WAVELET DOMAIN PARTITION-BASED IMAGE DENOISING

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ABSTRACT

Wavelet transforms are extensively used for image denoising and compression problems. The sparse property is the major reason for the effectiveness of the nonlinear operation such as thresholding of wavelet-transformed coefficients. Applying the thresholds to all coefficients uniformly, however, produces oversmoothing results on edges, or undersmoothing on uniform regions. Using statistical parameterized models often produces some artifacts because of overparameterizing. Recently, adaptive wavelet thresholding utilizing the correlation of space and adjacent scale has been introduced. This paper introduces a related, but more direct, technique of adaptively processing wavelet coefficients based on partitioning of the coefficient space. In wavelet domain, the coefficient space is partitioned by vector quantization method and the mask functions are used to obtain the denoised wavelet coefficients. Simulations show that the proposed technique yields the superior performance compared with current wavelet denoising methods.

1. INTRODUCTION

In the last decade, the wavelet thresholding method has attracted great attention from the image processing research community. Nonlinear operations based on the order of coefficients' magnitudes are nearly optimal for estimating signals with certain characteristics, such as piecewise smooth signals or bounded variations [1]. For denoising applications, Donoho proved that setting an appropriate universal threshold gives an upper bound of MSE risks assuming that the known noise statistics are Gaussian [2]. Other approaches are based on level dependent thresholds, i.e., utilizing a generalized cross validation criteria [3].

Applying thresholds uniformly to wavelet domains, however, does not utilize important characteristics of the wavelet transform, including the correlation of spatially adjacent coefficients and the correlation between adjacent scales' coefficients. Exploiting this information can significantly improve the performance of the denoising algorithm [4],[5] [6]. Statistical methods designed to exploit these correlations, however, often suffer from overparameterizing the wavelet coefficients models, thus yield artifacts along edges.

This paper introduces a wavelet domain denoising technique based on Partition-Based Filter (PBF) approaches [8]. Since wavelet transform detailed domain images all show directional high frequency information, such as edges, partitioning and classifying structures in the wavelet domains gives better performance in terms of finding true edge information in the underlying image. By applying partition (structure) based coefficient processing, wavelet domain partition based image denoising method shows better performance in both visual quality and error measures than current state of art wavelet denoising methods, such as GCV, hidden markov tree models.

The remaining sections are organized as follows. Section 2 reviews the image wavelet transform and thresholding methods. Section 3 explains the proposed wavelet domain partition-based image denoising method and it is followed by experimental results comparing with various wavelet denoising methods in section 4.

2. WAVELET TRANSFORM AND THRESHOLDING

The fast wavelet transform algorithm exploits the orthogonality of the bases and iterative nature of the transform such that

$$\begin{aligned} a_{j+1}(n) &= a_j \star hh(2n) \\ d^1_{j+1}(n) &= a_j \star hg(2n) \\ d^2_{j+1}(n) &= a_j \star gh(2n) \\ d^3_{j+1}(n) &= a_j \star gg(2n) \end{aligned} \tag{1}$$

where the $a_j(n)$ terms are coarse coefficients at scale j, $d^k_j(n)$'s are 3 directional detail coefficients at scale j for $1 \leq k \leq 3$, $\star$ is the convolution operator, h is the low pass filter for the scale function, g is the high pass filter for the wavelet functions, hh, hg, gh, hh are 2 dimensional scale and wavelet filters for images, and $2n$ corresponds to downsampling applied after the filtering.

Instead of downsampling prior to the next level decomposition, the Stationary Wavelet Transform (SWT) [1] upsamples the filters to produce a redundant signal decomposition [9]. The proposed algorithm uses the SWT to obtain redundant coefficients in order to estimate the uncorrupted coefficients.

Scharcanski *et al.* used the shrinkage function to obtain the thresholding effect [6] using Bayes theorem to acquire the probability of edge coefficients conditioned on the magnitude, where the conditional probabilities for each coefficient serves as the shrinkage functions. Since this algorithm depends on the magnitude of coefficients, the denoising performance significantly deteriorates as the noise variance increase. We use an approach similar to the shrinkage functions in order to adaptively reduce artifacts in neighborhoods of edges while successfully removing noise.

3. WAVELET PARTITION-BASED DENOISING

Partition based approaches exploits the structures that regularly occur in many signals [8] [10]. In the imaging case, vector quantization (VQ) is used to identify and define regularly occurring structures, such as edges at different orientations and uniform regions. Structures specific processing can then be applied. We extend these concepts to wavelet domain thresholding for denoisings.

