AN IMPROVED IMAGE DENOISING ALGORITHM BASED ON WEIGHTED ADAPTIVE LOCAL BOUNDS

Qi Li and Tania Stathaki

Communications and Signal Processing Group,
Department of Electrical and Electronic Engineering,
Imperial College London,
London SW7 2AZ, U.K.

ABSTRACT

In this paper we tackle the problem of image denoising. Given an image distorted by additive noise of unknown probability distribution, the intensity difference between the distorted and the original unknown images must be bounded. This idea is exploited here and we design a series of weighted adaptive local bounds based on local intensity information, such as the mean, variance, median and etc. The proposed method is tested and compared with other standard techniques, such as wavelet thresholding, for image denoising. Apart from the simplicity of implementation, the results are very encouraging, as far as both visual quality of the denoised images and quantitative metrics of improvement are concerned. More importantly it provides simultaneous desnoising of mixed noise, which is not obtainable by using single conventional denoising method.

1. INTRODUCTION

Assume the distorted image by additive noise is formulated as $g(x,y) = f(x,y) + n(x,y)$, $(x,y) \in \Omega$. f is the original image with each pixel coordinated as (x,y) in the image domain Ω, and n represents the unknown noise process. There is an extensive amount of published work in image de-noising. For example, one of the most popular recent approaches to the mentioned problem is the use of wavelet thresholding [1-5], [9-10].

The motivation for using the wavelet coefficients for denoising is that in most of the natural scenes, the signal level is much larger than the noise level, therefore small coefficients are more likely present due to the noise process, and large coefficients due to the important signal features (such as edges). Thus, thresholding could be applied on the image wavelet coefficients based on certain criteria. However, the criteria for selecting thresholds can not be determined universally, due to the uniqueness of each image, which is identified by the local intensity information. Besides, most of the methods [2-9] are only appliable to the image corrupted by the Gaussian noise. Therefore, most of the existing algorithms either blur the image severly in order to eliminate noise artifacts or cannot eliminate the multiple artifacts sufficiently. Moreover, they are often computationally expensive.

An alternative approach to the local thresholding of wavelet coefficients is the restriction of local space coefficients (image intensity values) within certain bounds. In most cases, the noise is more visible in smooth regions than regions of sharp intensity transitions. Thus, the idea is to exploit the possible range of intensity values to characterize the spatially varying local image nature. The contribution of this paper includes the development of the new weighted adaptive local bounds, which can retain more detail information for highly corrupted images. Moreover, the robustness of the algorithm to handle the outlier-like noise is improved by incorporating local median informaiton into the projection. Hence our proposed method, weighted adaptive local bounds estimator (WALBE) can be tuned to effecitively remove different types of noise, which is not feasible to other wavelet methods, while enough detial information is preserved. The implementation of WALBE is less complicated compared to the wavelet domain methods.

This paper is organised as follows. In Section II the concept of local bounds as were originally presented in [6-7] is reviewed. In Section III a modified formulation of the bounds is introduced. The latest is utilised in Section IV for the cases of images corrupted by different types of noise. In Section V the proposed techniques are compared with some other denoising algorithms, in terms of both subjective visual quality and quantitative measurements. Conclusions are presented in Section VI.

2. REVIEW OF THE LOCAL BOUNDS

The concept of local bounds was first used in the context of both blind and non-blind image restoration. In [6-7] it is suggested to the intensity difference between the distorted and the original unknown image must be bounded, and therefore the local intensity information available in the distorted image, can be brought into the image restoration problem. A set of spatially varying local bound constraints are initially determined by examining the local properties of the image. The estimated original image intensity at each iteration is constrained to lie within these predetermined bounds.

In the context of image denoising, a similar idea is exploited this time in a noniterative way. In accordance with the noise masking property [6], the local intensity variance $\sigma_g^2(x,y)$ still measures the spatial activity. Along with the local mean $m_g(x,y)$, and the maximum intensity variance $\sigma_{gmax}^2 = \max_{(x,y)}(\sigma_g^2(x,y))$, which are all calculated over a local neigorhood window containing a number of pixels equal to N, a set of local lower and upper bounds, $b_l(x,y)$ and $b_u(x,y)$ are constructed on the basis of the observed image:

$$\begin{aligned} b_l(x,y) &= m_g(x,y) - \beta\frac{\sigma_g^2(x,y)}{\sigma_{gmax}^2} \quad x,y \in \Omega \\ b_u(x,y) &= m_g(x,y) + \beta\frac{\sigma_g^2(x,y)}{\sigma_{gmax}^2} \quad x,y \in \Omega \end{aligned} \tag{1}$$

in which Ω represents the support of the image. From Eqs. (1), the denoised image is generated by the projection

$$P(g(x,y)) = \begin{cases} b_l(x,y) & g(x,y) < b_l(x,y) \\ b_u(x,y) & g(x,y) > b_u(x,y) \\ g(x,y) & \text{otherwise} \end{cases} \quad (2)$$

