IMAGE COMPOUNDING BASED ON INDEPENDENT NOISE CONSTRAINT

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ABSTRACT

Image restoration has been extensively studied in the past. But multi-image based restoration/compounding is still surprisingly primitive. It usually starts with weighted averaging of the multiple images followed by single-image based restoration methods, which discards the abundant information hinted in the multiple images that can help the restoration process. In this paper, we utilize the fact that the images are corrupted by independent noise and design a new independence measurement based on the properties of independent random variables. The new independence measurement can be efficiently evaluated and imposed as an energy term into the traditional Maximum a Posteriori (MAP) framework, compensating to the generative models of signal and noise. It can effectively prevent the signal from being smoothed out as noise and hence dramatically improve the restoration quality and robustness, especially when accurate noise/signal models are difficult to obtain. Experiments on real medical images show very promising results.

1. INTRODUCTION

Extensive research has been conducted to improve image quality based on a single image. Usually, the noise on neighboring pixels is independent and hence can be easily reduced by a low-pass filter. But this filter also blurs the sharp edges in the image. To preserve the high frequency signal in the image, e.g. edges or corners, prior knowledge has to be used to discriminate the signal from the noise. Various methods are proposed to model the high frequency signal in images, e.g. edge modelling based on Markov Random Field [3] or quadratic signal class [5]. The difficulties arise from the fact that accurate modelling of various signal and noise is usually very hard if not impossible.

Multiple images can be obtained in some cases to further improve imaging quality, which is also called image compounding [6, 7]. For example, in ultrasound B-scans, we can generate multiple images of the same tissue using different frequency ultrasound. The speckle noises on different images are proved to be independent [7, 9]. This is different from the traditional multiple-image based super-resolution techniques [4]. In image compounding, the images have the same underlying signal but are corrupted by independent noise. The multiple images do not provide more information for super-resolution but help reducing noise. This topic is much less studied than the single image based restoration methods and is usually handled with very simple weighted averaging (e.g. [6, 7]), followed by the traditional restoration methods on the averaged image. This scheme fails to fully utilize the abundant information hinted by the multiple images, which could be very effective in guiding the image restoration process.

In this paper, we propose a novel technique to fully utilize the multiple images and dramatically improve the restoration results with only a small number of compounding images (e.g. 2 or 3 images), which is very important when we cannot afford to scan many compounding images in reality. The main idea is to fully exploit the independency between the noise on different images. Independence analysis has been widely used in many different applications [2], but it is rarely seen in image restoration. We propose to utilize this constraint to reduce the possibility of misclassifying the signal as noise and hence preserve the sharp edges and corners without accurate modelling of them. This method is robust even when we have spatial variant noise and inaccurate signal/noise models. It is extremely useful for medical imaging, where the noise could be non-stationary and dependent on the underlying structures.

The rest of the paper is organized as follows. In Section 2, the detailed steps of our compounding technique are explained. In Section 3, we report very promising results in compounding some real medical images. Conclusion is given in Section 4.

2. IMAGE COMPOUNDING UNDER INDEPENDENCE CONSTRAINT

To illustrate the compounding algorithm, we start with two images (it will be extended to handle more images in subsection 2.3). Let's assume two images I_1, I_2 have the same underlying signal S (for simplicity, no motion or registration are considered in this paper) but are corrupted by independent noise N_1, N_2, i.e. $I_1 = S + N_1$ and $I_2 = S + N_2$.

Weighted averaging is widely used to estimate the true signal. Assuming N_1 and N_2 are zero mean and have the same variance, we have $\hat{I} = (I_1 + I_2)/2$. But a detailed study can show the problem of this scheme. If $\hat{I}$ is the true signal, the noises are: $\hat{N}_1 = I_1 - \hat{I}$ and $\hat{N}_2 = (I_2 - \hat{I})$. We can see that the two noise components are totally correlated (i.e. $\hat{N}_1 = (I_1 - I_2)/2 = -\hat{N}_2$), which is contradicted to the truth that N_1 and N_2 are independent. Only when we have a large number of images, the weighted averaging becomes more accurate and the residual error $\hat{N}_i$ more independent. From this example of compounding two images, it can be easily seen that, even though widely used, simple averaging is not an optimal way to combine the multiple images.

To better compound the multiple images, we propose to enforce the independency between the noise components. The intuition is simple. If the estimation of the signal is wrong, the error (i.e. $S - \hat{I}$) will be uniformly added to the residual errors (i.e. $\hat{N}_i = N_i + (S - \hat{I})$) on all images and increases the dependency between them. By enforcing the independence constraint, we can prevent misclassifying the signal as noise and hence achieve better restoration result.

