Using the sequency selection filter, we introduced the directional sequency spectrum (DSS) that is analogous to the 2D Fourier power spectrum. From the DSS, we can compute the radially averaged directional sequency spectrum (RADSS) and the directional asymmetry that are analogous, respectively, to the radially averaged power spectrum and anisotropy computed from the 2-D power spectrum.

## 6. REFERENCES


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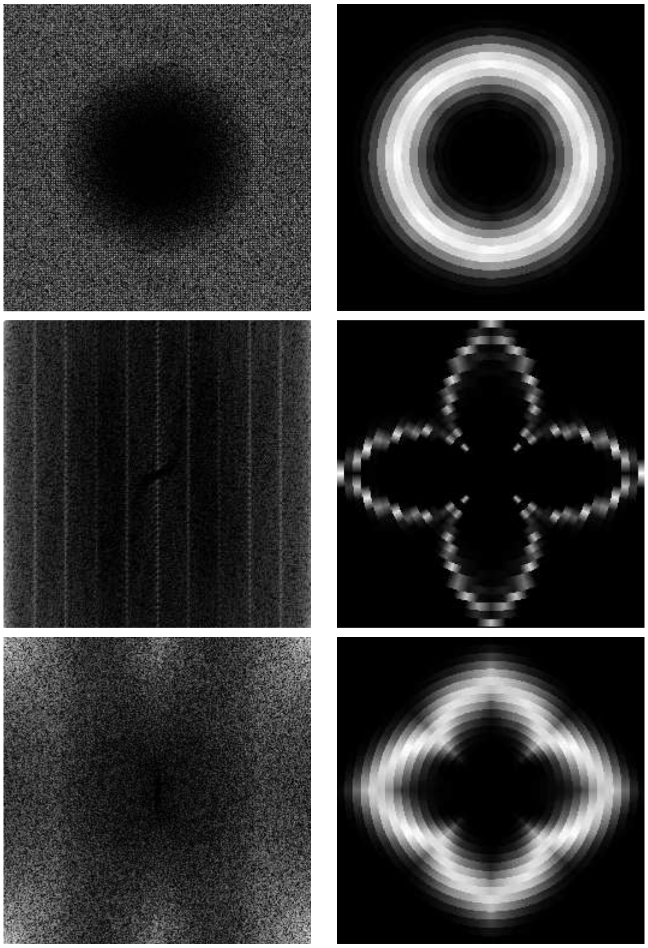

**Fig. 3**. Power spectra (left column) and directional sequency spectra (DSS) (right column) of halftone textures shown in Fig. 2 which correspond to (from top to bottom) DBS, FSED, and EQ-TDED. To better show low intensity features, these spectra were logarithmically compressed according to $i_{OUT} = \frac{255}{\log(K+1)} \log(K \frac{i_{IN}}{\max(i_{IN})} + 1)$, with selected $K$ values. For each DSS, the energy is linearly interpolated between the angles $\{\theta_i,\ i = 0, \cdots, N_d - 1\}$.


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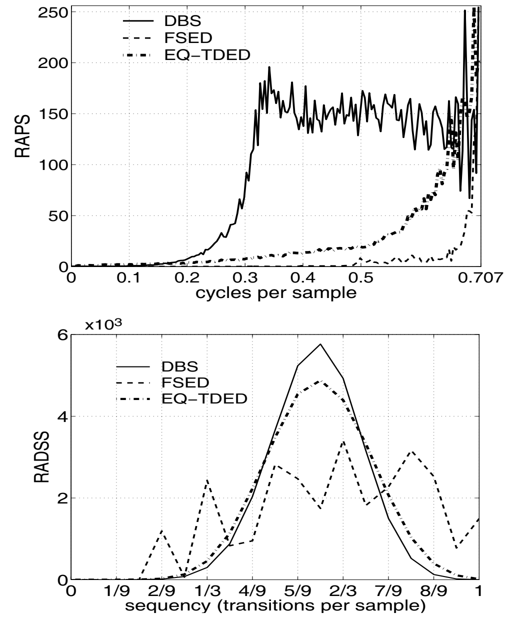


**Fig. 4**. The RAPS (top) and RADSS (bottom) of the halftone images in Fig. 2. We use the same logarithmic conversion equation as in Fig. 3 with $K = 1$ for all RAPS.

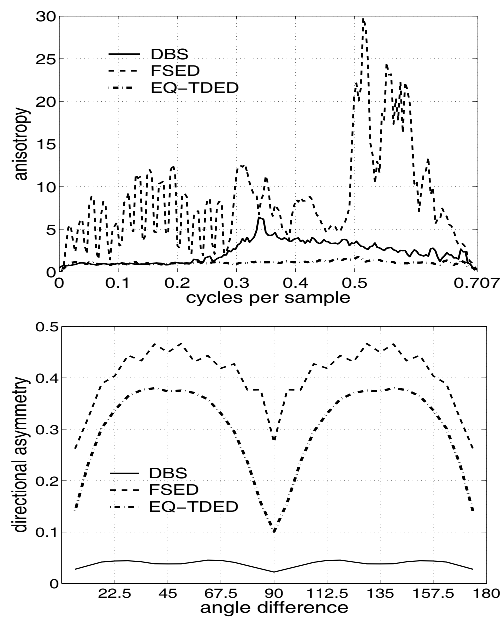


**Fig. 5**. Anisotropy measure (top) and directional asymmetry measures $\mathcal{D}(\delta)$ (bottom) for the halftone images in Fig. 2.


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