GRAVITATIONAL TRANSFORM FOR DATA CLUSTERING - APPLICATION TO MULTICOMPONENT IMAGE CLASSIFICATION

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ABSTRACT

In this communication, we introduce the concept of gravitational transform, with application to multicomponent image classification. This general concept, from which many settings can be implemented, takes into account both feature sets extracted from the image and the spatial distance between pixels in order to improve the further classification, and thus the partitioning into homogeneous regions. Examples of classification and comparative results with synthetic data are presented, showing that this approach improves the classification rates in all cases, independently of the chosen classification technique, and outperforms some recent approaches in contextual multidimensional unsupervised clustering. The encouraging results obtained make this technique a valuable tool to insert between the features extraction process and the unsupervised classification process.

1. INTRODUCTION

The clustering of multicomponent (color or multispectral) image data is a central step in every processing chain aiming at interpreting the image content. In this communication, we address the unsupervised clustering of multicomponent image data, assuming the property of similarity between local features available at neighboring image sites. Taking the example of multispectral aerial image analysis, we consider in the following that the production of thematic maps from this data is not only a problem of data clustering in high-dimensional spaces, but also a problem of partitioning the image into homogeneous regions (homogeneity including, if necessary, texture). Actually, a number of unsupervised classification techniques used for multispectral imagery rely on the assumption of pixels' statistical independance. This can be explained at least by two facts:

- The introduction of generalized second-order statistical features (e.g. auto-correlation or co-occurrence attributes) rapidly increases the dimensionality of the feature space, thus making the extraction of relevant information difficult. This problem is referred to as the *curse of dimensionality* in the literature (see [3] for example).
- In some applications, the risk of non-detection for some under-represented classes (i.e. rarely present in the image or corresponding to few very small regions) should absolutely be minimized. This is actually often the case for example in multispectral imagery.

This work is supported by the Regional Council of Brittany and the ERDF through the Interreg3B project PIMHAI.
The third author performed this work while at the IETR/TSI2M.

Recently there has been attempts to take into account the assumption of probable feature similarity (or correlation) between neigbouring pixels in multicomponent images [2], [9], [10], [11]. The processing technique which is proposed here aims at preparing the data before the unsupervised clustering step. It is a very general procedure since it can be applied to any multidimensional data representation and feature space. It is inspired from former research works on gravitational clustering [12], more recently revisited in a comparative study with fuzzy and hierarchical clustering techniques [5], and also in the framework of color image segmentation [6]. The last two works report better classification results credited to this approach in comparison with the classical k-means, fuzzy c-means and k-nearest neighbors methods. Also, it should be noted that the work presented herein is in some manner in direct link with inverse diffusion variational approaches, now widely used for histogram transform or edge-preserving image filtering [8].

In section 2, we introduce the gravitational transform approach and detail its practical setting as a pre-processing step before clustering. In section 3, we present the general methology which is used to assess this approach and give some comparative results obtained with other recently developed contextual clustering techniques, such as the Mean Shift [2], the fuzzy c-means method of Sittigorn *et al.* [9] or the k-means method of Theiler et Gisler [10]. We will conclude and provide some perspectives of this work in section 4.

2. GRAVITATIONAL TRANSFORM

2.1. Overview

The main purpose of the gravitational transform is to highlight the relevant modes of the data in the multidimensional space, by performing an adequate transform of the features distribution. Considering its application to multicomponent image data clustering, let N denote the number of image components. By choosing, for instance, as a feature vector (or point) $\mathbf{x} = [x_1 \ldots x_N]^T$ the intensity in each of the N spectral band (but many other choices are possible, including texture features), the principle of gravitational transform consists in applying to each point a force of attraction which depends on its distance to another point or to a set of other points of the distribution.

2.2. Model of attraction

Consider the set of N-dimensional data $\{\mathbf{x}_s\}_{s \in S \subset \mathbb{N}\times\mathbb{N}}$ which are located at the different pixel sites s. From a general point of view, one can define the attraction of the point $\mathbf{x}_t$ on the point $\mathbf{x}_s$ through the following model:

$$\overrightarrow{F_{st}} = -\alpha \frac{\mathbf{x}_s - \mathbf{x}_t}{d(\mathbf{x}_s, \mathbf{x}_t)} \quad , \tag{1}$$

where $d(\mathbf{x}_s, \mathbf{x}_t)$ is a distance to be defined and $\alpha > 0$ is a constant analog to the gravitational constant in classical mechanics. In this model, the force of attraction is obviously greater as the two points are closer to each other.