3.1. Partition-Based Methods

Let **x** be an observation vector and R^N be the observation space to be segmented into a set of M mutually exclusive partitions, defined as $\Omega_1, \Omega_2, \cdots, \Omega_M$.

$$\begin{aligned} \Omega_i = \{\mathbf{x} \in R^N : \| \mathbf{x} - \mathbf{z}_i \|^2 \leq \| \mathbf{x} - \mathbf{z}_j \|^2, \\ \text{for } j = 1, 2, \cdots, M, j \neq i\}. \end{aligned} \tag{2}$$

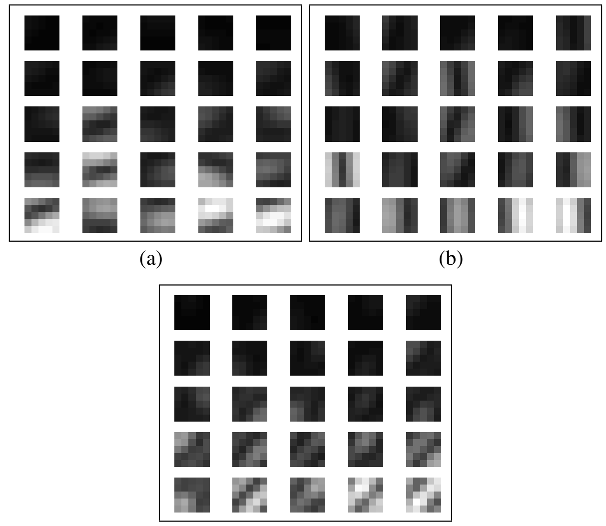
where $C = \{\mathbf{z_i}, i = 1, \cdots, M\}$ is a VQ codebooks and the partition index function is defined as $p(\mathbf{x}) = \arg\min_i \|\mathbf{x} - \mathbf{z}_i\|^2$.

3.2. Wavelet Domain Partition-Based Denoising

The Wavelet domain Partition-based Image Denoising (WPID) proposed here is a structure and coefficient magnitude specific thresholding approach.

3.2.1. Partitioning and Adaptive Thresholding in Wavelet Domains

The WPID exploits the structures of directional edge components by partitioning the space of SWT coefficients in the detail domains. Figure 1 shows the VQ generated codebooks of the level 3 detail wavelet coefficients where **x** is defined as the samples in a 5 by 5 window and $M = 30$ partitions are utilized. Since the range of the wavelet transform coefficients varies depending on the levels, we normalized values and arranged the partitions, numbered 1 to 30, based on the magnitude order of center codeword pixel values. Directional structures can be seen in each codebook. The significant signal components are concentrated in a small number of codewords that have large magnitudes in horizontal, vertical, and diagonal directions.



(a) (b) (c)

Fig. 1. Normalized codebooks of 256 by 256 lenna with 5 by 5 window. 6 - 30 partitions are numbered by increasing orders of center pixel magnitudes. (a)-(c) level 3 horizontal, vertical, and diagonal codebook.

We order codewords as noted above, by absolute magnitudes, in increasing order, and assign observed SWT vectors to partition based on distance to codebook vectors. The partition number 30 thus produces the largest signal energy. Hence if the coefficient belong to partition number 30, it most likely represents the underlying signal. In contrast, if the coefficient correspond to a low numbered partition, it likely correspond to noise and should be thresholded. This heuristic indicates that higher weights should to be assigned to higher partition numbers. The weight setting for the ordered partitions is explained in the next section.

3.2.2. Nonlinear Adaptive Denoising Mask for WPID

An adaptive thresholding approach is required to attain the crisp edges while maintaining smoother edge lines in an denoised image. Here, we utilize the concept of a Mask, $[0, 1]^{N_1 \times N_2}$ where $N_1 \times N_2$ is the dimension of the coefficients. This Mask is multiplied by the coefficients, producing coefficients shrinking. For example, if $M(i, j) = 1$ no shrinkage is applied, and if $M(i, j) = 0$, the corresponding coefficient is changed to 0. Hence the Mask function, $M(w)$, is a generalization of thresholding with greater freedom.

