3-D TEXTURE CHARACTERIZATION BASED ON WOLD DECOMPOSITION AND HIGHER ORDER STATISTICS

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ABSTRACT

This paper deals with the parametric 3-D models for volumes and block of images based on the so-called Wold decomposition theory. This decomposition raises purely indeterministic, harmonic and evanescent components. Based on this theory, we recently proposed new explicit parametric models able to describe both the spectral support and spatial structure of each component. When the purely indeterministic is assumed Gaussian and the evanescent component vanishes, we state the problem of separating the 3-D texture into deterministic component and indeterministic one as a 3-D harmonics retrieval problem in a noisy background. We propose to achieve such task using a new 3-D spectral method based on Higher Order Statistics (HOS).

1. Introduction

Textured images have been often classified into two categories, deterministic textures and stochastic textures. A deterministic texture is characterized by a set of primitives and placement rule. A stochastic texture does not have easily identifiable primitives. Many real-world textures have some mixture of these two characteristics. The texture model based on a 3-D Wold-like decomposition theory for homogenous random fields is able of representing both types of characteristics [1]. Indeed, the Wold decomposition theory allows textured image blocks i.e. 3-D texture, to be modeled as a finite realization of a 3-D stationary random field, which is a superposition of three orthogonal component fields: the harmonic, the evanescent and the purely indeterministic ones [2]. The perceptual characteristics of theses components can be described as "randomness" for the purely non deterministic, "periodicity in all directions" for the harmonic field and "periodicity in particular directions" for the evanescent components [1][3]. However, a main challenge in Wold- based texture modeling is to develop an efficient and robust Wold decomposition algorithm for extracting all components from a 3-D texture.

In the 2-D case, some decomposition methods have been proposed in the literature. The first one is based on the conditional Maximum-Likelihood algorithm which jointly estimates the parameters of all components of the texture [4]. The second approach is a spectral decomposition procedure presented in [5-6]. These methods extract the deterministic component by detecting the frequencies of the largest and sharpest isolated peaks of the periodogram. Thus, this approach will cause some error since the second-order statistics are colored noise sensitive and the power spectrum is a mixed one. However, this method is successful when the purely indeterministic component is assumed to be white, or if its correlation is known.

The aim of this paper is to present a new 3-D spectral decomposition method based on HOS. We treat only the case where the purely indeterministic component is a Gaussian zero mean field, and the evanescent component is zero. In this case, the texture decomposition into deterministic and indeterministic parts is equivalent to the 3-D harmonics retrieval problem in colored linear Gaussian noise of unknown covariance. By exploiting the fact that higher-than-second order cumulants are zero we show that a theoretical diagonal slice of the fourth-order cumulants (DSFOC) of the observed data is proportional to the autocorrelation of the deterministic component with identical frequencies. Hence, the decomposition of 3-D texture will be carried out in the spectrum of this DSFOC by estimating the deterministic spectral support.

The remainder of this paper is organized as follows: in Section 2 we recall the 3-D texture model based on the Wold decomposition theory. In Section 3, a new spectral 3-D Wold decomposition algorithm for stationary random fields is presented. Numerical example is included to illustrate the effectiveness of the proposed method in section 4.

2. PREVIOUS WORK

2.1 The 3-D Wold-like decomposition

From the 3-D Wold-like decomposition theory [2], any 3-D stationary random field $\{y(m,n,t)\}$ can be uniquely represented by the following orthogonal decomposition:

$$y(m,n,t)=w(m,n,t)+h(m,n,t)+e(m,n,t) \quad (1)$$

such $w(\ )$ is a purely indeterministic, $h(\ )$ is a harmonic field and $e(.)$ is an evanescent component. Moreover it is shown in [2] that, the evanescent components itself can be decomposed into two orthogonal components called the evanescent of type 1 and the evanescent of type 2. Consequently the 3-D texture field is modeled as a 3-D stationary random field, which is a superposition of four orthogonal fields:

$$y(m,n,t)=w(m,n,t)+h(m,n,t)+e_1(m,n,t)+e_2(m,n,t) \quad (2)$$

This decomposition involves the decomposition of the spectral distribution function (SDF) of $y(\ )$ into four mutually singular parts:

$$F_y(\omega,\nu,u)=F_u(\omega,\nu,\eta)+F_h(\omega,\nu,\eta)+F_{e_1}(\omega,\nu,\eta)+F_{e_2}(\omega,\nu,\eta) \quad (3)$$

Furthermore, if $F_y = f\,d\lambda_3 + F_y^s$ is the Lebesgue decomposition of F_y into absolutely continuous part $F_a = f\,d\lambda_3$ and singular one F_y^s, then $F_w = F_a$ and, $F_h + F_{e_1} + F_{e_2} = F_y^s$. Therefore, the SDF F_h, F_{e_1} and F_{e_2} are supported by sets of Lebesgue measure zero in the frequency space.

