A EDGE-PRESERVING MINIMUM CROSS-ENTROPY ALGORITHM FOR PET IMAGE RECONSTRUCTION USING MULTIPHASE LEVEL SET METHOD

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ABSTRACT

Due to the inherent ill-posedness of PET reconstruction, the reconstructed images usually have noise and edge artifacts, and regularization techniques are needed to produce reasonable results. In this paper, we propose a new minimum cross-entropy (MXE) image reconstruction method for PET based on the total variation (TV) norm constraint. The use of TV is due to the fact that it can effectively reduce the noise in 2D images while preserving edges. In addition, a multiple level set method was incorporated into image reconstruction to identify the shape of the radioactive objects. It is important for some special applications where the shape of tumors should be identified. The initial emission rates which used by multiphase level set method were estimated by using discrete reconstruction method. The experimental results show that the proposed method is more effective.

1. INTRODUCTION

The problem of reconstructing an unknown image from a measurement vector is usually ill-posed and the reconstructed images will have noise and edge artifacts. Therefore, the regularization techniques or Bayesian approaches is needed to produce reasonable reconstructions. The regularization of the ill-posed problem requires smoothing homogeneous areas of the object without degrading the edges, which are very important attributes of the image. In the past decades, there are abundant literatures that reported the edge-preserved regularization methods, but most of them rely on information from a local neighborhood to determine the presence of edge[1, 2]. In this paper, we proposed a minimum cross-entropy algorithm for PET image reconstruction using level set method (LS-MXE). The method described here uses the total variation (TV) as a prior to regularize the MXE algorithm. The motivation for us to utilize the TV is that it can suppress effectively the noise while capturing the sharp edges without oscillation, since both noise and edges contribute to high energy component. In addition, this paper incorporates the multiphase level set method into PET image construction. By this way, sharp boundaries between different tissues are directly given for PET image. It is mention that our algorithm improve Lysaker's algorithm [3], in which all intensity values are assumed to be approximately known. But in actual PET reconstruction, we only know the projection data rather than the emission rates (density) and the shape of radioactive. Our algorithm estimates the radioactivity densities by discrete reconstruction in advance, and then level set method was used to describe the geometry of the different tissues in the reconstruction image. To demonstrate the effectiveness of the proposed method, some results of the application of LS-MXE reconstruction to simulated data are presented.

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2. METHODOLOGY

In 1982, Shepp and Vardi [4] developed a Poisson model for the PET process. In the Poisson model, the emissions are modeled as a spatial inhomogeneous Poisson process with unknown intensity. Maximum likelihood (ML) estimation is a well established method in the field of statistics for estimating the value of an unknown parameter. The ML estimate of the activity $\mathbf{x} = \{x_j\}$ is obtained by maximizing the log-likelihood function of the measured emission data $\mathbf{y} = \{y_i\}$, which is given by

$$L(\mathbf{x}) = \sum_i [-\sum_j p_{ij} x_j + y_i \ln(\sum_j p_{ij} x_j) - \ln(y_i!)] \quad (1)$$

where p_{ij} is the probability that a positron emitted at pixel j will be detected by detector pair i.

2.1. The cross-entropy algorithm with total variation regularization

The goal of iterative image reconstruction in emission tomography may be considered as minimizing a given distance measure between the measured photons and the forward projections of the image estimate. This distance function can be shown to be the cross-entropy or Kullback-Leiber (KL) distance between $\mathbf{y}$ and $\mathbf{Px}$[5]. The KL distance between $\mathbf{y}$ and $\mathbf{Px}$ is given by

$$D(\mathbf{y}, \mathbf{Px}) = \sum_i [y_i \ln y_i - y_i \ln(\mathbf{Px})_i - y_i + (\mathbf{Px})_i] \quad (2)$$

where $(\mathbf{Px})_i$ denotes the ith component of vector $\mathbf{Px}$.

$$(\mathbf{Px})_i = \sum\nolimits_j p_{ij} x_j \quad (3)$$

The TV norm for two-dimensional case is defined as:

$$TV(f) = \int_\Omega |\nabla f| dxdy = \int_\Omega \sqrt{f_x^2 + f_y^2} dxdy \quad (4)$$

where $f_x = \frac{\partial}{\partial x} f$, $f_y = \frac{\partial}{\partial y} f$. Pania evaluated the energy function using a two-point difference in 2D as[6]:

$$U_{TV} = \sum_{i,j} \sqrt{(f_{i+1,j} - f_{i,j})^2 + (f_{i,j+1} - f_{i,j})^2 + \sigma^2} \quad (5)$$

The parameter σ is equal to approximately 1% or less of the expected maximum value of f. Too large value of σ will smooth out the edges in the image [6].