Three questions of diverse importance may then arise:

1. What kind of distance should be used ? To our knowledge, this question is still widely open, although the classical mechanics would point out a choice of $d(\mathbf{x}_s, \mathbf{x}_t) = \|\mathbf{x}_s - \mathbf{x}_t\|^3$, and that passive optical multispectral imagery also indicate the spectral angle distance as a potential and now well-adopted alternative to the Euclidean distance [4].
2. What kind of simple attraction scheme should be used ? The ones proposed in [12] and [5], in spite of their relevance, both have the drawback of a heavy computational burden since the force of attraction applied on one point is the sum of the contributions of all the other points' individual forces.
3. Should the adopted scheme be global or local ? In [12] and [5], the notion of features spatially local similitude is not taken into account, which in our sense does not help obtaining relevant classification results, particularly in the field of image processing.

Through these questions, we can see that the principle of gravitational transform may be practically derived in a wide variety of ways. However, the gravitational transform algorithms that are generally considered, all share a common iterative setting: the feature data update is performed within a few number of iterations. Moreover, a so-called *Markovian* evolution scheme can be set up [12], which disables the time integration of the force of attraction, or equivalently resets to zero the speeds of points at each iteration, thus avoiding unstable evolution.

2.3. The proposed scheme

The scheme we propose here simply consists in conditioning the gravitational transform process to the spatial distance between pixel sites. In other words, the mutual attraction of corresponding features vectors (or points) is enabled if and only if two sites are neighbors to each other. Note that this approach has the benefit to dramatically reduce the computational complexity with respect to global approaches while taking into account the reality of probable similar class dependance at neighboring sites. Figure 1 sketches in a 2-D feature space only, the contribution of the force of attraction of a point $\mathbf{x}_t$ onto a point $\mathbf{x}_s$. The sum of the individual contributions of neighbors' forces of attraction then yields a modification of vector $\mathbf{x}_s$, the evolution of which is given by the following equation:

$$\dot{\mathbf{x}}_s = -\frac{1}{4} \sum_{i=1}^{4} \overrightarrow{F_{st_i}} \quad , \tag{2}$$

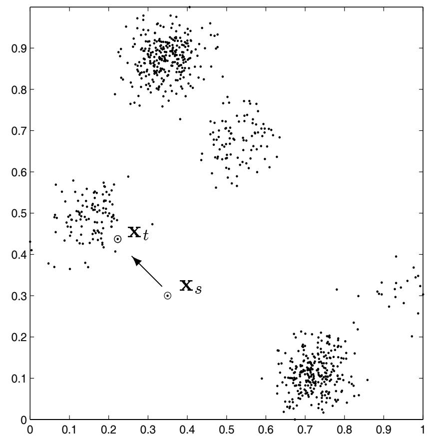

Fig. 1. *Force of attraction of point $\mathbf{x}_s$ by point $\mathbf{x}_t$, both corresponding to spatially neighboring sites in the image.*

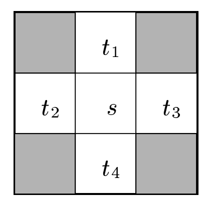

Fig. 2. *First order neighborhood (4-connexity) used in the evolution Eq. (2).*

where $t_i, 1 \leq i \leq 4$ represent the neighbors of site s (see Figure 2). Moreover, in our experiments, we made the choice of an *a priori* distance $d(\mathbf{x}_s, \mathbf{x}_t) = \|\mathbf{x}_s - \mathbf{x}_t\|^2$. Finally, a close attention should be paid to the expression of the force of attraction in Eq. (1), due to the indetermination of the denominator if $d(\mathbf{x}_s, \mathbf{x}_t) = 0$. This is why we have chosen a modified distance (which is actually no longer a distance in the mathematical sense) $d(\mathbf{x}_s, \mathbf{x}_t) = \|\mathbf{x}_s - \mathbf{x}_t\|^2 + \varepsilon$, where $\varepsilon > 0$ is negligible compared to the sole distance term.