In this section, we first explain the limitation of the traditional MAP based image restoration method. Then an efficient measurement of the independency is designed for the traditional energy minimization framework to enforce noise independency. An iterative minimization method can be used to find the optimal restoration result.

2.1. Traditional MAP image restoration

In the traditional MAP estimation based image restoration, generative models are used. It assumes that we have the probability distribution models of both noise and signal. Then a MAP estimation can be obtained. It can be easily extended to multiple images. With the assumption that N_1 and N_2 are independent, we have:

$$\begin{aligned} P(\hat{I}|I_1, I_2) =& c \cdot P(I_1, I_2|\hat{I}) P(\hat{I}) \\ =& c \cdot P_{N_1}(I_1 - \hat{I}) P_{N_2}(I_2 - \hat{I}) P_S(\hat{I}) \end{aligned} \tag{1}$$

where c is a normalization constant. $P_{N_i}()$ and $P_S()$ are the prior noise/signal models respectively. Based on Gaussian noise modelling and smooth signal constraint, we can define the cost function $C(\hat{I})$ as follows:

$$\begin{aligned} \hat{I} &= argmin_{\hat{I}} C(\hat{I}) = argmin_{\hat{I}} \log(P(\hat{I}|I_1, I_2) \qquad (2) \\ &= argmin_{\hat{I}} (\lambda_1 (I_1 - \hat{I})^2 + \lambda_2 (I_2 - \hat{I})^2 + \lambda_3 (\hat{I} - \bar{I})^2) \end{aligned}$$

where $\bar{I}$ is the average intensity of the neighborhood. The first two terms enforce that the estimated image should look like the observed images. The 3rd term models the signal property and prefers smooth signal. Many studies has been focused on the signal modelling based on different energy functions [3, 5]. The MAP estimation can be obtained iteratively based on the derivative of $C(\hat{I})$:

$$\frac{\partial C(\hat{I})}{2 \cdot \partial \hat{I}} = \lambda_1(\hat{I} - I_1) + \lambda_2(\hat{I} - I_2) + \lambda_3(\hat{I} - \bar{I}) = 0 \tag{3}$$

Because the $\bar{I}$ is unknown and has to be estimated based on the solution on previous iteration. We can use the iterative minimization process:

$$\hat{I}^{(k+1)} = \frac{(\lambda_1 + \lambda_2)(\hat{I} - I_{avg}) + \lambda_3(\hat{I} - \bar{I}^{(k)})}{\lambda_1 + \lambda_2 + \lambda_3} \tag{4}$$

where $I_{avg} = (\lambda_1 I_1 + \lambda_2 I_2)/(\lambda_1 + \lambda_2)$. This leads directly to the traditional averaging and filtering scheme. This scheme is the optimal solution if all the assumptions are right and we have very accurate signal/noise models.

However, in reality, obtaining accurate prior models is very difficult if not impossible. For example, we can see that in the CT image shown in Fig. 1 (a), there is non-isotropic noise which is very hard to be distinguished from the edges or lines. In ultrasound, the speckle noise is non-stationary and dependent on the underlying structures and sub-resolution scatterers. Considering all this, this generative model based method cannot provide satisfactory results.

On the other hand, the prior knowledge of the independence between the noises, even though used in the derivation of the MAP framework, is never enforced in the optimization. In fact, this knowledge can be very helpful in distinguishing the signal and the noise. To show the robustness and effectiveness of the independent noise constraint, we impose it with the simple smooth signal model and Gaussian noise model in our experiments and still achieve surprisingly sharp compounding results.

2.2. Independent noise constraint

When only a single image is available, the only thing we can do is to rely on some assumptions and the prior noise/signal models as in the traditional MAP framework. However, when multiple images of the same scene are available, averaging the images followed by the traditional restoration methods won't give us the optimal solution because it ignores the abundant information in the correlation between different images.

As we have pointed out, in real-world applications, it is almost impossible to obtain very accurate models of all the possible signal and noise. If we have inaccurate models, it will be reflected in the restored image $\hat{I}$, which ideally should be equal to the true signal S. The restoration error (i.e. $S - \hat{I}$) will appear in all the residual errors:

$$\hat{N}_i = I_i - \hat{I} = N_i + (S - \hat{I}) \tag{5}$$

Because the $(S - \hat{I})$ is common for all the images, the correlation between the residual errors will increase with the restoration error. So the independency between the residual errors provides an elegant way for us to detect and correct the restoration error caused by invalid assumptions or inaccurate signal/noise models.