The method described here is a new method with two terms: a KL distance term and a TV penalty term for regularization. The estimated image $\hat{x}$ is given by

$$\hat{x} = \arg\min_x (J_\beta(x)) \quad (6)$$

where $J_\beta(x)$ is the new cost function given by

$$J_\beta(x) = D(\mathbf{y}, \mathbf{Px}) + \beta U_{TV} \quad (7)$$

where β is a weight parameter, which influences the regularization term. Substituting Eq.(2) into Eq.(7), the first partial derivative of J_β for a given pixel x_j is given as follows:

$$\frac{\partial J_\beta(x)}{\partial x_j} = \sum_i \left(\frac{-y_i p_{ij}}{(\mathbf{Px})_i} + p_{ij}\right) + \beta \frac{\partial U_{TV}}{\partial x_j} \quad (8)$$

2.2. Multiphase level set method

The level set method proposed by Osher and Sethian is a versatile tool for tracing interfaces separating a domain Ω into subdomains[7]. Moving the interfaces can be done by evolving the level set functions instead of moving the interfaces directly. Let Γ be a closed curve in $\Omega \subset R^2$. We define a ϕ as a signed distance function by :

$$\phi(x) = \begin{cases} dist(x, \Gamma), & x \in interior \quad of \quad \Gamma \\ -dist(x, \Gamma), & x \in exterior \quad of \quad \Gamma \end{cases} \quad (9)$$

Γ is the zero level set of the function ϕ. Once the level set function is defined, we can use it to represent many piecewise constant functions. Assume that we have two closed curves $\Gamma_1$and Γ_2, and we associate the two level set functions ϕ_1 and ϕ_2. Then the domain Ω is divided into four subdomains[8]:

$$\begin{aligned} \Omega_1 &= \{x \in \Omega, \phi_1(x) > 0, \phi_2(x) > 0\} \\ \Omega_2 &= \{x \in \Omega, \phi_1(x) > 0, \phi_2(x) < 0\} \\ \Omega_3 &= \{x \in \Omega, \phi_1(x) < 0, \phi_2(x) > 0\} \\ \Omega_4 &= \{x \in \Omega, \phi_1(x) < 0, \phi_2(x) < 0\} \end{aligned} \quad (10)$$

Using the Heaviside function, we can express x with possibly up to four pieces of constant values as:

$$x = \lambda_1 H(\phi_1)H(\phi_2) + \lambda_2 H(\phi_1)(1 - H(\phi_2)) + \lambda_3(1 - H(\phi_1))H(\phi_2) + \lambda_4(1 - H(\phi_1))(1 - H(\phi_2)) \quad (11)$$

where

$$H(\phi) = \frac{1}{2}\left(1 + \frac{2}{\pi}\arctan\left(\frac{\phi}{\varepsilon}\right)\right) \quad (12)$$

$\varepsilon \in (0, 1)$. We see that n level set function give the possibility of 2^n regions. If the true x needs less than 2^n regions, we can still use n level set functions since some subdomains are allowed to be empty. The multiregional case is given by [8]. Using chain rule [3], it is easy to see that

$$\frac{\partial J_\beta}{\partial \phi_n} = \frac{\partial J_\beta}{\partial x}\frac{\partial x}{\partial \phi_n}, \quad n = 1, 2 \quad (13)$$

here

$$\begin{aligned} \frac{\partial x}{\partial \phi_1} &= ((\lambda_1 - \lambda_2 - \lambda_3 + \lambda_4)H(\phi_2) + \lambda_2 - \lambda_4)\delta(\phi_1) \\ \frac{\partial x}{\partial \phi_2} &= ((\lambda_1 - \lambda_2 - \lambda_3 + \lambda_4)H(\phi_1) + \lambda_3 - \lambda_4)\delta(\phi_2) \end{aligned} \quad (14)$$

where the Delta function $\delta(\phi) = H'(\phi)$

$$\delta(\phi) = \frac{\varepsilon}{\pi(\phi^2 + \varepsilon^2)} \quad (15)$$

In this method, all intensity values λ must be approximately known. Generally speaking, we only know the projection data. Therefore, accurate estimation of the emission rates is essential for the LS-MXE reconstruction performance.