2.2 The 3-D texture model

The orthogonal property of the four Wold components leads to independent models of each one separately [1]. The purely indeterministic component can be modeled by a three-dimensional autoregressive model (3-D AR):

$$w(m,n,t)=\sum_{(j,k,l)\in D} a(j,k,l)w(m-j,n-k,t-l)+e(m,n,t) \quad (4)$$

where D is the support of the AR model, and $\{a_{k_1,k_2,k_3}\}$ are the transversal model's parameters.

The evanescent component of type 1 can be modeled by a separable model, given by the product of 2-D purely indeterministic process $S(.,.)$ in two dimensions and 1-D sinusoidal in the orthogonal one

$$e_1(m,n,t) = S(m,n)\sum_{i=1}^{I_1} b_i Exp\left[j(2\pi \nu_i t + \varphi_i)\right] \quad (5)$$

The 3-D texture generated by this model is random looking in two dimensions and structured in the third orthogonal dimension. Its corresponding spectral support has the form of parallel plans in the frequency space [1]. However, the evanescent component of type 2 is represented also by separable model, given by the product of 1-D purely indeterministic process $S(.)$ and a randomly 2-D sinusoidal in two other orthogonal dimensions

$$e_2(m,n,t) = S(m)\sum_{i=1}^{I_2} a_i\, Exp\left[j2\pi(n f_{1_i} + t f_{2_i}) + j\varphi_i\right] \quad (6)$$

This model generates 3-D texture which is structured in two dimensions and random looking in the third orthogonal dimension. In the frequency space, its spectral measure is carried by parallel lines.

Finally, the deterministic harmonic component is modeled by

$$h(n,m,t) = \sum_{i=1}^{P} a_p\, Exp\left[j2\pi(n f_{1_i} + m f_{2_i} + t f_{3_i}) + j\phi_i\right] \quad (7)$$

where the phase ϕ_i are i.i.d uniformly distributed over $(-\pi,\pi)$ and the amplitude a_i are each nonzero deterministic constant. This model generates periodic textures in all directions and its spectral support contains isolated points i.e. 3-D peaks.

3. NEW DECOMPOSITION ALGORITHM

Through the remainder of this paper we use the following notations:

$$m=(m_1,m_2,m_3); \quad w=(w_1,w_2,w_3), \quad \alpha=(\alpha_1,\alpha_2,\alpha_3),$$
$$mf^T = m_1 f_1 + m_2 f_2 + m_3 f_3, \quad u_i(m) = a_i Exp\left[j2\pi m f^T\right],$$
$$S_i = Exp(j\varphi_i). \quad (8)$$

3.1 Cumulants and polyspectra

The cumulants of complex processes may be defined in several ways; the particular definition to be used in a given problem depends upon the nature of the random processes involved [7]. Here, the second- third- and fourth-order cumulants of zero mean stationary complex fields, are defined by

$$c_{2y}(\alpha) = E[y^*(m)y(m+\alpha)],$$
$$c_{3y}(\alpha,\beta) = E[y^*(m)y(m+\alpha)y(m+\beta)],$$
$$c_{4y}(\alpha,\beta,\gamma) = cum[y^*(m)y^*(m+\alpha)y(m+\beta)y(m+\gamma)]$$

Where, E is the expectation operator and the fourth order cum operator is given by

$$cum[y_i,y_j,y_k,y_l] = E[y_i y_j y_k y_l] - E[y_i y_j]E[y_k y_l] - E[y_i y_k]E[y_j y_l] - E[y_i y_l]E[y_j y_k] \quad (9)$$

The k^th^-order spectrum $S_{ky}(\omega_1,\omega_2,...,\omega_{k-1})$ is defined as the discreet Fourier transform (DFT) of $c_{ky}(i_1,i_2,...,i_{k-1})$. Then, The $S_{2y}(\omega)$, $S_{3y}(\omega,\nu)$, and $S_{4y}(\omega,\nu,\mu)$ are called respectively the spectrum, the bispectrum, and the trispectrum of y. Only, since the processes are assumed to be a zero mean, we will need the second and fourth order moments. Thus, the third-order cumulant is identically zero and the bispectrum approach cannot be used in this work.