However, the strict independency (i.e. $p(N_1, N_2) = p(N_1)p(N_2)$) is very expensive to evaluate [1]. So we propose to rely on one of the important properties of the independent random variables:

$$E(h_1(N_1)h_2(N_2)) = E(h_1(N_1))E(h_2(N_2)) \quad (6)$$

where $h_1()$ and $h_2()$ are any kind of functions of N_1 and N_2 respectively. $E()$ is the expectation value.

If we choose $h_1(N_1) = N_1$ and $h_2(N_2) = N_2$, the independent noise constraint is reduced to uncorrelated noise constraint. When the noise is Gaussian distributed, they are equivalent. In our experiment, we use this approximated constraint instead of enforcing real independency and achieve very good results. More complex $h_1()$ or $h_2()$ (e.g. higher order moments) can be used for more accurate approximation of independence constraint if necessary.

2.3. Multiple image compounding

As we have explained in previous sections, we should explicitly enforce the independence between the noises to reduce the restoration errors caused by the invalid assumptions or inaccurate signal/noise models. We propose to use the traditional energy minimization framework, with an additional energy term derived from Eq. (6). The new energy term regularizing the independence constraint between the residual errors of image i and j (i.e. $\hat{N}_i = I_i - \hat{I}$ and $\hat{N}_j = I_j - \hat{I}$) can be defined as:

$$\begin{aligned} e_{i,j}(\hat{N}_i, \hat{N}_j) = \|&E(h_1(\hat{N}_i)h_2(\hat{N}_j)) \\ &- E(h_1(\hat{N}_i))E(h_2(\hat{N}_j))\|^2 \end{aligned} \quad (7)$$

where the $E()$ is the expectation and can be calculated in a small neighborhood (e.g. 15 by 15 in our experiments). For accuracy, we could also add several terms based on different $h_1()$ and $h_2()$ to better measure the independency.

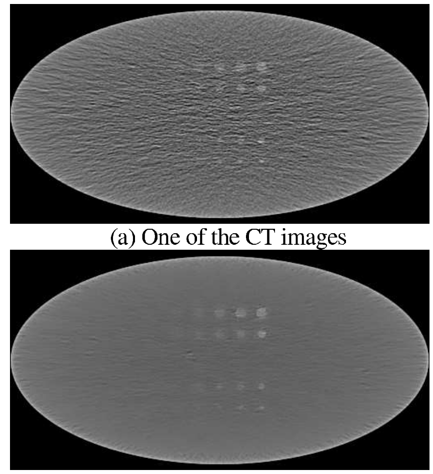
For multiple images, we use the sum of the pairwise independency to approximate the joint independency between the residual errors. The new objective function with independence constraint is now defined as:

$$\begin{aligned} \hat{I} &= argmin_{\hat{I}} C(\hat{I}) = argmin_{\hat{I}}(\lambda_0(\hat{I} - \bar{I})^2 \\ &+ \sum_{i=1}^{N} \lambda_i (I_i - \hat{I})^2 + \lambda_{N+1} \sum_{j=1}^{N-1} \sum_{k=j+1}^{N} e_{j,k}) \end{aligned} \quad (8)$$

To find the optimal solution, an iterative optimization process can be easily designed similar to the traditional framework shown in sub-section 2.1. The detailed derivation is omitted due to the space.

3. EXPERIMENTS

To show the robustness of the new compounding method, we apply it on some difficult medical images, where accurate signal/noise models are hard to obtain.