2.3. Estimating the emission rate

The basic problem behind discrete reconstruction is to determine the specific levels present in a reconstruction and then classifying each pixel to one of these discrete levels [9]. We borrow this way to estimate emission rate of each pixel. This method includes two steps:

1. Optimization step by discrete method

   We assume that each pixel has one of a fixed set of known emission rates. The update equation for x_j can then be written as

   $$x_j^{(k+1)} = \arg\min_q \Big[\sum_i (p_{ij}q - y_i \log(p_{i+}x^{(k)} + p_{ij}(q - x_j^{(k)}))) + (r_1 v_1(q, x_j) + r_2 v_2(q, x_j))\Big] \quad (16)$$

   The elicit of Eq.(16) by using the log-likelihood difference of Eq.(1). $v_1(q, x_j)$ is the number of horizontal and vertical neighbors of x_j which do not have emission rate q , and $v_2(q, x_j)$ are the number of diagonal neighbors of x_j which do not have emission rate q. $r_2 = r_1/\sqrt{2}$. we try all initial q, and select the new density for pixel x_j by a method which make Eq.(16) minimization.

2. Densities estimation

   If we define a region as the collection of all pixels with the same emission rate, we obtain 2^n different regions in the reconstruction. Regions are allowed to be empty. Let $\theta_1, \ldots \theta_{2^n}$ denote the discrete emission rates. We define projection matrix Q for the regions such that $Q_{i\tau}$ is the probability that an emission from the region τ is registered by the ith detector.

$$p_{i+}x = \sum_j p_{ij} x_j = \sum_{\tau=1}^{2^n} \lambda_\tau Q_{i\tau} \qquad (17)$$

   where $Q_i\tau = \sum_{\{j:x_j=\lambda_\tau\}} p_{ij}$ The update of emission rate is given by

$$\theta_\tau^{(k+1)} = \arg\min_{\overline{x}} \{ \sum_i (Q_{i\tau}\overline{x} - y_i \log(Q_{i+}\theta^{(k)} + Q_{i\tau}(\overline{x} - \theta_\tau^{(k)}))\} \qquad (18)$$

   Where $\overline{x} \geq 0$ insures the non-negativity of the emission rates. Emission rate θ will be returned to the first step instead of q. After several iterations, one can get some steady emission rates by these two steps.

The LS-MXE algorithm is written as follows:

1. Choose initial level set functions ϕ_n and $\triangle t$
2. Estimating initial emission rate λ using Eq.(18)
3. Update the level set functions

$$\phi_n^{(k+1)} = \phi_n^{(k)} - \triangle t \frac{\partial J_\beta}{\partial \phi_n^{(k)}} \qquad (19)$$

4. Reinitialize the functions ϕ_n according to [10].

3. SIMULATION

In our simulation experiments, we used a 96×96 pixels thorax phantom to test the feasibility of our proposed method. The relative activities of the elements are shown in Fig.1. The projection space is supposed to be 139 bins and 180 angle evenly spaced over $180°$. The final reconstructed image is set to a size of 96×96 pixel matrices. Fig.2(left) shows the FBP image. About four density values and basic shape were approximately known from the FBP image. Fig.2(right) shows the result of discrete reconstruction described by section 2.3. The phenomenon of densities overlapping using discrete reconstruction is obvious in it. But this method provides a good way to obtain the densities of PET. The reconstructed images using LS-MXE algorithm are shown in Fig.3. The iteration numbers are 140 for each algorithm. The time step is 5, σ is 0.001, r_1 is 1 and ε is 0.5 for all LS-MXE. We can see from Fig.3, with increasing of parameter β, the smoothing effect is also more obvious. TV regularization term smooths the image while preserving the edges. LS-MXE reconstruction significantly improved the quality of the image. The initial level set functions were shown in Fig.4. The evolution of the two function ϕ_1 and ϕ_2 are given in Fig.5.

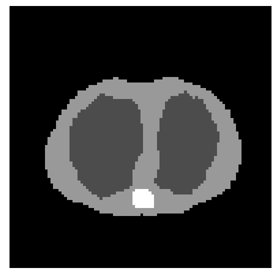

Fig. 1. A simulated emission thorax phantom.

