MRI Image Reconstruction via Homomorphic Signal Processing

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***Abstract*— We consider the use of homomorphic signal processing for image reconstruction in phased-array magnetic resonance imaging (MRI). Based on the prior information provided from the spectral analysis of the estimated true pixels and the coil sensitivities, homomorphic signal processing is used to partly filter out the coil sensitivities based on the criterion of image contrast in the reconstructed image in specified pixel locations. The image quality, quantized as the entropy of the pixel value distribution, demonstrates a better image contrast compared with the widely used Sum-of-Squares (SoS) method.**

I. Introduction

MRI image reconstruction with phased-array coils was first studied by Roemer et al. [1]. They proposed a pixel by pixel reconstruction method, the sum-of-squares (SoS) Method, to reconstruct coil images. They showed that this method loses only 10% of the maximum possible signal-to-noise-ratio (SNR) with no prior information of the coils' positions or RF field maps. We proved an SNR optimality result of the SoS method compared by the optimal linear combination with known coil sensitivities in the high SNR region of all the input coil images [2]. However, this high SNR condition is not always satisfied in noisy coil images. A number of somewhat more sophisticated techniques for image construction with phased-array coils have appeared in recent years. For example, as an alternative to the sum-of-squares reconstruction, Debbins et al. suggested to add the images *coherently* after their relative phase was properly adjusted by another calibration scan [3]. This method increased imaging rate by reducing the demands such as bandwidth and memory while it still kept much of the SNR performance compared to SoS. Walsh et al. used adaptive filters to improve SNR in the image [4]. Kellman and McVeigh proposed a method that can use the degrees of freedom inherent to the phased array for ghost artifact cancellation by a constrained SNR optimization [5]. This method also needed a prior information of reference images without distortion to estimate coil sensitivities. Bydder et al. proposed a reconstruction method that estimated the coil sensitivities from the smoothed coil images to reduce noise effects [6]. Finally, we developed a Bayesian method using the iterative maximum likelihood estimate with prior information in coil sensitivities [7].

All the previous attempts to improve the image quality focused on the SNR performance, with or without prior information. We know that the objective reconstruction image quality is hard to identify due to lack of a true reference image. A reduction in the background noise will increase the computed SNR but affects the image quality little in the interesting signal region. Moreover, the SNR can be changed by a nonlinear transform, in some cases opposed to the image quality, which is implemented in many reconstruction methods. We propose to use image contrast as an objective image quality measure in MRI. It provides more detailed pixel brightness information in the desired signal area and is easily perceived by human eyes. Based on this criterion, we introduce a signal processing method, homomorphic signal processing, to split the two multiplied signals in spatial domain while their spectrum can be separated in the frequency domain. Thus the true pixel values and the coil sensitivities are separated based on their different statistical properties at the level of the whole image.

In this paper, we analyze the spectral distribution of the estimated true pixels and the coil sensitivities for each coil signal. The effect of sensitivities is mainly filtered out by homomorphic signal processing. Some standard image processing methods are implemented to increase the reconstruction image contrast. The image quality is compared with that of SoS.

II. Data Model

Consider a phased-array MRI system with N coils and let s_k be the observed pixel value from coil k:

$$s_k = \rho c_k + e_k, \quad k = 1, 2, \cdots, N \tag{1}$$

where ρ is the (real-valued) object density (viz. the MR contrast), c_k is the (in general complex-valued) sensitivity associated with coil k for the image voxel under consideration, and e_k is zero-mean noise with variance σ_k^2. We assume in this paper that the noise is white; at the price of some additional notation all our results can easily be extended to noise with a general covariance structure.

III. Sum-of-Squares Reconstruction

The SoS method is applicable when $\{c_k\}$ are unknown. The reconstructed pixel is obtained via:

$$\hat{\rho} = \sqrt{\sum_{k=1}^{N} |s_k|^2} \qquad (2)$$

This SoS estimate can be interpreted as an optimal linear combination

$$\hat{\rho} = \frac{\sum_{k=1}^{N} \hat{c}_k^* s_k}{\sum_{k=1}^{N} |\hat{c}_k|^2} \qquad (3)$$

where the coil sensitivity is estimated as $\hat{c}_k$

$$\hat{c}_k = s_k / \sqrt{\sum_{k=1}^{N} |s_k|^2} \qquad (4)$$

[6]. Clearly, if the noise level goes to zero the SoS estimate converges to $\hat{\rho} \rightarrow \rho\sqrt{\sum_{k=1}^{N} |c_k|^2}$ which is in general *not* equal to ρ.