The above scheme implies that the intensity of the denoised image is bounded. Actual values of bounds differ at different regions of interest and are anti-propotional to the amount of noise removed. An in-depth explanation can be found in [6] and [7].

3. DEVELOPMENT OF THE NEW WEIGHTED ADAPTIVE LOCAL BOUNDS

One important drawback to the previous denoising approach using traditional local bounds is that some detail information could be lost, particularly for highly corrupted images.

Therefore our idea is to further adjust the amount of noise-suppression to the local information and produce the weighted adaptive local bounds, which is relevant to local features of a center pixel. This results in more detail information to be preserved.

In this paper, Let $|M[g(x,y)]|^p$ denote the local mean related measurement. One reliable example exploited in this paper is equal to $|e^{m_g}|^p$. Unlike the constant weight β in the traditional bounds, the new weight for the local bounds is tuned to be much larger if a local image feature is detected, such as edge or other highly structured parts, but slightly smaller for those flat uniform areas thus to remain the tightness in flat area but increase the looseness in other image details. The new local bounds hence could be demonstrated by

$$\begin{aligned} b_l(x,y) &= m_g(x,y) - |e^{m_g}|^p \, \frac{\sigma_g^2(x,y)}{\sigma_{gmax}^2} \qquad x,y \in \Omega \\ b_u(x,y) &= m_g(x,y) + |e^{m_g}|^p \, \frac{\sigma_g^2(x,y)}{\sigma_{gmax}^2} \qquad x,y \in \Omega \end{aligned} \quad (3)$$

in which the exponent $p \geq 1$ controls the relative strength of weights in different regions. Typically p should be selected up to such a value that $|e^{m_g}|^p$ shows an approximate linear weight for small values and weights significantly for large values. Ideally, the weighted local bounds-based denoising provides the same level of noise suppression as the original denoising in the uniform regions, while reducing loss of detail in other areas.

As it stands, the projection based on weighted adaptive local bounds proposed above can not produce reasonable solutions for outlier-like noise. Then a redefinition of projection is needed to improve the robustness of the algorithm.

4. REDEFINITION OF THE PROJECTION

Imposing bounds on the local image intensity [6-7] or thesholding the wavelet domain coefficients in alternative mathematical terms [1-5, 9-10] is equivalent to low pass filtering. It is indeed the case that most of the image denoising methods use a low pass filtering-type approach. This is not efficient when the additive noise has the form of very large or very small intensity values (spots). A representative example is the "Salt and Pepper" noise. This type of noise is removed effectively by the use of median filtering. In this section an extension of the bound formulation is presented, that aims to denoise an image that is corrupted by a combination of Gaussian and "Salt and Pepper" noise. The new pair of bounds and the resulting projection are defined as follows:

$$\begin{aligned} b_l(x,y) &= median_g(x,y) - |e^{m_g}|^p \, \frac{\sigma_g^2(x,y)}{\sigma_{gmax}^2} \qquad x,y \in \Omega \\ b_u(x,y) &= median_g(x,y) + |e^{m_g}|^p \, \frac{\sigma_g^2(x,y)}{\sigma_{gmax}^2} \qquad x,y \in \Omega \end{aligned} \quad (4)$$

$$G(x,y) = P_{new}(g(x,y)) = \begin{cases} b_l(x,y) & g(x,y) < b_l(x,y) \\ b_u(x,y) & g(x,y) > b_u(x,y) \\ g(x,y) & \text{otherwise} \end{cases} \quad (5)$$

Consistent with the idea of incorporating more representative local information into the bounds, the first term of local bounds, the mean value is replaced by the median. It is believed that for most of the local intensity distributions the median distorts the local intensity less than the mean. Moreover, the second term, efficient in removing Gaussian noise is kept same as the traditional bounds. By incorporating the median factor with the weighted component, the local bounds can also act to remove the Gaussian and "Salt and Pepper" noise and preserve sharp edges.