3. EXPERIMENTAL STUDY

3.1. Methodological framework

Our aim being the objective assessment of the gravitational transform for unsupervised clustering and segmentation of multicomponent image data, we have focused our attention and designed our experimental study regarding two questions:

– What is the intrinsic value added by this pre-processing step with respect to the direct use of any unsupervised clustering technique such as k-means or fuzzy c-means ?

– What are the performances of this approach compared to other recent unsupervised clustering techniques integrating

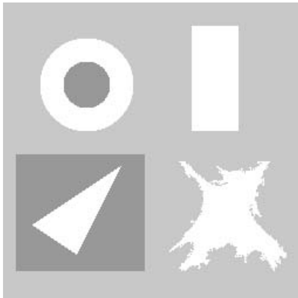

Fig. 3. *Ground truth image (3 classes) used for evaluation.*

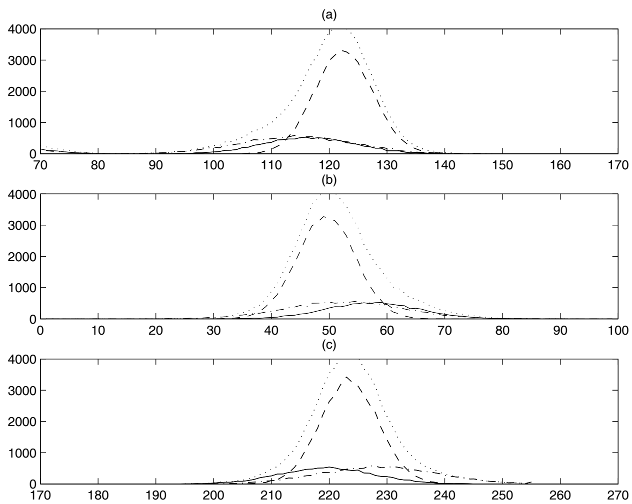

Fig. 4. *Class-conditional and total histograms computed from the 3-components image to be classified. Solid line: class 1 ; dashed: class 2 ; dashdot: class 3; dotted: total. (a) first, (b) second, (c) third component.*

a spatial distance criterion?

The objective evaluation of our results was performed in all cases after clustering pixel values of a synthetic image with three components ($N = 3$), the ground truth of which was given, and containing three classes (see Figure 3). This multicomponent image is not shown here because the distribution of associated color levels prevents any visual distinction between classes. This can be seen from Figure 4 where are given the class-conditional and total histograms of this image: in each image component, a single mode is present, and some class-conditional distributions are very close to each other with a very large overlap. Class-conditional distributions were obtained by independant random outcomes from three-dimensional Gaussian probability density functions. The scalar between-class / within-class ratio for this data set is approximately 0.6, which is very low. The choice of such a difficult clustering problem is motivated by practical situations, for example in multispectral or hyperspectral imagery where objects with very close reflectance values have to be discriminated in a noisy context.

3.2. Choice of basic unsupervised clustering techniques

The classical unsupervised clustering techniques which were chosen to follow the gravitational transform step are the k-means algorithm [7] [3] and the fuzzy c-means (FCM) algorithm [1] [3]. These techniques will not be described herein, and the reader is referred to the adequate literature for details. These approaches belong to the family of partitioning techniques based upon a distance criterion. In their basic implementation, both require an estimate or an upper bound of the cluster number K, and try to estimate the optimal class centroid/membership in an iterative manner. Only the FCM algorithm requires an additional coefficient called the fuzziness parameter $m > 1$ for which we have set $m = 2$ as is often the case in the literature. In our study, we have taken $K = 3$, i.e. we have assumed that the correct class number is known. Note that both techniques start with a random initialization of cluster centroids.

3.3. Intrinsic contribution of gravitational transform

Correct classification rates were computed for each clustering result, and the results which are reported here represent mean classification rates over 20 sample trials. Table 1 shows the mean classification rates obtained for both clustering techniques without and with the application of the gravitational transform. The gravitational transform (GT) was run in 10 iterations. These global results may look relatively poor, but recall that the image under study is very noisy. What can be pointed out is the increase of the classification rate in both cases, which reaches 15% with the use of the k-means algorithm. However, the increase in classification rate, while significant, is smaller for the FCM technique. From this experimental study, we can conclude that the gravitational transform clearly highlights the modes of class-conditional distributions, as can be seen from Figure 5.