IV. Homomorphic signal processing

Homomorphic signal processing, as a nonlinear signal processing method based on a generalized superposition principle, is widely applied in image enhancement, speech analysis, etc. [8]. A signal modeled as a product of two components can be splitted by using homomorphic signal processing. The MRI signal $|s_k|$ is represented by the product of two positive components, the true pixel ρ and the sensitivity $|c_k|$ $(0 < |c_k| < 1)$ in noise-free case. Fig. 1 shows the canonic form of the discrete homomorphic signal processor.

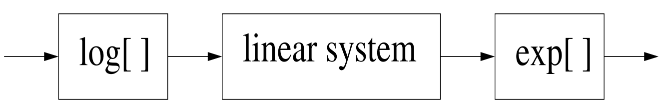


Fig. 1. Canonic form for homomorphic signal processor.

The logarithm function firstly transforms the multiplication of ρ and c_k into an addition.

$$\log|s_k| = \log\rho + \log|c_k| \qquad (5)$$

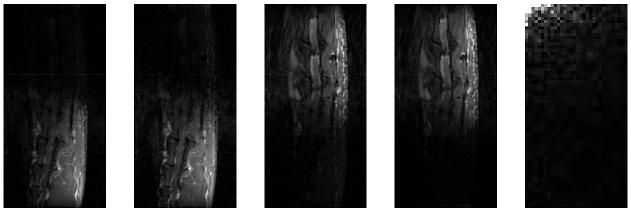
The linear system separates ρ and $|c_k|$ by assuming different spectral contents for each component. The most effective information in the true pixel image is at the sharp boundary between bones and muscles or between bones and tissues because of different water percentages inside. Thus the effective ρ is mostly a high-frequency signal. The magnitude of the coil sensitivities $|c_k|$, related to the coil signals, is relatively slow-varying in signal area and mostly a low-frequency signal. Though they may have some overlap in the low frequency domain, one could partly filter out the coil sensitivities by passing the two-dimensional Fourier transform of the logarithm of the coil image through a high-pass filter. Then an inverse Fourier transform recovers the true pixel signal from the frequency domain to the original spatial domain. The third step is an exponential function that eliminates the effect of the logarithm. The output $\hat{s}_k$ from the homomorphic signal processor for each coil is considered as a multiple sample of pixel image. Thus, the reconstruction is simplified to average $\hat{s}_k$

Fig. 2. Photograph of the phased array coil, transmit coil, and cabling.

$$\hat{\rho} = \frac{1}{N}\sum_{k=1}^{N} \hat{s}_k \qquad (6)$$

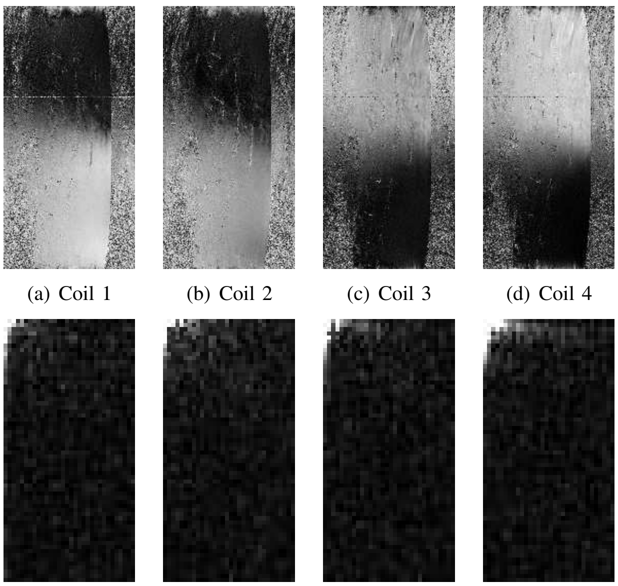
Some image processing methods are implemented to improve the reconstructed image quality. The Gaussian shaped frequency domain filter, which has the same shape in the spatial and frequency domains, is used to remove noise in the noise area. A nonlinear gamma function is used to weight toward the higher pixels. Though the nonlinear transform introduces bias, it increases the image contrast.