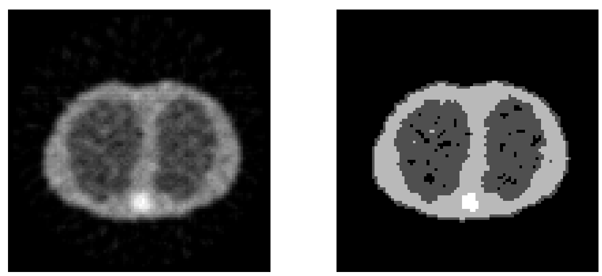

Fig. 2. (from left to right) Reconstruction image using FBP and discrete method.

4. CONCLUSIONS

In this paper, we proposed a novel algorithm (LS-MXE) that combines the level set approach with minimum cross-entropy algorithm based on TV norm regularization for PET image reconstruction. This algorithm has three distinct characteristics. First, the TV norm smoothes the image while preserving the edges. Second, the multiple level sets were used to represent the topology shape of interesting objects. The last is the accurate estimation of the emission rate. The experimental studies clearly show that the LS-MXE method yields the reconstructed images with better contrast and resolution and the geometry is also sketched by two zero level

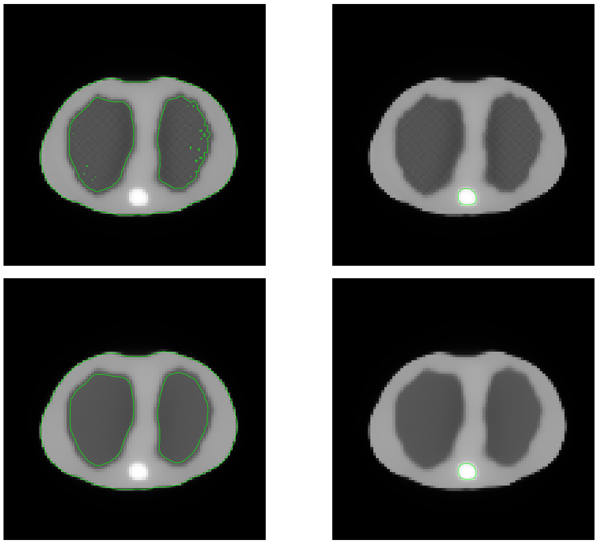

Fig. 3. Reconstruction images using LS-MXE algorithm. First row with $\beta = 0$ and the second row with $\beta = 1.5$.

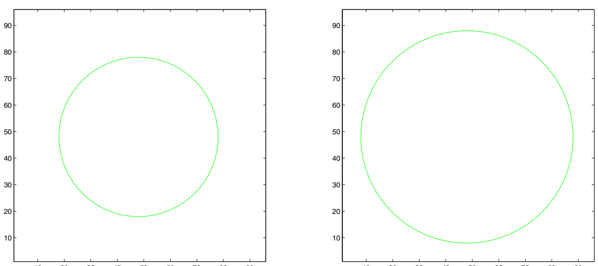

Fig. 4. (from left to right)Initial level set functions ϕ_1 and ϕ_2, respectively.

set functions.

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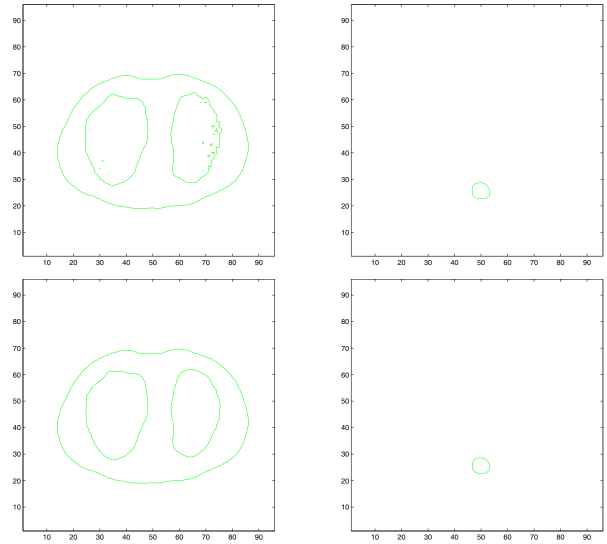

Fig. 5. Interfaces given by the zero level set of the function ϕ_1 and ϕ_2, respectively. First row with $\beta = 0$ and the second row with $\beta = 1.5$.

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