Proposition
The FOC of the zero mean complex harmonic fields in (7) is given by

$$c_{4h}(\alpha,\beta,\gamma) = -\sum_{i=1}^{P} a_i^4 \, Exp\left[j2\pi(-\alpha+\beta+\gamma)f_i^T\right] \qquad (10)$$

Proof: Since the φ_i's are i.i.d random variables uniformly distributed over $(-\pi,\pi)$, we have

$$E\{S_i S_j^*\} = \delta(i-j),\ E\{S_i S_j\} = 0,\ E\{S_i^* S_j^*\} = 0, \text{ and}$$

$$E\{S_i^* S_j^* S_k S_l\} = \delta(k-i+l-j). \qquad (11)$$

Using (8), (9), and (11), the FOC can be expressed as

$$\begin{aligned} c_{4h}(\alpha,\beta,\gamma) = &\sum_{i,j,s,l} u_i^*(m)u_j^*(m+\alpha)u_k(m+\beta)u_l(m+\gamma)\delta(k-i+l-j) \\ &-\left[\sum_{i,k} u_i^*(m)u_k(m+\beta)\delta(k-i)\right]\left[\sum_{j,l} u_j^*(m+\alpha)u_l(m+\gamma)\delta(l-j)\right] \\ &-\left[\sum_{i,l} u_i^*(m)u_l(m+\gamma)\delta(l-i)\right]\left[\sum_{j,k} u_j^*(m+\alpha)u_k(m+\beta)\delta(k-j)\right] \end{aligned}$$

After some manipulations we have

$$c_{4h}(\alpha,\beta,\gamma) = -\sum_i u_i^*(m)u_i^*(m+\alpha)u_i(m+\beta)u_i(m+\gamma).$$

The proposition is established by replacing u_i by its expression in (8).

From the above proposition we conclude that when $\alpha=\beta=\gamma$ the DSFOC given by $c_{4h}(\alpha)=c_{4h}(\alpha,\alpha,\alpha)$ satisfied

$$c_{4h}(\alpha) = \sum_i -a_i^4 \, Exp\left[j2\pi(\alpha_1 f_{1i}+\alpha_2 f_{2i}+\alpha_3 f_{3i})\right] \qquad (12)$$

Therefore, the considered DSFOC retains all the pertinent information about the number of harmonics, their amplitude and frequencies. Then, the decomposition procedure can be carried out in the power spectrum of this DSFOC.

3.2 Spectral 3-D Wold decomposition algorithm

Let the observed 3-D real valued textured field be $y(m)=w(m)+h(m)$, where $w(m)$ is the Gaussian purely indeterministic components and $h(m)$ the harmonic field

$$h(m) = \sum_{i=1}^{P} a_i \cos\left(mf_i^T + \phi_i\right) \qquad (13)$$

The harmonics frequencies correspond to the 3-D peaks in the spectrum. However, the autocorrelation of $y(m)$ is given by

$$c_{2y}(\alpha)=c_{2w}(\alpha)+c_{2h}(\alpha), \qquad (14)$$

where $c_{2w}(\alpha)$ is unknown. So, the spectrum of $y(m)$ is a mixed one:

$$S_{2y}(\omega)=S_{2w}(\omega)+S_{2h}(\omega). \qquad (15)$$

Consequently, the detection of harmonic peaks in the power spectrum is not easy and can select false peaks due to the colored Gaussian component. To avoid this problem we will use a new power spectrum estimated from the DSFOC. In fact, since the FOC of any Gaussian field is zero, we have $c_{4y}(.)=c_{4h}(.)$, and $S_{4y}(\omega)=S_{4h}(\omega)$.

From the above proposition it can be seen easily that the DSFOC of the real harmonic field is given by

$$c_{4h}(\alpha) = -\frac{3}{8}\sum_{i=1}^{P} a_i^4\left[\cos(\alpha_1 f_{1i}+\alpha_2 f_{2i}+\alpha_3 f_{3i})\right]. \qquad (16)$$

Hence, the harmonics frequencies can be identified from the spectrum $S_{4y}(\omega)$ or as a the 3-D peaks in the power density spectrum

$$P_y(\omega)=\frac{1}{N^3}\left|S_{4y}(\omega)\right|^2. \qquad (17)$$

This 3-D peaks can be localized as a local maxima in this power spectrum. After detecting spectral support of the deterministic component, the texture decomposition problem can be achieved by performing a spectral Lebesgue decomposition i.e. by separating the singular and the absolutely continuous components in the spectrum of the texture field.
The proposed decomposition algorithm is summarized in the following steps:

- Estimate the DSFOC $c_{4y}(\alpha)$, and its power spectrum given by (17).
- Detect the spectral singularities which appear in the power spectrum in (17) as a 3-D peaks and extract the corresponding frequencies Λ_s.
- Compute the 3-D DFT $F(w)$ of y.
- Decompose the DFT $F(w)$ into the singular part and the absolutely continuous one such as $\begin{cases} F_s(w)=F(w) & if\ \ w\in\Lambda_s \\ F_s(w)=0 & otherwise \end{cases}$, and $\begin{cases} F_a(w)=0 & if\ w\in\Lambda_s \\ F_a(w)=F(w) & otherwise \end{cases}$.
- Compute the 3-D inverse DFT of $F_s(w)$ and $F_a(w)$, to obtain the deterministic and the purely indeterministic components respectively.