Better results can be obtained if applying a pre-processing prior to the local bounds projection. That procedure is called as outlier detector, based on the Mahalanobis distance $r = (\frac{G-m_G}{\sigma_G})^2$. The Mahalanobis distance quantifies the "inconsistency" between the center pixel G and neighbours, which have the values of mean and standard deviation equal to m_G and σ_G. The comparison of the Mahalanobis distance and the threshold will determine the existence of the outlier. If $r > T$ (T is a threshold determined empirically), the pixel, determined as a outlier, is replaced by the median in the output image, otherwise the pixel is used directly in the output image. Based on the pre-processing and two projections above mentioned, a comprpehensive denoising rule(**Weighted Adaptive Local Bounds Estimator (WALBE)**) is designed and can be tuned to produce more robust result in the case of different noise types.

5. EXPERIMENTAL RESULTS

In order to demonstrate the performance of the proposed Weighted Adaptive Local Bounds Estimator (WALBE) in situations of single noise and the mixed noise, we compared it with the following denoising methods: the linear wavelet estimator (LWE) [2], the bishrink dualtree CWT estimator (BDCE) [9], and the traditional local bounds Estimator (LBE) [6], on both real and synthetic images.

Since edge details represent most of the useful information present in an image, edge-preservation is always a basic indicator of the performance of all denoising algorithms. Besides, the Improvement in Signal to Noise Ratio (ISNR) is used as another quantative measurement, which is defined by $ISNR = 10log_{10} \frac{\|f-g\|^2}{\|f-\hat{f}\|^2}$, in which f, g, $\hat{f}$ represent the original image, noisy image and the denoised one respectively.

- Case I: The Single Noise Situation

In this case, an improvement in denoising quality is observed in all the test images. Both noisy images, as shown in (Figure 1 (a) and (f)) are corrupted by Gaussian noise with SNR equal to 20dB. It is bbserved from Figures 1 (a)-(j) that the poorest results in terms of noise amplification occur when using the LWE. Improvements are made when using BDCE, but the results are sub-optimal, particularly noticeable ringing artifacts around edges are present. Moreover, it is observed that the denoising method (LBE) based on the traditional local bounds enables a more accurate recovery of edge details than the BDCE. Furthermore, the introduction of WALBE (Figure 1 (e) and (j)) results in the removal of more artifacts still present in the resulf of LBE (Figure 1 (d) and (i)), while the detail-preservation is not jeopardized but more highlited.

Table 1 gives the comparison of quantative measurements ISNR

Estimator	Cameraman	Testpat1
LWE	5.11(dB)	6.06(dB)
BDCE	6.49(dB)	7.13(dB)
LBE	9.72(dB)	10.98(dB)
WALBE	**10.17(dB)**	**11.79(dB)**

Table 1. Performance Camparison

for all five algorithms in case I. It is evident that the WALBE outperforms the other methods in the case of single noise.

- Case II: The Mixed Noise Situation

When both Gaussian and "Salt & Pepper" noise corrupt the original image, the resulting noisy images are shown in Figure 2 (a) and (f). Consistently the performance of the LWE algorithm is still very poor, and moreover, the use of BDCE yields to the suppression of the Gaussian noise only. On the contrary, the use of sole median filtering, shown in Figure 2 (d) and (i) yields mainly to the suppression of the "Salt and Pepper" noise and it severly blurs the image. However, Figure 2 (e) and (j) demonstrates the superiority of the second proposed technique (WALBE) in removing both types of noise.

Estimator	Cameraman	Testpat1
LWE	2.45(dB)	2.80(dB)
BDCE	3.69(dB)	4.73(dB)
LBE	5.34(dB)	6.15(dB)
WALBE	**7.23(dB)**	**8.05(dB)**

Table 2. Performance Camparison for Case II

Similar with Table 1, Table 2 gives the comparison of quantative measurements ISNR for all five algorithms in case II, which demonstrates the satisfactory performance of , even in the case of mixed noise.

6. CONCLUSIONS

We have proposed a simple and effective use of weighted adaptive local bounds for denoising. The use of local information, such as mean, median and variance, enables an adaptive local estimation of the tightness of bounds. Besides, by redefining the local bounds and adding the pre-pocessing, the method WALBE is able to remove the mixed noise Gaussian and "Salt and Pepper" simultaneously, and the results show substantial improvement in terms of both visual quality and ISNR. As shown in all the examples, adapting the range of bounds allows us to keep the details, while eliminating most of the noise in smooth regions, regardless of the type of noise. This is usually not possible with other denoising methods, such as the wavelet thresholding. Thus, the modified WALBE provides a more robust and efficient framework for image denoising.