The criterion of the filter selection in homomorphic signal processor is a key problem. SNR in the homomorphic signal processing method is not a suitable criterion; on the contrary, the lower SNR is the cost of the method to gain higher image contrast because part of the energy is filtered out in the signal area while the noise is not affected much due to its approximately uniform spectral in the frequency domain. Besides, the MMSE criterion $(min \sum_k |s_k - \hat{\rho}c_k|^2)$ doesn't give the optimal solution because of the computational cancellation due to the splitting of ρ and c_k. We propose the effective maximum image contrast in the reconstructed image as a criterion to choose the high-pass filter. The automatic selection of the highpass filter characteristics is performed via the use of a normalized entropy criterion.



(a) 1 (b) 2 (c) 3 (d) 4 (e) SoS

Fig. 3. Vivo sagittal images of cat spinal cord from coil 1-4 and the spectral estimate of SoS.



(a) Coil 1 (b) Coil 2 (c) Coil 3 (d) Coil 4

(e) Coil 1 (f) Coil 2 (g) Coil 3 (h) Coil 4

Fig. 4. (Upper row) Spatial distribution of the coil sensitivities for four coil signals. (Lower row) Spectral distribution of the coil sensitivities for four coil signals.

V. Numerical Results

The cat spinal cord data is collected by a four-coil phased array showing in Fig. 2 (TR=1000ms, TE=15ms, FOV=10×5cm, matrix=256×128, slice thickness=2mm,sweep width=26khz, 1 average) [9]. Figs. 3(a),3(b),3(c), and 3(d) show the collected four coil images, where coils 1, 2 focus on the upper part of the image and coils 3, 4 emphasize the lower part of the image due to different coil locations. The spectral distribution of the SoS estimate of the true pixel image (Eq. 2) is shown in Fig. 3(e) (all the figures in the frequency domain are shown in $[0\ \frac{\pi}{4}]$ and the upper left corner is the origin). Though the strongest spectral components are in low-pass band, they come from the flat reflection area from muscles and tissues which don't represent the desired high contrast area from the spinal cord and bone structure parts. The coil sensitivities $\log|\hat{c}_k|$ are estimated by Eq. 4, and their spectral distributions are shown in Fig. 4. We can see that the coil sensitivities are slow-varying in the effective signal area and show their low-pass property. Thus a high-pass filter is designed to filter the coil sensitivities. The cutoff frequency and the stopband magnitude of the filter are chosen based on the effective maximum image contrast criterion (the filter order adjustment is not considered for simplicity). Fig. 5 shows that the image contrast surface has a global maximum and the magnitude at the peak is over two times higher than that in SoS. Based on the filter with peak contrast, the true pixel image is reconstructed by the filter outputs for each coil. The proposed method demonstrates visually better reconstruction results than SoS method in Fig. 7. Fig. 7(d) shows the reconstructed filtered coil sensitivities, indicating that the effective information of the high contrast image is not filtered out by the proposed method. This is because though the energy is dominant in low-pass band, the effective information of the image is mainly in high-pass band. The probability density function distributions of these reconstructed pixels are shown in Fig. 8. It shows that the contrast-enhanced homomorphic signal processing method which has the flattest pixel distribution in the middle of intensity scale (between 50 and 100) gives the best image contrast (similar to histogram equalization). This method also shows a gain of 10% in normalized entropy compared to the SoS method computed by (Table. I),