(a) One of the CT images

(b) Compounding Result

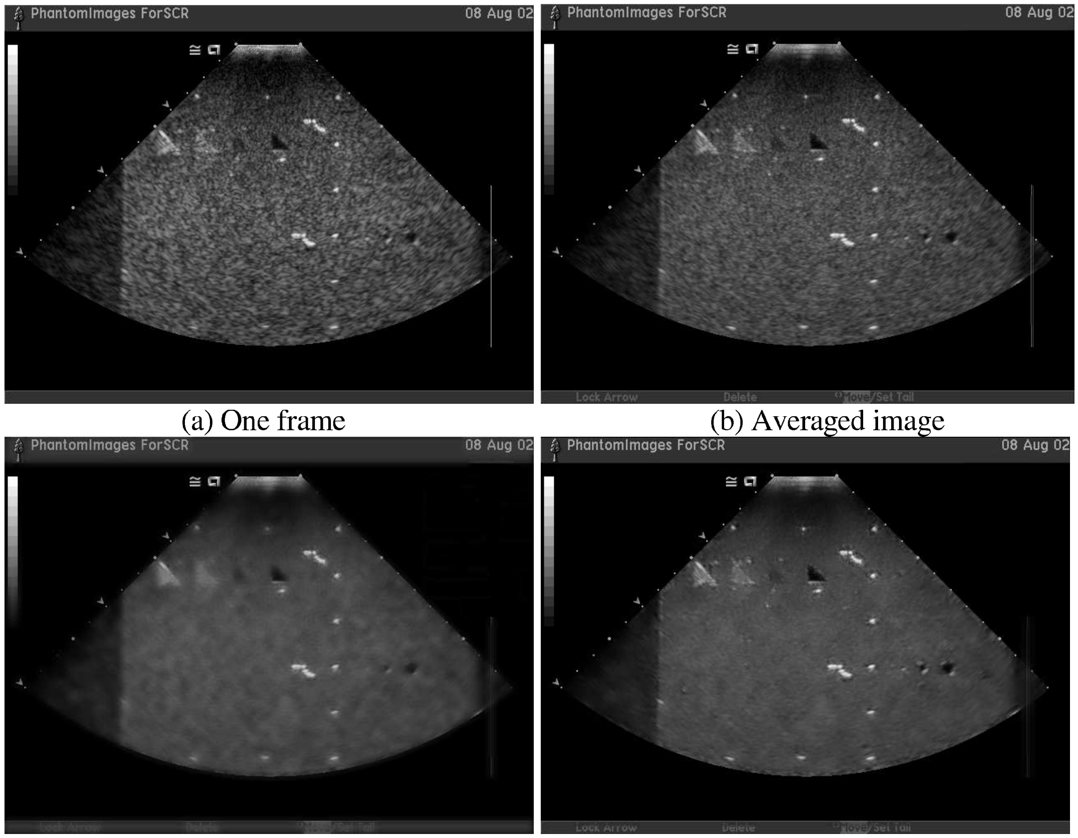
Fig. 1. CT image compounding

First, a set of three CT images are tested. In the CT images, the noise is not isotropic but looks more like line structures, while the non-isotropy can hardly be predicted. The traditional edge modelling cannot distinguish such noise from the signal. We just apply the simple smooth signal model in the objective function in Eq. (8) and achieve very good restoration as shown in Fig. 1 (b). We can see that the noise is dramatically removed while the weak signals (i.e. the various circles) are well preserved with sharp boundaries except the very weak ones.

We also test our algorithm for ultrasound frequency compounding. Three images are scanned using different frequency ultrasound. Because the acoustic signal has been taken $log()$ before converting to the display images, we assume the multiplicative speckle noise in acoustic signal is now additive and apply our compounding technique to restore the underlying signal. One of the three original images is shown in Fig. 2 (a). Averaging the three images does not reduce the noise dramatically due to the small number of compounding images (as shown in Fig. 2 (b)).

To apply the adaptive image filtering on the averaged image, the main difficulty is the non-stationary noise, which is also dependent on the structure [8], making accurate modelling almost impossible. We use the adaptive wiener filter (with 15 by 15 neighborhood) in matlab for comparison. The result is shown in Fig. 2 (c). As we can see, strong signal can be detected and preserved well. But the weak features are severely blurred while the noise region is not smoothed enough.

Our algorithm exploits the correlations between the residual errors with the simple noise/signal models as shown in Eq. (8). The result is shown in Fig. 2 (d). Without the independence constraint, this simple noise/signal modelling will severely blur all the structures in the image due to the lack of edge modelling, generating much worse filtering result than



(a) One frame (b) Averaged image

(c) Wiener filter on the averaged image (d) Our compounding result

Fig. 2. Ultrasound frequency compounding

the adaptive Wiener filter. But with the independence constraint, it prevents misclassifying signal to noise. The weak edges are preserved much better, while the noise regions are smoother than the adaptive Wiener filter.

4. CONCLUSION

Various independence assumptions are assumed in traditional MAP frameworks to factorize the joint distribution terms, but never seriously investigated and enforced in the optimization process. In this paper, we study the independency between noise components for multiple-image compounding. An effective measurement is designed to regularizing the independency between noise in the traditional generative model based filtering framework, resulting a new algorithm much more robust to inaccurate signal/noise modelling. Comparisons with some traditional methods (e.g. MAP filtering scheme and adaptive Wiener filter) clearly demonstrate the strength of the new method.

5. REFERENCES

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