4. EXPREMENTAL RESULTS

In order to illustrate the effectiveness of the DSFOC based method, a numerical example using synthetic data is presented in this section. Consider a real valued 3-D synthetics texture composed of deterministic field and colored Gaussian component. The deterministic component consists of three unity amplitude sinusoids and the purely indeterministic component is generated by a 3-DAR model with quarter space support

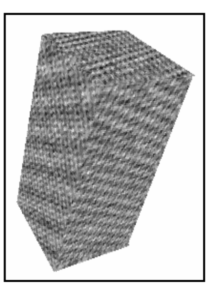
$$w(m,n,t) = -\sum_{\substack{k_1=0 \\ (k_1,k_2,k_3)\neq(0,0,0)}}^{p_1}\sum_{k_2=0}^{p_2}\sum_{k_3=0}^{p_3} a_{k_1,k_2,k_3} w(m-k_1,n-k_2,t-k_3)+e(m,n,t)$$

Where $e(m,n,t)$ is a white Gaussian process with variance $\sigma^2=4$ and $(p_1,p_2,p_3)=(2,2,2)$. Figure (a) shows the generated 3-D texture of size $64\times64\times128$, and Figure (b) shows the power spectrum computed from the autocorrelation function $c_{2y}(\alpha)$. Note that $c_{2y}(\alpha)$, and $c_{4y}(\alpha)$, are estimated by a sample averaging. As we can check from figure (b) the power spectrum computed from the estimated autocorrelation contains the harmonics peaks and also many powerful peaks due to the colored Gaussian component. However, in Figure (c), the power spectrum obtained from the DSFOC shows clearly the number of the harmonic components present in the texture field. So, the harmonic frequencies can be easily extracted from this spectrum. Appling a local threshold to this spectrum we obtain the spectral support of the deterministic component as in figure (d). Extracting this support from the spectrum in figure (b) we take the spectral support of the indeterministic component (figure (e). At this stage the frequencies corresponding to this support are identified, the 3-D Fourier block can be then decomposed into deterministic $F_s(w)$ and indeterministic component $F_a(w)$. The final decomposition results obtained are depicted in figure (g) and (h).

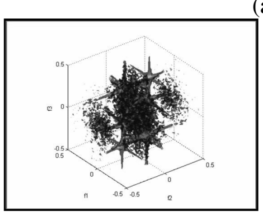
5. CONCLUSION

The purpose of this paper is to exploit the properties of higher order statistics and Wold decomposition theory for the sake of finding an algorithm for 3-D texture decomposition. The proposed method is based on the Lebesgue decomposition of the 3-D Fourier spectral. To separate the spectral supports of deterministic and purely indeterministic component we use the power spectrum estimated from a particular diagonal slice of the fourth-order cumulant. The final decomposition of observed texture is achieved by computing the 3-D inverse Fourier transform of each spectral component. Experimental results showed that this method is effective for 3-D texture decomposition.

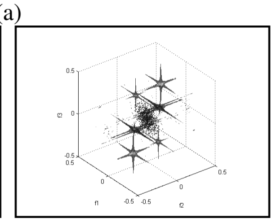
4. FIGURES



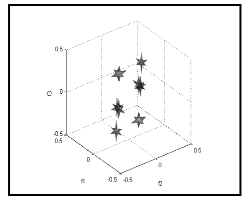
(a)



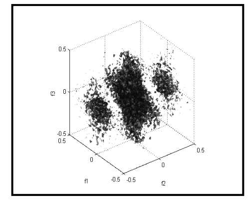
(b)



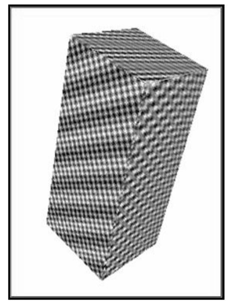
(c)



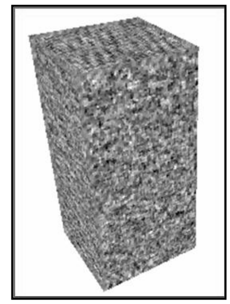
(d)



(e)



(g)



(h)

Figure 1: Experiments on 3-D real value synthetic texture:
(a) original block of images; (b) the power spectrum obtained from autocorrelation function (c) the power spectrum obtained from the DSFOCS; (d); spectral support of deterministic component; (e) spectral support of purely indeterministic component; (g) deterministic component; (h) purely indeterministic component.

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