The Mask function can be considered as a Fuzzy mem-

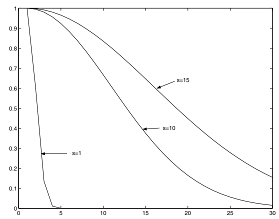


Fig. 2. Gaussian membership functions.

bership function, $FM[0,1]$. The membership function most widely used is a gaussian membership function,

$$\mu_G(\mathbf{x}, M) = e^{-(M-p(\mathbf{x}))^2/2\sigma^2} \tag{3}$$

where, in the case of here, M is the number of partitions, $p(\mathbf{x})$ is the partition index of the coefficient vector $\mathbf{x}$, and σ controls the spread of the membership function.

Figure 2 shows the gaussian membership function for σ = 1, 10 and 15. This approach yields a smoother shrinkage function than conventional hard or soft thresholding methods, which is controlled by the value of σ. The σ is obtained by training, and is dependent on the noise variance and the level of wavelet transform domain.

As noted earlier, the detail coefficients in the first level of the wavelet transform correspond mostly to noise. To capture this level dependent phenomena, we introduce a parameter, Confidence Value (CV_j) into the algorithm, where j is the level of wavelet transform. The CV_j monotonically increases as j goes ∞.

The Fuzzy Mask (FM_j) is thus calculated as

$$FM_j^k = \mu_G{}_j^k(\mathbf{x}, N) \cdot CV_j \tag{4}$$

where $j \geq 1$ indicates the level, and $1 \leq k \leq 3$ represent 3 detail domains.

The denoised wavelet coefficients are calculated as

$$\hat{\mathbf{w}}_\mathbf{j}^\mathbf{k} = FM_j^k \cdot \mathbf{w}_\mathbf{j}^\mathbf{k} \tag{5}$$

4. RESULTS

To illustrate the performance of WPID, we use several images comprising different characteristics. We compare our technique with the Wiener filter, which is a locally optimal FIR filter, the wavelet GCV (Generalized Cross Validation) technique [3], which uses GCV to obtain level dependent thresholds, and the Hidden Markov Models (HMM) method for estimating the denoised wavelet coefficients [10].

Although visual quality is the most important as a denoising performance measure, we use signal-to-noise ratio (SNR) as a numerical experimental performance measure, $SNR = 10 \cdot log_{10} \frac{|\mathbf{y}|^2}{|\mathbf{y}-\hat{\mathbf{y}}|^2} dB$. where $\mathbf{y}$ is an uncorrupted image, and $\hat{\mathbf{y}}$ is a denoisied image.

Table 1. SNR performance comparison of various methods

	σ = 20		σ =25	
	Lenna	**Aerial**	**Lenna**	**Aerial**
Noisy	6.6152	5.5920	5.08350	4.0665
Wiener	13.1035	8.6241	12.2739	8.1796
HMT	13.3871	9.5958	12.5129	8.7409
GCV	13.3104	7.6617	12.4115	7.1709
WPID	**13.8712**	**10.0411**	**12.9806**	**9.5230**

The parameter values are set heuristically in this study, and adaptive optimization methods are currently an area of study. For moderate SNR cases, $CV_1 = 0.1$, $CV_2 = 1$, and $CV_3 = 1$ are appropriate and utilized here. Also the spread parameters were set as σ_j, $\sigma_1 = 1$, $\sigma_2 = 5$, $\sigma_3 = 10$. Extensive experiments show that these parameters produce good denoising results in various natural images.

In all experiments, the Wiener filter result is the least effective for both visual quality and SNR. This indicates the effectiveness of multiresolutional techniques such as wavelet transform applications. Table 1 shows the experimental results of various i.i.d. Gaussian noise standard deviations.

Figure 3 shows that GCV is oversmoothing in the hairs of Lenna and HMM has some broken edges while having crisp edges than GCV. WPID shows crisp edges with reduced artifacts.

5. CONCLUSION

A new image denoising algorithm in wavelet transform domains, WPID, was developed using a partition-based approach. The effectiveness of the partition-base approach on non-stationary signals such as images combining with wavelet directional detail characteristics produce the adaptive thresholding capability for the denoising application.

Experiments show promising results both quantitatively and qualitatively, compared with some well known wavelet denoising techniques. Our result shows that the WPID can suppress noises in regular regions while preserving edges in the denoised images.

6. REFERENCES

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Fig. 3. Denoised image of 256 by 256 lenna. (a) Original Lenna (b) Noised with i.i.d Gaussian of s.d. 25 (c) Wiener filtering with 5 by 5 window (d)GCV estimation (e)HMT estimation (f) WPID estimation

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