7. REFERENCES

[1] D. L. Donoho *Denoising by soft-thresholding* in *IEEE Trans. Inform. Th.* 41, pp. 613-627, May 1995

[2] E. P. Simoncelli, *Bayesian denoising of visual images in the wavelet domain*, in Bayesian Inference in Wavelet Based Models (P. Muller and B. Vidakovic, eds.), ch. 18, pp. 291-308, New York: Springer-Verlag, June 1999.

[3] S. G. Mallat, *A theory for multiresolution signal decomposition: The wavelet representation*, IEEE Pat. Anal. Mach. Intell., vol. 11, pp. 674-693, July. 1989.

[4] E. P. Simoncelli, and E. H. Adelson, *Noise removal via Bayesian wavelet coring*, In Third Int'l Conf on Image Proc, vol. I, pp. 379-382, Lausanne, Sptember 1996, IEEE Sig Proc Society.

[5] D. L. Donoho and I. M. Johnstone, *Ideal Spatial Adaptation via Wavelet Shrinkage*, Biometrika 81, pp. 425-455, 1994

[6] K. L. May, T. Stathaki and A. K. Katsggelos, *Spatially Adaptive Intensity Bounds for Image Restoration*. EURASIP Jounal on Applied Signal Processing Vol. 12, pp. 1167-1180, 2003

[7] M. C. Hong, T. Stathaki and A. K. Katsggelos, *Iterative Regularized Image Restoration Using Local Constraints*. In IEEE Workshop on Nonlinear Signal and Image Processing, Mackinac Island, ME, September 1997.

[8] G. Ramponi, *A simple cubic operator for sharpening an image*. Proc. 1995 IEEE Workshop on Nonlinear Signal and Image Processing, pp. 963-966, June 1995.

[9] L. Sendur and I. W. Selesnick. *Bivariate shrinkage functions for wavelet-based denoising exploiting interscale dependency*. IEEE Trans. on Signal Processing, vol. 50, no. 11, pp. 2744-2756, November 2002.

[10] S. G. Chang, B. Yu, and M. Vetterli, Fellow *Spatially Adaptive Wavelet Thresholding with Context Modeling for Image Denoising*. IEEE Trans. on Image Processing, vol. 9, no. 9, September 2000