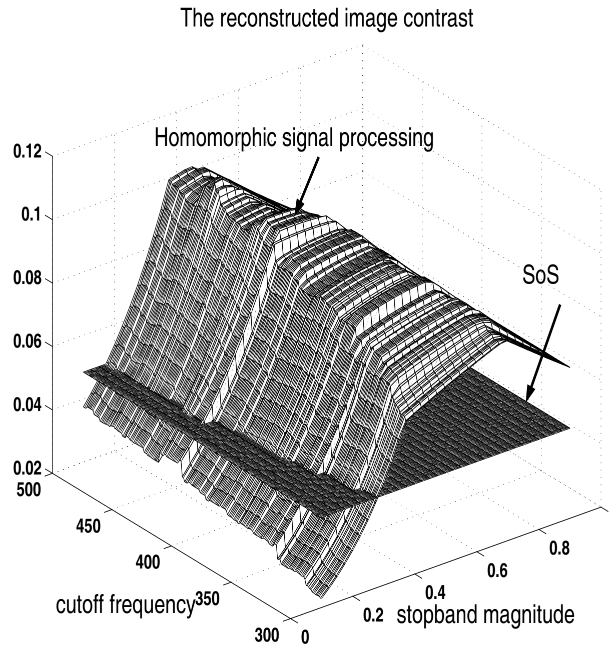


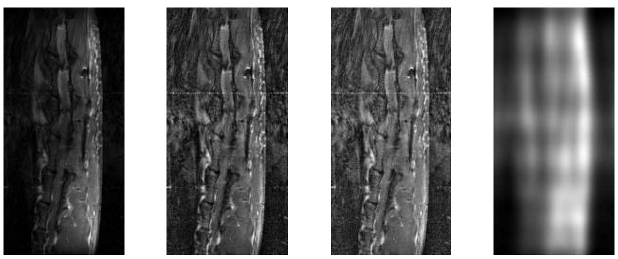
Fig. 5. The reconstruction image contrast versus the high-pass filter cutoff frequency and the stopband magnitude.

$$E = \frac{1}{\log N_{scale}} \sum -f(\hat{\rho}) log(f(\hat{\rho})) \qquad (7)$$

where $\hat{\rho}$ is the reconstructed pixel, $f(\cdot)$ is the pixel distribution, N_{scale} is the pixel intensity upper bound and E is the normalized entropy.

(a) Coil 1 (b) Coil 2 (c) Coil 3 (d) Coil 4

Fig. 6. High-pass filter to eliminate coil sensitivities.



(a) a (b) b (c) c (d) d

Fig. 7. Reconstructed images. (a) Sum-of-squares (sos), (b) homomorphic signal processing, (c) contrast-enhanced homomorphic signal processing, and (d) reconstruction from the filtered coil sensitivities.

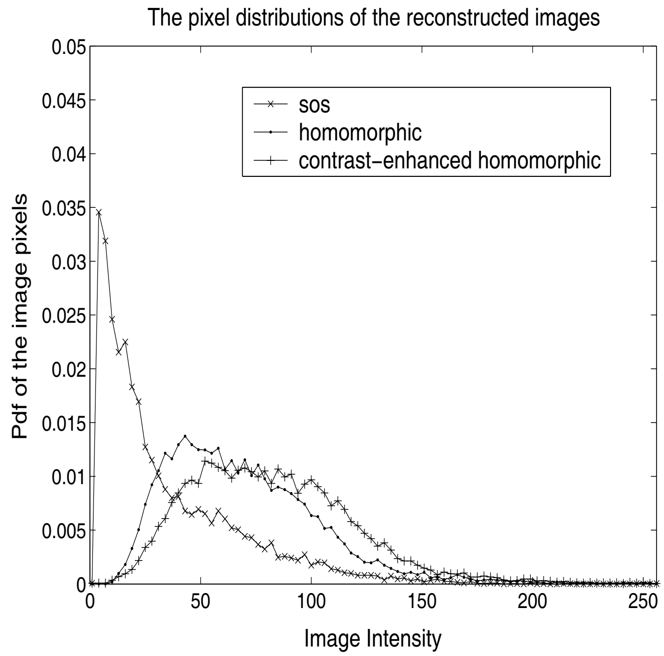


Fig. 8. The pdf distribution of the reconstructed images.

TABLE I

NORMALIZED ENTROPY OF (A) SOS, (B) HOMOMORPHIC SIGNAL PROCESSING, AND (C) CONTRAST-ENHANCED HOMOMORPHIC SIGNAL PROCESSING.

Method	(a)	(b)	(c)
Entropy	0.8064	0.8714	0.8924

VI. CONCLUDING REMARKS

In summary, the proposed homomorphic signal processing method effectively splits the spectral of the effective coil signal and coil sensitivity and the following nonlinear transform increases the image contrast. The reconstructed image quality is enhanced not only visually but also in terms of image contrast and entropy compared with the widely implemented MRI reconstructed method, Sum-of-squares (SoS) method. The disadvantage of this method lies at the computed SNR decrease due to the enlargement of noise in the background region compared to SoS in the same dynamic range. However, the image quality is not affected in the desired signal area with high contrast.

VII. ACKNOWLEDGMENT

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