Method	Classification rate	
	without GT	with GT
k-means	63.20%	78.23%
fuzzy c-means	48.32%	52.91%

Table 1. *Classification rates obtained by different clustering techniques with and without application of the gravitational transform to the image data.*

3.4. Comparison with recent contextual image segmentation approaches

Recently, several unsupervised contextual classification techniques were proposed in the context of multicomponent image clustering. Among others, one can point out the modified k-means of Theiler et Gisler [10], the modified FCM of Sittigorn *et al.* [9], or the Mean Shift method [2]. The first two methods are based upon the original algorithms upon which is added a ”spatiality” criterion. The modified k-means algorithm consists in minimizing a linear combination of a density criterion (i.e. the mean intraclass variance) and (dis)contiguity (which relates to the dispersion of classes within a local neighborhood). The modified FCM is based upon the minimization of a functional in which appears a dissimilarity index between neighboring pixels. The Mean Shift method is

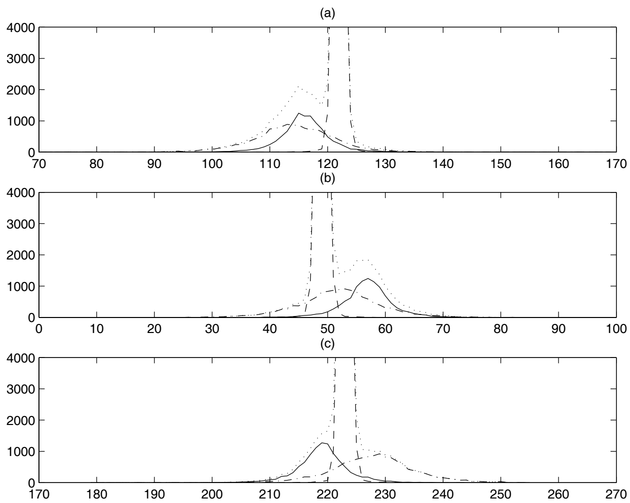

Fig. 5. *Class-conditional and total histograms after 40 iterations of the gravitational transform. Solid line: class 1 ; dashed: class 2 ; dashdot: class 3; dotted: total. (a) first, (b) second, (c) third component. The vertical axis is truncated to enhance the weakest modes.*

a nonparametric method which considers that the features distribution follows an empirical probability density function, and aims at finding the modes of the distribution in an iterative manner, jointly producing a segmentation map. It is important to note that each of these techniques were set up using a first order neighborhood into the spatial criteria in order to compare with our proposed approach.

Table 2 presents the comparative results which were obtained. The best result was obtained by using our approach (reported from Table 1), showing its relevance with respect to more sophisticated techniques designed for the same objective.

Method	Classification rate
Modified k-means [10]	74.34%
Modified fuzzy c-means [9]	66.31%
Mean Shift [2]	65.52%
GT + basic k-means	78.23%

Table 2. *Comparison of classification rates obtained by different contextual clustering techniques.*

4. CONCLUSION AND FUTURE WORK

In this communication, we have addressed gravitational transform as a pre-processing tool to be used prior to classical unsupervised clustering techniques for multicomponent image data. We have shown that gravitational transform, in spite of the data modification it generates, improves the classification task and provides a simple and efficient way to take into account the probable similarity between neighboring pixels which is a property of most image data, especially in multispectral imaging. When compared to other recent contextual clustering algorithms, this methodology shows slightly better results in classification rates, especially when using the k-means algorithm as the unsupervised classifier.

From a general point of view, one could argue that gravitational transform is much more a filtering process than a tool for further image data clustering, which is somewhat true. Actually, this poses the problem of the image processing chain structure in general. To our opinion, the question of the relevance of mixing several processings such as filtering and classification into a single algorithm is still open and requires further investigation.

Further work is currently under study about the use of gravitational transform in feature clustering. Since feature extraction (e.g. local empirical first and second order statistics) can provide large amounts of data to classify, gravitational transform can help in discriminating the salient distribution modes in the data, without merging neighboring pixels which are too far from each other in the feature space. For instance, a classification rate of 98.18% was obtained on the same image data by applying the gravitational transform on local empirical mean and standard deviation for each component (6 features), and using the k-nearest neighbors algorithm in [11] which moreover provided the correct number of classes.

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