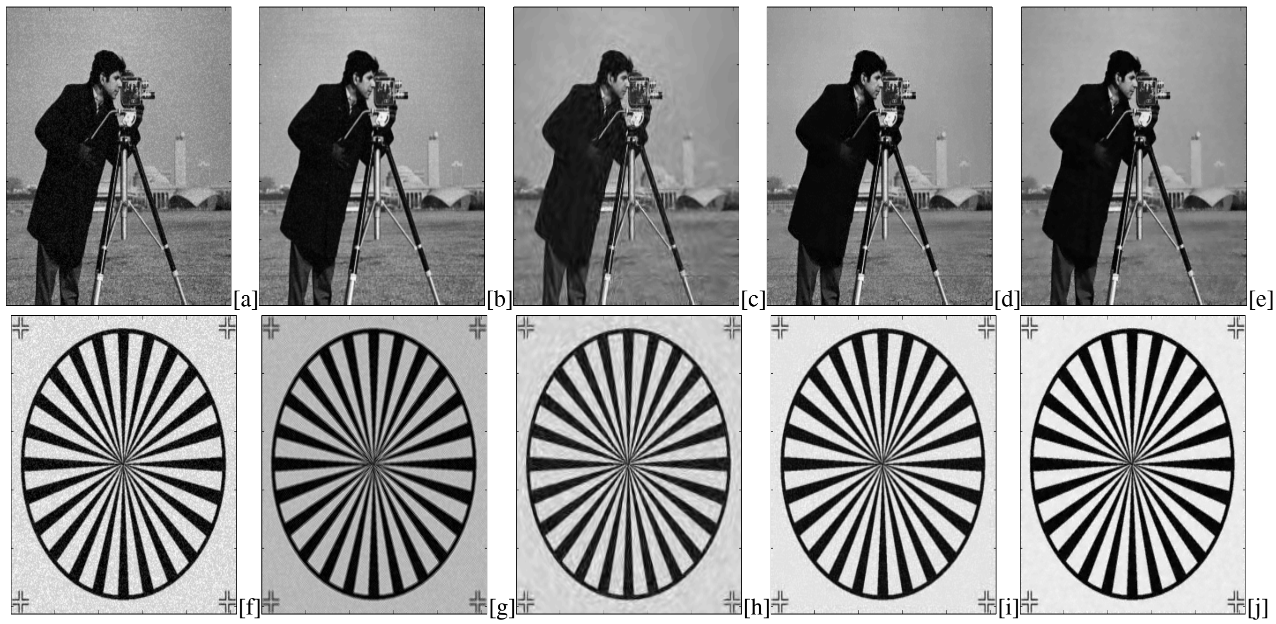

Fig. 1. For Case I: (a) and (f) The real and synthetic images corrupted by Gaussian noise. (b) and (g) The Linear Wavelet Estimator (LWE) for the real and synthetic image; (c) and (h) The Bishrink Dualtree CWT Estimator (BDCE) for the real and synthetic image; (d) and (i) The Local Bound Estimator (LBE) for the real and synthetic image; (e) and (j) The Weighted Adaptive Local Bound Estimator (WALBE) for the real and synthetic image.

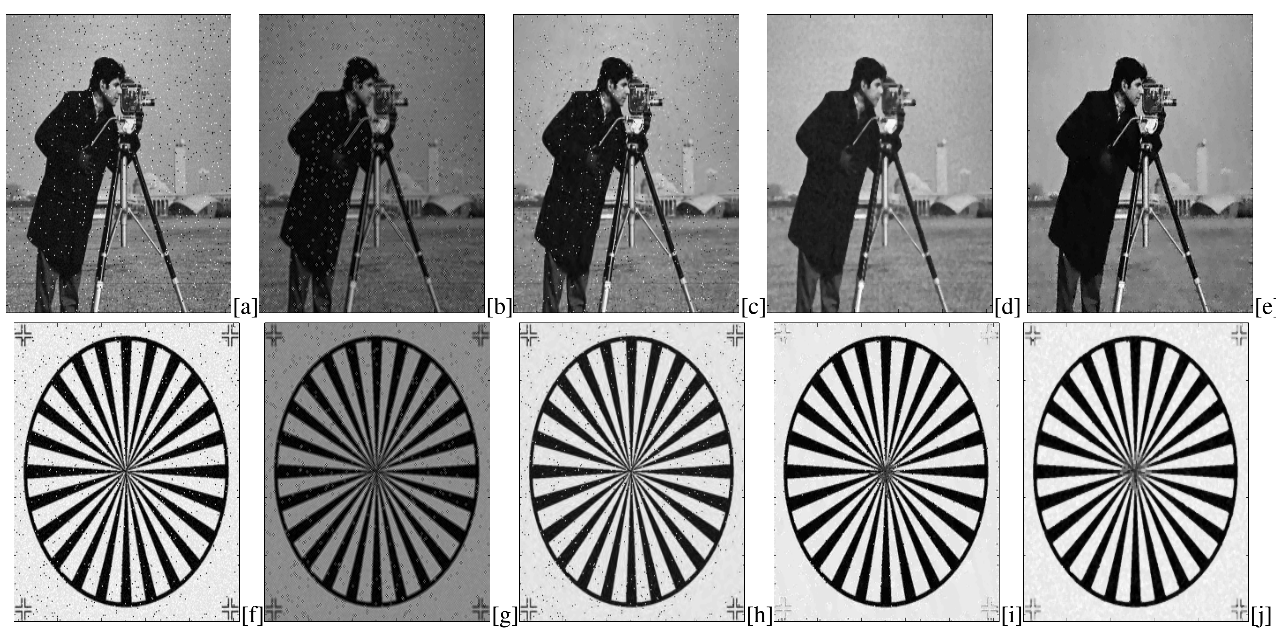

Fig. 2. For Case II: (a) and (f) The real and synthetic images corrupted by mixed noise. (b) and (g) The Linear Wavelet Estimator (LWE) for the real and synthetic image; (c) and (h) The Bishrink Dualtree CWT Estimator (BDCE) for the real and synthetic image; (d) and (i) The Local Bound Estimator (LBE) for the real and synthetic image; (e) and (j) The Weighted Adaptive Local Bound Estimator (WALBE) for the real and synthetic image.